

Improved Water Cycle Algorithm with Chaotic Levy Distribution Applied to Engineering Problem

Weiguo Zhang and Jinzhong Zhang*

School of Computer Science, Xi'an University of Science and Technology, China

*Corresponding author

Abstract—The water cycle algorithm(WCA) is a simple and effective global optimization algorithm, mainly used for engineering optimization. However, It is easy to fall into the local optimal solution when solving some constrained problems. Therefore, this paper proposes a improved water cycle algorithm based on chaotic Levy distribution. The improved water cycle algorithm was named as CLWCA, and chaos sequence was used as step control parameter based on Levy distribution in combination with the perturbation of Levy distribution and the boundedness of chaotic sequence, in order to enhance the global searching ability of the algorithm. In this paper, CLWCA is applied in the field of engineering model design and compared with the traditional water cycle algorithm(WCA) through experiments. The result verified that the CLWCA algorithm has better global search ability and stability than WCA algorithm in the field of industrial model design.

Keywords—water cycle algorithm; chaotic sequence; levy distribution; engineering model

I. INTRODUCTION

In the past decades, the research of intelligent algorithms has been widely concerned by scholars [1]. The concepts of intelligent algorithms stems from the biological characteristics of nature or the theory of natural phenomena, and the optimal solution is obtained through the interaction between groups. Its application fields are extensive, and the fields involved are machine learning, artificial intelligence, data mining and engineering optimization [1]. Among them, the intersection of engineering and mathematics has produced many mathematical models, so the solution of mathematical models has become the mainly focus, such as: function extremum problem [2], nonlinear equations solving problem [3], more Target optimization problem [4] and so on. The traditional algorithm has strong dependence on the initial point and is easily affected by the nature of the function, which makes it impossible to obtain a more accurate solution. It also has no way to deal with the problem of the root of the nonlinear equations [3]. In order to solve the problems above, in the 1860s, many researchers began to conduct in-depth research in the field of intelligent optimization algorithms [5].

At present, mature intelligent algorithms include genetic algorithms, evolutionary algorithms, particle swarm optimization algorithms, simulated annealing algorithms, moths and fire-fighting algorithms, and water cycle algorithms [5-9]. Compared with other algorithms, the water circulation algorithm has the advantage of using evaporation conditions to prevent the algorithm from falling into the local optimum, so the

water cycle algorithm has attracted the attention of many researchers. The idea of water cycle algorithm comes from the natural water cycle process. The core of the algorithm is to use evaporation conditions to prevent the search of the optimal solution from falling into the local optimum. And water cycle algorithm has many advantages, such as fewer parameters, simple ideas, easily to understand, and the relatively simple process of program implementation, but the global search ability is poor when solving some problems [6]. In 2015, Ali Sadollah et al. proposed the water cycle algorithm with evaporation rate, which adopted the idea that partial evaporation exists in the process of stream flowing into river, and further improved the global search ability of the water circulation algorithm through this intentions [7]. Based on the previous studies, this paper proposes a water cycle algorithm (CLWCA) optimized by chaotic Levy distribution, the chaotic sequence is used as the step size control parameter of Levy distribution, with the help of a Levy distribution of perturbation to update streams, which is regarded as the evaporation condition of water cycle algorithm, aimed at strengthening water cycle algorithm in global search ability. This algorithm was applied to the two engineering examples in order to verify CLWCA global search ability of the algorithm.

II. WATER CYCLE ALGORITHM

The water cycle algorithm [6] (WCA) simulates the water cycle process of natural rainfall, confluence, and evaporation. The initial population N_{pop} is generated by the rainfall process, and the initial population is divided into three grades, which are sea, river and stream from the optimal to the worst. And the sea is the optimal solution in the confluence process. In the confluence process, some better rivers flow directly into the sea, and the rest of the streams are Into the river, the river flows directly into the ocean, and the ocean is continuously updated through the confluence process. If the evaporation condition is reached, the stream will be regenerated and then confluent, Repeat the above process until the termination condition is met.

Flow direction of streams is determined by flow intensity calculation, and the calculation formula is as shown in equation (1):

$$NS_n = \text{rond} \left\{ \frac{Cost_n}{\sum_{i=1}^{N_{sr}} Cost_i} \times N_{Raindrops} \right\}, n = 1, 2, \dots, N_{sr} \quad (1)$$

The process of the stream flowing into the river, each stream flowing into the corresponding river, the iterative formula is as shown in equation (2):

$$X_{Stream}^{i+1} = X_{Stream}^i + rand \times C \times (X_{River}^i - X_{Stream}^i) \quad (2)$$

The process of the river flowing into the sea gradually approximates the optimal solution, and the iterative formula is as shown in equation (3):

$$X_{River}^{i+1} = X_{River}^i + rand \times C \times (X_{Sea}^i - X_{River}^i) \quad (3)$$

III. LEVY DISTRIBUTION AND CHAOTIC SEQUENCE

A. Levy Distribution

The Levy distribution was proposed by the French mathematician Paul Lévy in 1937 [11], and the Levy distribution has been widely used in the iterative process of intelligent algorithms. The jump step size of the Levy distribution is random, and the short-distance jump with large probability is accompanied with a small probability of long-distance jump. The short-range jump can enhance the local search ability of algorithm, and the long-distance jump helps to jump out the local optimal solution and improve global exploration ability [12]. The calculation of the original formula of Levy distribution is more complicated, so this paper uses the Levy distribution calculation formula derived by Mantegna algorithm [11]:

Levy distribution calculation formula:

$$Levy(\lambda) = \frac{r_1 \times \delta}{|r_2|} \quad (4)$$

$$\delta = \left(\frac{\Gamma(1+\beta) \times \sin \frac{\pi\beta}{2}}{\Gamma(\frac{1+\beta}{2}) \times \beta \times 2^{\frac{\beta-1}{2}}} \right) \quad (5)$$

Where: r_1 and r_2 obey the standard normal distribution, $\Gamma(x)$ is the standard Gaussian function, and β takes the empirical value of 1.5. Figure I shows the change of levy step varies with the number of flights within 100 flights.

B. Chaotic Sequence

The chaotic sequence is a deterministic process that produces sequence values between 0 and 1, which are bounded, non-periodic, and non-convergent. The chaotic sequence used in this paper is the classical Logistic map sequence [12]. The variation of the chaotic sequence with the number of iterations is shown in Figure II, and the formula is shown in equation (6).

$$x = 4x(1-x) \quad (6)$$

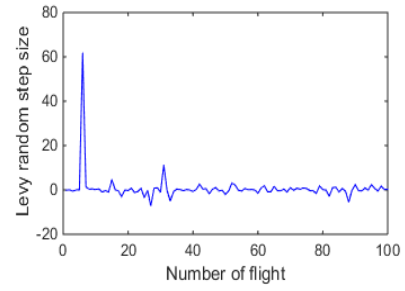


FIGURE I. LEVY STEPS CHANGE WITH THE NUMBER OF FLIGHTS

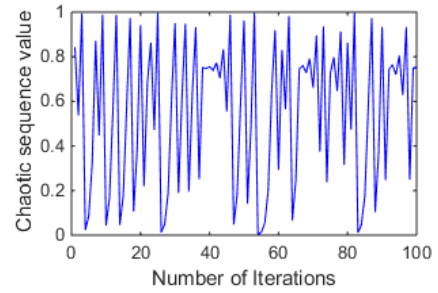


FIGURE II. THE CHAOTIC SEQUENCE CHANGES WITH THE NUMBER OF ITERATIONS

IV. WATER CYCLE ALGORITHM FOR CHAOTIC LEVY DISTRIBUTION (CLWCA)

Compared with other intelligent algorithms, WCA has mature technology in solving constrained and unconstrained problems, but it occasionally falls into local optimization in solving certain problems. In order to solve these problems. This paper introduces a evaporation condition based on chaotic levy distribution, because of the levy strong disrupting, it can promote the randomness of the water cycle algorithm, and it is also beneficial to improve the global search ability. Using chaotic sequences as step control parameters of Levy distribution, the position of some streams is updated by using chaotic Levy distribution. The updated formula for stream location is as follows:

$$X_{Stream}^{i+1} = X_{Stream}^i + Levy(\lambda) \otimes x \quad (7)$$

The specific description of the algorithm as follows:

Step1: Initialize related parameters: \max_it , N_{sr} , N_{pop} , d_{\max} ;

Step2: Randomly generate initial populations, calculate their fitness values, and form sea, rivers, and streams.

Step3: Use equation(8) to calculate the flow intensity.

$$NS_n = \text{rond} \left\{ \frac{Cost_n}{\sum_{i=1}^{N_{sr}} Cost_i} \times N_{Raindrops} \right\}, n = 1, 2, \dots, N_{sr} \quad (8)$$

Step4: Use equation (9) to realize the process of some streams flowing into the river.

$$X_{Stream}^{i+1} = X_{Stream}^i + rand \times C \times (X_{sea}^i - X_{Stream}^i) \quad (9)$$

Step5: If the stream flowing into the sea gives a better solution, exchange the position of the stream and the sea, otherwise, do not exchange.

Step6: Use equation (10) to cause part of the stream to flow into the corresponding river.

$$X_{Stream}^{i+1} = X_{Stream}^i + rand \times C \times (X_{River}^i - X_{Stream}^i) \quad (10)$$

Step7: If the stream gives the optimal solution, exchange the position of the stream and the river, otherwise, do not exchange.

Step8: Use equation (11) to realize the process of rivers which are entering the sea.

$$X_{River}^{i+1} = X_{River}^i + rand \times C \times (X_{Sea}^i - X_{River}^i) \quad (11)$$

Step9: Compared with the sea, if the river gives a better solution, exchange the position of river and sea, otherwise, do not exchange.

Step 10:Form evaporation condition, update the location of stream flowing into the ocean by equation(12).

$$X_{Stream}^{i+1} = X_{Stream}^i + Levy(\lambda) \otimes x \quad (12)$$

Step 10: Determine whether the evaporation condition is satisfied or a random number between 0 and 1. If it is satisfied, the rain process is realized, and the river is regenerated by the formula (13).

$$X_{Stream}^{new} = LB + (UB - LB) \times rand(1, nvars) \quad (13)$$

Step11: Determine whether the evaporation condition is met. If it is satisfied, it will enter the raining process and regenerate the stream through formula (14).

$$X_{Stream}^{new} = sea + \sqrt{0.1} \times randn(1, nvars) \quad (14)$$

Step12: Use equation(15) to decrease the value of d_{max} .

$$d_{max}^{i+1} = d_{max}^i - \frac{d_{max}^i}{max_it} \quad (15)$$

Step13: If the convergence condition is met, the iteration will be stopped, otherwise return to step 4.

V. ENGINEERING APPLICATIONS

The following two experiments of engineering model design optimization are implemented based on matlab. The Runtime environment is Inter(R) Core(TM) i5-6300u CPU @2.40ghz 2.50ghz, and 8.00gb RAM. In these experimental processes, for each optimization model experiment, the model was iterated for 1000 times as the result of one run. In order to guarantee the generalization and accuracy of the data, CLWCA algorithm and WCA algorithm were used to run independently for 30 times and record the experimental results, and their average value and variance were calculated.

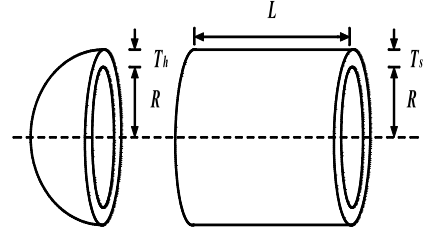


FIGURE III. PRESSURE VESSEL MODEL

A. Pressure Vessel Model

Pressure vessel is a commonly used engineering optimization model whose main purpose is to reduce the cost. The schematic diagram of the model is shown in Figure III. The main body of the vessel consists of a hollow cylinder and a hemispherical top cover. The inner radius of the bottom surface of the hollow cylinder is R , the wall thickness is T_s , the cylinder length is L , the hemispherical top cover has an inner radius of R and a wall thickness of T_h . The mathematical model is as follows:

$$f(x) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3$$

$$g_1(x) = -x_1 + 0.0193x$$

$$g_2(x) = -x_2 + 0.00952x_3 \leq 0$$

$$g_3(x) = -\pi x_3^2 x_4 - 4/3\pi x_3^3 + 1296,000 \leq 0$$

$$0 \leq x_i \leq 100(i=1,2)$$

$$10 \leq x_i \leq 200(i=3,4)$$

Figure IV shows the tendency about the iteration number and function value of CLWCA and WCA in the process of optimizing pressure vessel model design. The function value of the CLWCA is always lower than WCA within 100 iterations.

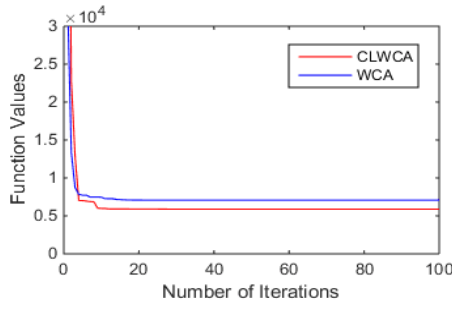


FIGURE IV. OPTIMIZATION COMPARISON OF THE PRESSURE VESSEL MODEL

Table I shows the average experimental results after 30 times operation of CLWCA and WCA in the process of pressure vessel model design. According to results, the maximum value and minimum value of CLWCA are similar to WCA, however, the mean value and the variance of CLWCA are better than WCA.

TABLE I. PRESSURE VESSEL MODEL DESIGN ITERATION 30 TIMES AFTER THE EXPERIMENTAL RESULTS

Methods	Wrost	Best	Mean	SD
WCA	7.319001e+3	5.885333e+3	6.294742e+3	4.606492e+2
CLWCA	7.319012e+3	5.885341e+3	6.205963e+3	3.450080e+2

B. Multi-disc Clutch Brake Model

The multi-disc clutch brake model is primarily designed to solve (evaluate) the dynamic bearing capacity of rolling bearings. The better performance of brake model related to greater bearing capacity, so the results obtained by the brake model are as large as possible. The schematic diagram of the model is shown in Figure V. The pitch diameter is D_m , the diameter of the small ball is D_b , the number of small balls is Z , the curvature coefficients of the inner diameter and the outer track are f_i and f_o , respectively, the other correlation coefficients are: K_{dim} , $K_{D_{max}}$, ε , e And ζ , the constraints stem from the knowledge of dynamic principle and manufacturing, the specific mathematical model is as follows:

$$f(x) = \pi(r_o - r_i)t(Z + 1)\rho$$

$$g_1(x) = r_o - r_i - \Delta r \geq 0$$

$$g_2(x) = l_{max} - (Z + 1)(t + \delta) \geq 0$$

$$g_3(x) = p_{max} - p_{rz} \geq 0$$

$$g_4(x) = p_{max}v_{srmax} - p_{rz}v_{sr} \geq 0$$

$$g_5(x) = v_{srmax} - v_{sr} \geq 0$$

$$g_7(x) = M_h - sM_s \geq 0$$

$$g_8(x) = T \geq 0$$

$$M_h = \frac{2}{3}\mu FZ \frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}, \quad p_{rz} = \frac{F}{\pi(r_o^2 - r_i^2)}$$

$$v_{rz} = \frac{2\pi n(r_o^3 - r_i^3)}{90(r_o^3 - r_i^2)}, \quad T = \frac{I_z \pi n}{30(M_h - M_f)}$$

$$\Delta r = 20mm, I_z = 55kgmm^2, p_{max} = 1Mpa,$$

$$F_{max} = 1000N, T_{max} = 15s, \mu = 0.5, s = 1.5,$$

$$M_s = 40Nm, M_f = 3Nm, n = 250rpm,$$

$$v_{srmax} = 10m/s, l_{max} = 30mm, r_{imin} = 60, r_{imax} = 80,$$

$$r_{omin} = 90, r_{omax} = 110, t_{min} = 1.5, t_{max} = 3,$$

$$F_{min} = 600, F_{max} = 1000, Z_{min} = 2, Z_{max} = 9.$$

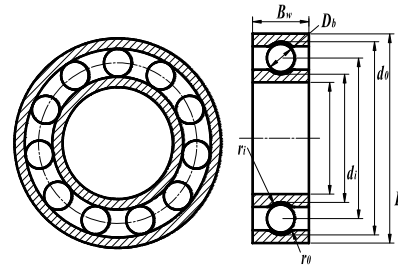


FIGURE V. MULTI-DISC CLUTCH BRAKE MODEL

Figure VI shows the tendency of iteration numbers and function values of CLWCA algorithm and WCA algorithm in the process of optimizing the multi-disc clutch brake model design. The results shows that the function value of CLWCA algorithm is significantly better than WCA algorithm within 100 iterations.

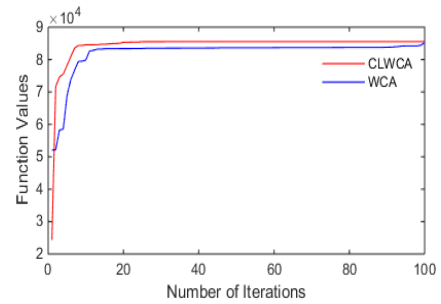


FIGURE VI. OPTIMIZATION COMPARISON OF MULTI-DISC CLUTCH BRAKE MODEL

The result in Table II are the average experimental data of the CLWCA algorithm and WCA algorithm after running for 30 times in the process of optimizing the multi-disc clutch brake model design. According to the data in Table II, the maximum value and minimum value of CLWCA are the same as WCA, and the average value and variance are both better than WCA, which are $8.552703e+4$ and $5.822139e+1$ respectively.

TABLE II. COMPARISON OF 30 EXPERIMENTAL RESULTS OF MULTI-DISC CLUTCH BRAKE MODEL

Methods	Best	Worst	Mean	SD
WCA	$8.553919e+4$	$8.522068e+4$	$8.547548e+4$	$1.295784e+2$
CLWCA	$8.553919e+4$	$8.522068e+4$	$8.552703e+3$	$5.822139e+1$

VI. CONCLUSION

According to the experimental results in Figure IV and Figure VI, CLWCA is not easy to fall into local optimization in the iterative process. The global exploration ability is stronger than WCA, and the convergence speed is faster than WCA. WCA is easy to fall into the local optimum in the iteration process and the convergence speed is slower. From the comparison of Table I, CLWCA has a little weaker ability to in local search. According to the results of Table I and Table II, the average value and the variance of CLWCA are smaller, indicated that the solutions obtained by CLWCA after running several times are closer to the global optimal solution, that is, the stability is better than WCA.

In general, CLWCA offers stronger global search capability and higher stability, but its local search capability is relatively weak. In the further studies, a better local search capability is need to be seek while ensuring the high global search capability.

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