# New Proportional Integral Controller for N<sup>th</sup> Order Transfer Function Model

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Hoo Choon Lih

Department of Mechanical and Materials Engineering Universiti Kebangsaan Malaysia Bangi, Malaysia e-mail: steven85@vlsi.eng.ukm.my

Edwin Chung Chin Yau
School of Engineering
Taylor's University
Subang Jaya, Malaysia
e-mail: edwin.chung@taylors.edu.my

Abstract—The windup phenomenon in control has results in performance degradation or even system instability. The existing methods manipulate the integral control in their own respective way when saturation occurred in order to bring the system back into the linear region. The proposed new proportional integral controller consists of a separately controlled integration which functions to seek for steady state value with respective to the input command and stored the value throughout the system operation. Simulations done for unloading cases for induction motor plant and also a higher order transfer function plant. The results show that the proposed controller gives a promising performance when compared with the existing methods.

Keywords-component; Anti-windup; proportional-integral (PI) control; tracking back calculation; steady state.

# I. INTRODUCTION

In variable speed motor drive, inner feedback loop is conducted by current control in order to obtain a fast dynamics and peak current protection, where its command is generated by the outer speed controller. The current command must be within a prescribed maximum value for the sake of preventing magnetic saturation, overheating of the motor and converter protection. However, in many cases such as sudden input change or when the motor encounter the disturbance or torque, the current command generated outbound the maxima range. For proportional integral derivative (PID) controller that works disregarding the input limit, windup phenomenon or saturation nonlinearity will occur in the speed control. The PID linear control will deteriorate when the saturation happened and motor performance exhibits large overshoot, longer time settling and even instability [1].

Although saturation problem has gained awareness among researchers, formal treatment on it only started from 1940s. As described in a review, researchers tried to investigate on the local stability or enforcing stability and the performance properties of the anti-windup compensator [1].

Sallehuddin Mohamed Haris
Department of Mechanical and Materials Engineering
Universiti Kebangsaan Malaysia
Bangi, Malaysia

e-mail: salleh@eng.ukm.my

Nik Abdullah Nik Mohamed

Department of Mechanical and Materials Engineering Universiti Kebangsaan Malaysia Bangi, Malaysia e-mail: enikkei@eng.ukm.my

Reference [2] discussed the details about conditioning technique and emphasis on the modified set point and back calculation. Filtered setpoint was introduced to overcome the short-sightness problem and other conditioning drawbacks. In the paper [3], they suggested a Variable-Speed PID (VSPID) Control to prevent integrator windup. Even though they received comment from [4] which stated that a better comparison should be done between other techniques at their respective tuning parameter performance. Yet, the VSPID is claimed to be able to return the control law to linear operation faster compared to the anti-windup bumpless transfer (AWBT) technique [5].

Among the techniques, conditional integration, tracking back calculation and integral state prediction schemes have been commonly discussed among researchers. The conditional integration switches off the integral control and remains the same value when controller output saturation occurred. The integral controller will again back to duty after the saturation back to the linear region again. In the meanwhile, tracking back calculation scheme integrates the error by using the difference between the saturated and the unsaturated control signals to generate a feedback signal to properly control the integral state to ensure the integral state did not exceed the control limit [6]. In the paper, the PI plane is very useful in evaluating the anti-windup PI controller as it denotes the behaviour of the system based on the PI controller output path. From the PI plane also, it is noted that the overshoot and the setting time greatly depend on the touching condition rather than the PI gains. For any antiwindup scheme, the rise time is mainly related to the control input limit during the saturation range.

In this paper, a new proposed PI controller utilising steady state integral controller is suggested for variable speed motor drives. The approach independently controls the integral controller with an inner PI controller to ensure the integral to carry a steady state value with regards to the respective speed condition. The proposed minimises or prevents from integral windup phenomenon which is the concern of many, as the integral will seek and stay at the

steady state. The integral will not fluctuates as the normal integral control does, and will remain in steady state value or the current limiting value when falls into the saturation region. However, this is still in the fundamental stage and in this paper, only variable speed unload condition is considered and compared with conditional integration, tracking back calculation and integral state prediction schemes through simulation.

# II. PI PLANE

PI plane is a plot of integral control against the proportional control value which indicates the controller trajectory at any instant of particular proportional and integral values. If the PI control output falls within the higher boundary,  $B_h$  and lower boundary,  $B_l$  lines, the controller is deemed to be in linear or unsaturated state and saturated if out bounded. As been discussed [7], the overshoot is influenced by the relative position of the integral state at the linear range boundary and the steady state value. The higher the integral state when touching the boundary compared to the steady state will apt to large overshoot. In the meanwhile, if the integral state is lower, a slow error response occurs. This is shown by the respective PI controller value trajectory in Fig. 1 where  $q_o$  and  $q_{ss}$  denote the linear range boundary integral error and steady state integral error respectively.

It is expected to have an overshoot whenever the trajectory moves into the negative  $k_p e$  region given a positive input. For the  $q_o \approx q_{ss}$ , the trajectory becomes shorter to reach the steady state condition.

# III. STEADY STATE INTEGRAL

By the state of the art of PI controller, the controller will have its tuning parameters that bring the system response into the desired input reference. The error obtained from the feedback signal reaches zero when the system attain the required output and the controller will also carry a steady state value as depicted below.

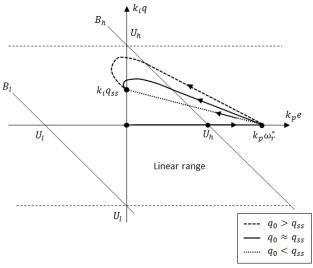


Figure 1. LINEAR PLANE AND ERROR TRAJECTORIES ON A PI PLANE.

# A. Common Closed Loop System

Fig. 2 shows the common block diagram for a closed loop system with its controller and system plant. The system response transfer function is:

G(s) = Transfer function for plant

 $k_p$  = Proportional tuning parameter

 $\vec{k_i}$  = Integral tuning parameter

K(s) = Transfer function for PI controller

 $Y_p(s)$  = Transfer function for plant output

 $\dot{Y}(s)$  = Transfer function for PI controller output

U(S) = Transfer function for step input

$$Y_p(s) = \frac{U(s)K(s)G(s)}{I + K(s)G(s)}$$
 (1)

The controller output signal can be obtained as follow.

$$Y(s) = \frac{U(s)K(s)}{I + K(s)G(s)}$$
(2)

Investigate the pi controller output value when reaching steady state for different plant transfer function types.

Case 1: 
$$G(s) = \frac{A}{Bs + C}$$

$$\frac{Y(s)}{U(s)} = \frac{(k_p s + k_i)(B s + C)}{s(B s + C) + (k_p s + k_i)A}$$
(3)

When steady state occurs:

$$\lim_{s \to 0} sY(s) = \frac{UC}{A} \tag{4}$$

Case 2: 
$$G(s) = \frac{As+B}{Cs^2+Ds+E}$$

$$\frac{Y(s)}{U(s)} = \frac{(k_p s + k_i)(Cs^2 + Ds + E)}{s(Cs^2 + Ds + E) + (k_p s + k_i)(As + B)}$$
(5)

When steady state occurs:

$$\lim_{s \to 0} sY(s) = \frac{UE}{B} \tag{6}$$

In general for any n<sup>th</sup> order transfer function plant:

$$G(s) = \frac{a_{m+l}s^m + a_{m}s^{m-l} + a_{m-l}s^{m-2} + \dots + a_{l}s^0}{b_{n+l}s^n + b_{m}s^{m-l} + b_{n-l}s^{m-2} + \dots + b_{l}s^0}$$
(7)



Figure 2. BLOCK DIAGRAM OF A CLOSED LOOP SYSTEM.

$$\lim_{s \to 0} sY(s) = \frac{Ub_I}{a_I} \tag{8}$$

Where  $a_i$  and  $b_j$  with i = 1, 2, ..., m+1, j = 1, 2, ..., n+1 are the coefficients, m and  $n \in \mathbb{N}^+$ ,  $m \supseteq n$ .

Equations (4), and (6) show that only the integral control will carry a steady state value that comprises the input reference and the constant proportion for any system plant as given by (7) and (8).

# IV. PROPOSED NEW PI CONTROLLER

The integral as analysed in PI plane shows different trajectory characteristics when the integral carries certain value relative to the steady state value. As evaluated and proved in integral state prediction scheme [7] where a better performance can be obtained if the integral reaches the steady state value when touching the linear range boundary before back into the unsaturated region. The proposed tends to bring the same concept by carrying the steady state value with respect to the input command.

The proposed will have a conventional PI controller but channeled with different reference input. The proportional control will be directed with the error between the input reference and system response, while the integral control input is sourced with the steady state value obtained from section III and controlled by another PI controller in a closed loop. Only the integral control part of the closed loop will be transferred as the integral control of the whole proposed system.

In order to prevent the integral windup phenomenon, the tracking back calculation is adopted in the integral control closed loop as depicted in Fig. 3. The integral is controlled using a modified tracking back calculation controller where only the integral part is sourcing out to the system. The closed loop and P controller work to prevent or reduce the integral windup which differs from the existing that corrects the integral whenever the PI controller saturated.

$$\frac{Y_{p}(s)}{U(s)} = \frac{Z(\frac{k_{i}}{s(1+k_{p})+k_{i}})G(s)+k_{p}G(s)}{1+k_{p}G(s)}$$
(9)

# V. SIMULATION RESULTS

The proposed is applied for speed control of an induction motor and compared with the conditional integration, tracking back calculation and integral state prediction schemes through simulation in MATLAB simulink. The specification of the induction motor is as given in Table 1.

Simulations were done for two different plant transfer function. The first simulation was tested for speed control which consists of a first order transfer function with specification given in Table 1.

The speed command is set to 1500 rpm from the start and then stepped down to -1500 rpm after t = 5 s. The PI gains used are  $k_p = 12.3$  and  $k_i = 130$ . The bandwidth parameter  $\omega_i$  is chosen as 1/(0.5\*rise time) and with a current limiter of

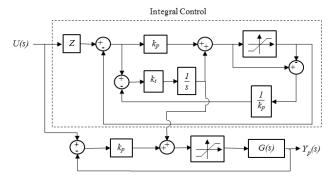


Figure 3. BLOCK DIAGRAM FOR PROPOSED PI CONTROLLER WITH MODIFIED TRACKING BACK CALCULATION IN THE INTEGRAL CONTROL.

5.54 A. The plant transfer function is given in the form of (10) which is the usual variable speed motor drive.

$$G(s) = \frac{1}{J_s + R} \tag{10}$$

Fig. 4 shows the simulation result of the proposed controller compared with the existing tracking back calculation, conditional integration and integral state prediction schemes. In general, all the methods behave similarly and approach the desired input command. Each technique differs in terms of controlling their integral part during saturation. The integral of the tracking back operates at the maximum limiting value as set by the limiter. The conditional integration switches off its integral and the integral state prediction will seek for the steady state value before the system resume back to the linear region. In the meanwhile, the proposed will always look for the steady state value with respect to the input command and carry the value regardless of the linearity region. Hence, the proposed PI controller's output attains the steady state condition faster compared to the others.

An attempt also been done on testing the feasibility of the proposed method on higher order transfer function plant. The same parameters were utilised and the system plant used is given by (11).

$$G(s) = \frac{1}{0.005s^2 + 0.005s + 0.0008}$$
 (11)

Results are shown in Fig 5. The system undergoes nonlinearity when encountering sudden ramp up to such a high step input. All the system responses converge, however the proposed is seen to converge faster compared to the others. This is due to the consistent steady state value of the integral term of the controller, whereas the rest are fluctuating as in tracking back calculation and integral state

TABLE I. PARAMETERS OF INDUCTION MOTOR

Characteristics	Value
Moment of inertia ( <i>J</i> )	$5.0 \times 10^{-3} [\text{kg.m}^2]$
Viscous damping coefficient (B)	$0.8 \times 10^{-3} [kg.m^2/s]$
Time constant $(\tau_m)$	6.25 s
Torque constant $(k_T)$	1.27 [N.m/A]

prediction and switched off as for conditional integration. The proposed back into the linear unsaturated region earlier than the rest.

# VI. CONCLUSION

The proposed PI controller introduces another antiwindup controlling method. The separately sourced integral control closed loop incorporated with the modified tracking back calculation scheme ensure a steady state value of the respective input command be found and keep throughout the operation under unloading condition. Only the integral signal will be transferred into the system from the above closed loop and engaging with the P controller directed with the error, will make a PI controller. As compared with the existing methods, the proposed shows some promising results. The proposed reaches the steady state condition earlier and the integral control will not fluctuate as seen in the concerned methods. The method can be applied to higher order transfer function as well and this indicates the flexibility in implementing on other applications. However, this is still in the fundamental stage and test only been done on unloading cases. With the promising result, the proposed PI controller will be studied and expand to loading case and tested in hardware.

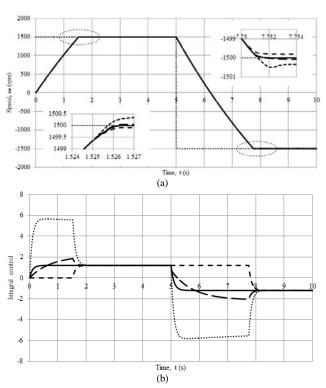


Figure 4. SIMULATED COMPARISON OF ANTI-WINDUP SCHEMES FOR FIRST ORDER PLANT SYSTEM WITH CHANGING STEP INPUT (SQUARE-DOTTED: TRACKING BACK CALCULATION, SHORT-DASH: CONDITIONAL INTEGRATION, LONG-DASH: INTEGRAL STATE PREDICTION, SOLID: PROPOSED SCHEME). (A). SPEED. (B). INTEGRAL CONTROL.

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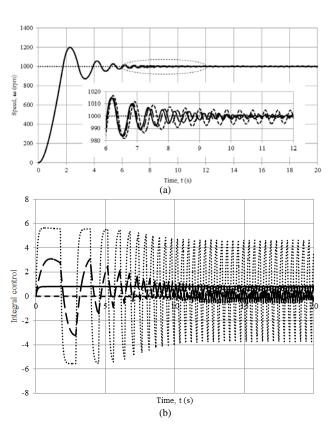


Figure 5. SIMULATED COMPARISON OF ANTI-WINDUP SCHEMES FOR SECOND ORDER PLANT SYSTEM WITH CONSTANT STEP INPUT (SQUARE-DOTTED: TRACKING BACK CALCULATION, SHORT-DASH: CONDITIONAL INTEGRATION, LONG-DASH: INTEGRAL STATE PREDICTION, SOLID: PROPOSED SCHEME). (A). SPEED. (B). INTEGRAL CONTROL.