

# Optimization of Costs Function for Prevention of Firms' Industrial Risks With Penalties

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**Abstract**—The paper investigates the problem of searching for firm's costs function which minimizes the industrial risk of the firm. The framework considers the fiscal penalties for the environmental damage and the civil action penalties for individuals' property damages. We analyze an influence of the production costs function's parameters on the internal and external damage functions. Additionally, for the solution of profit maximizing problem, we analyze the influence of the product price on the risk costs.

**Index Terms**—industrial risk, industrial damages, costs, optimization, risk management

## I. INTRODUCTION

In industrial production, the problem of the risk management is relevant. The risk costs can be caused by the penalties and the environmental damage, which may be classified under the internal damage and the external damage. The internal damage causes a reduction in the firm's assets. The external damage is the property wastes of other firms, individuals and environment. Additionally, the fiscal penalties depend on the value of the external damage, and it is the inverse relationship.

Problems of the risk management efficiency were investigated by the following authors. Some authors investigated the risk of industrial firms in the technical aspect [1], and some authors analyzed this risk in the economic aspect [2]–[6]. Finally, this risk was considered as an abstract mathematical problem [7]. Additionally, the risk problems were studied at macroeconomic level [8], the industrial risk was considered at regional level in comparison with individual firms [9]–[11]. For example, Shelkov A. B. analyzed the risk as the regional security problem and carried out an analysis by scenario method [12]–[14]. The risks of industrial firms were analyzed, considering a factor of human errors [15], on the basis of the business games [7], and the penalty mechanisms [9]. Additionally, industrial risk was minimized by the implementation of the innovation [10], [11]. The economic results of risk factors were investigated [16], [17].

In this paper the risk is considered at individual firm level. The penalties, caused by the external damage, include the following types the ecology payments; the penalties for damage to the health and the property of other firms and individuals; the compensations which are paid for the non-fulfillment of contracts.

The internal damage influences the external damage because the large-scale accidents increase in a probability of the external damage. The external damage, the internal damage

and the penalties can be reduced if probability of accidents decreases. For example, the harmful emissions volume of firms depends on the production volume and on the capacity of cleansing structures. The production growth leads to increasing in the harmful emissions. The improvement of the cleansing structures efficiency depends on their capacity and operation costs. A decrease in the possible internal damage can be provided by additional expenses on the risk reduction. Henceforth, these expenses are named the voluntary risk costs (VRC). We consider the problem of searching for the VRC function which optimizes the total costs with penalties. Additionally, we investigate the optimal production volume, accounting the penalties and the damages.

## II. PROBLEM FRAMEWORK

We search for a nonnegative, real, and limited from above VRC function, minimizing the total costs function of the firm with the following conditions:

$$f^*(Q) = \arg \min_{f(\cdot) \in A_f} C_{\Sigma}(Q, f(\cdot)), \quad (1)$$

$$A_f = \{f(\cdot) \in R^+ : f(\cdot) \leq f^{\max}, f^{\max} \in (0, C^{\max})\},$$

$$\begin{aligned} C_{\Sigma}(Q, f) &= C(Q) + f(Q) + H(Y) + X(Q, f), \\ C_{\Sigma}(\cdot) &\leq C^{\max}, \end{aligned} \quad (2)$$

where  $Q$  is the production volume,  $C_{\Sigma}(Q, f)$  is the firm's total costs function,  $C(Q)$  is the production costs function,  $f(Q)$  is the VRC function,  $f^{\max}$  is the VRC maximum possible value,  $X(Q, f)$  is the internal damage function, and  $C^{\max}$  is the maximum possible total costs. The symbol “\*” denotes the optimum values.

We formulate a problem of firm's choice as follows: to search for the production volume that maximizes the profit, considering the optimal VRC function (1), that is:

$$Q^* = \arg \max_{Q \in A_Q} (Q, f^*(Q)), \quad (3)$$

$$A_Q = \{Q \in R^+ : Q \leq Q^{\max}, Q^{\max} > 0\},$$

$$\Pi(Q, f) = R(Q) - C_{\Sigma}(Q, f^*(Q)) \quad (4)$$

$$R(Q) = pQ \quad (5)$$

where  $\Pi, R$  are the firm's profit and revenue,  $p$  is the product price of product,  $Q^{\max}$  is the maximum possible production volume, considering the firm's production capacity.

We introduce the following assumptions that determine the applicability limits of models (1), (3).

The hypothesis of the perfect competition [18]: the product price is an exogenous variable, that is, the firm does not affect the price:

$$p'_Q = 0.$$

The hypothesis of a decreasing in return from the production growth, which corresponds to the relatively large company [19]:

$$C''_Q(Q) > 0.$$

The hypothesis of the control parameters influence on the internal damage and the external damage [18]: an increase in the production assets leads to an increase in the possible damage; the internal damage and the external damage are reduced with an increase in the VRC; the external damage is proportional to the internal damage; the internal damage is limited from above due to technology features and production volume:

$$\begin{aligned} X'_Q(Q, f) > 0, \quad X_{f'}(Q, f) < 0, \\ X(Q, f) \in [0, X^{\max}], \quad X^{\max} > 0, \end{aligned} \quad (6)$$

$$\begin{aligned} Y'_Q(Q, f) > 0, \quad Y_{f'}(Q, f) < 0, \\ Y(Q, f) = \omega X(Q, f), \end{aligned} \quad (7)$$

where  $X^{\max}$  is the maximum possible internal damage,  $\omega$  is the coefficient of the accident consequences expansion to the environment. This coefficient depends on the technological features and the firm's location.

The hypothesis of the monotone nondecreasing in the penalty function:

$$h'_Y(Y) \geq 0, \quad h(Y) \in [0, h^{\max}], \quad h^{\max} > 0,$$

where  $h^{\max}$  is the maximum possible penalty, which corresponds to the maximum possible damage.

The production costs function and the damage function, satisfying the hypotheses 2, 3, have the following forms [18], [19]:

$$\begin{aligned} C(Q) = BQ^\beta, \quad \beta \in (1, \beta^{\max}], \\ \beta^{\max} \in (1, 2], \quad B > 0, \end{aligned} \quad (8)$$

$$\begin{aligned} X(Q, f) = \chi(Q)e^{-\xi f}, \quad \xi \in (0, \xi^{\max}], \\ \xi^{\max} \in (0, 1], \quad \chi(Q)'_Q \geq 0. \end{aligned} \quad (9)$$

The function (8) is used for the long-term period; therefore, it includes only variable costs. The damage function (9) expresses an exponential distribution of the damage [20], which corresponds to man-made accidents.

According to the hypothesis 4, we consider the linear penalty function of the following type:

$$h(Y) = a + bY, \quad a > 0, \quad b > 0.$$

The penalty values are different, because the external damage can be incurred to various objects (the environment, the health

and the property of other firms). The total penalty function is a sum of the penalty functions of  $m$  objects:

$$\begin{aligned} H(Y, M) &= \sum_{j=1}^m h_j(Y_j) = \sum_{j=1}^m (a_j Y_j + b_j) = \\ &= \sum_{j=1}^m (a_j \mu_j Y + b_j) = \sum_{j=1}^m (a_j \mu_j \omega X + b_j), \end{aligned} \quad (10)$$

where  $M = \{\mu_j, j = 1, \dots, m\}$  is a vector of the external damage structure,  $\mu_j > 0, j = \overline{1, m}$  is the component of  $M$ , characterizing the damage, incurred to the  $j$ -th object,  $a_j > 0, b_j > 0, j = \overline{1, m}$  are the coefficients of the penalty function.

We consider the following problem of optimal control: to search for a pair  $\langle f^*(\cdot), Q^* \rangle$ , which is optimal by criteria (1), (3) on admissible sets for the functions of costs and damages (6), (7), (9), and the penalty function (10), respectively.

### III. RESULTS

In the first stage, we search for the VRC function  $f^*(\cdot)$  under condition (1), (2) in the following proposition.

**Proposition 1.** *The function*

$$f^*(Q) = \frac{1}{\xi} \ln \left[ \xi \chi(Q) \left( 1 + \omega \sum_{i=1}^m a_j \mu_j \right) \right] \quad (11)$$

is a solution of the problem (1) for the continuously differentiable functions  $C(\cdot), f(\cdot), H(\cdot)$  and under condition

$$\begin{aligned} \chi \chi'' - \chi'^2 > 0 \vee \\ \vee \{ \Omega > |\chi \chi'' - \chi'^2| \wedge \chi \chi'' - \chi'^2 < 0 \}, \\ \Omega = \frac{B\beta(\beta-1)Q^{\beta-2} \left( 1 + \xi^3 \chi^2 \left( 1 + \omega \sum_{j=1}^m a_j \mu_j \right)^2 \right)}{\xi^4 \chi^2 \left( 1 + \omega \sum_{j=1}^m a_j \mu_j \right)} + \\ + \frac{\chi'' \xi \chi \left( 1 + \omega \sum_{j=1}^m a_j \mu_j \right)}{\xi^4 \chi^2 \left( 1 + \omega \sum_{j=1}^m a_j \mu_j \right)}. \end{aligned} \quad (12)$$

*Proof.* We solve the problem (1) as a problem of searching for the extremum point of a function of several variables. We write function of total costs:

$$\begin{aligned} C_\Sigma(Q, f) &= C(Q) + f(Q) + H(Y) + X(Q, f) = \\ &= BQ^\beta + f(Q) + \sum_{j=1}^m h_j(Y_j) + \chi(Q)e^{-\xi f}. \end{aligned}$$

We search for the partial derivatives with respect to  $f$ :

$$C'_{\Sigma f} = 1 - \xi \chi e^{-\xi f} \omega \sum_{j=1}^m a_j \mu_j - \xi \chi e^{-\xi f} = 0.$$

We solve this equation

$$f^*(Q) = \frac{1}{\xi} \ln \left[ \xi \chi(Q) \left( 1 + \omega \sum_{i=1}^m a_j \mu_j \right) \right].$$

In the function  $f^*(Q)$ , the logarithm's argument is positive, in result of conditions (9) and (10).

We prove a fulfillment of the sufficient conditions of the minimum at  $f = f^*$ .

$$\begin{aligned} \Delta = & B\beta(\beta - 1)Q^{\beta-2} \left( 1 + \xi^3 \chi^2 \left( 1 + \omega \sum_{j=1}^m a_j \mu_j \right)^2 \right) + \\ & + \chi'' \xi \chi \left( 1 + \omega \sum_{j=1}^m a_j \mu_j \right) + \\ & + \xi^4 \chi^2 \left( 1 + \omega \sum_{j=1}^m a_j \mu_j \right)^4 (\chi \chi'' - \chi'^2). \end{aligned}$$

$\Delta|_{f=f^*} > 0$  under condition (12). Thus, the function  $f^*(Q)$  is a solution of the problem  $\min_{f \in A_f} C_{\Sigma}(Q, f)$ .  $\square$

Next stage, we determine the production volume in the case of the firm's profit is maximum, taking into account functions (11), in the following proposition.

**Proposition 2.** *The equation*

$$p = B\beta Q^{\beta-1} + \frac{2\chi'}{\xi\chi} \quad (13)$$

is a solution of problem (3) for continuously differentiable functions  $C(\cdot)$ ,  $f(\cdot)$ ,  $H(\cdot)$ , (12) and under condition

$$\begin{aligned} \chi \chi'' - \chi'^2 > 0 \vee \\ \vee \{ \Theta > |\chi \chi'' - \chi'^2| \wedge \chi \chi'' - \chi'^2 < 0 \}, \quad (14) \\ \Theta = \frac{B\beta(\beta - 1)Q^{\beta-2}\xi\chi^2}{2}. \end{aligned}$$

for  $Q \in A_Q$ .

*Proof.* We determine a value of the production volume  $Q^*$ , for which the criterion profit function (4) reaches the maximum value. We search for the partial derivatives (4) with respect to  $Q$

$$\begin{aligned} \Pi'_Q(Q, f) = R'_Q(Q) - C'_{\Sigma Q}(Q, f) = 0, \\ p - (C(Q)'_Q + f^*(Q)'_Q + H(Y)'_Q + X(Q, f)'_Q) = 0, \\ p - \left( B\beta Q^{\beta-1} + \frac{\chi'}{\xi\chi} + \chi' e^{-\xi f} \omega \sum_{j=1}^m a_j \mu_j + \chi' e^{-\xi f} \right) = 0. \end{aligned}$$

If  $f = f^*$  (11), we have

$$\begin{aligned} p - \left( B\beta Q^{\beta-1} + \frac{2\chi'}{\xi\chi} \right) = 0, \\ p = \left( B\beta Q^{\beta-1} + \frac{2\chi'}{\xi\chi} \right). \end{aligned}$$

We prove a sufficient conditions of the function maximum for  $\Pi(Q, f)$ :

$$\Pi''_{QQ}(Q, f) = R''_{QQ}(Q) - C''_{\Sigma QQ}(Q, f) < 0.$$

If  $f = f^*$  (11), we have:

$$\Pi''_{QQ}(Q, f) = -B\beta(\beta - 1)Q^{\beta-2} - \frac{2(\chi''\chi - \chi'^2)}{\xi\chi^2}.$$

$\Pi''_{QQ}(Q, f) < 0$  for (14),  $Q \in A_Q$ .  $\square$

Profit function (1) has maximum value, if  $q^*$  satisfies (12), (13).

#### IV. SIMULATION

We analyze functions (11), (13) and formulate the mathematical models with graphic illustrations for the most visual representation of the results.

The parameter  $\xi$  demonstrates the greatest impact on the VRC function. This parameter shows the influence of costs on the damage reduction. If  $\xi$  increases, then the VRC and total costs decrease.

We substitute into model initial data, corresponding to electric power industry

$$\begin{aligned} \xi = 4 \cdot 10^{-6}, \quad \beta = 1.5, \quad \omega = 19 \cdot 10^{-5}, \\ \mu_1 = 0.1, \quad \mu_2 = 0.05, \\ a_1 = 1, \quad a_2 = 0.1, \quad b_1 = 10^4, \quad b_2 = 50. \end{aligned}$$

The problem (3) does not have a solution, if the product price is minor than in variant 2 (Table I). In this case, a value of the firm's revenue does not enable to make the VRC. The problem (3) does not have a solution in variant 3 (Table I) with great coefficient  $B$ , because the firm has great production costs and low revenue. If the production volume ascends, the production costs increase, and there is low profit for the VRC, but, in this case, the production capacity ascends, that requires the additional safety measures.

Table I

Variant	$p$ , rubles	$B$	$Q^*$ , thing	$\Pi$ , million rubles
1	800	230	84259	2.65
2	$\leq 738$	230	$\notin R$	$\notin R$
3	800	$\geq 252$	$\notin R$	$\notin R$

We consider the profit function, and we analyze its dependence on the production volume  $Q$  and the VRC.

The dependences of the profit on production volume and the VRC function are shown in Fig. 1, *a*. The effect of the VRC is low perceptible in this graph, because the influence of the variable  $Q$  is greater. We limit the variable  $Q$  ( $Q \in [9.5; 10.5]$ ) to show the VRC influence on the firm's profit. In this case (Fig. 1, *b, c*), we see that the function has a maximum of the variable  $f$ . This kind of graph is explained by the costs function because the revenue function is a linear.

An increase in the VRC results in a decrease in  $C_{\Sigma}$ , because there is a reduction of the damage  $X$ , and the penalty  $H$ . We believe that the total costs are decreased, because the firm makes VRC (Fig. 2).

The simulation has the following practical importance for businesses. An increase in the production capacity leads to an ascendancy in the VRC, because the damage can exceed the firm's revenue. Additionally, there is an optimal value of the VRC when the profit has a maximum.

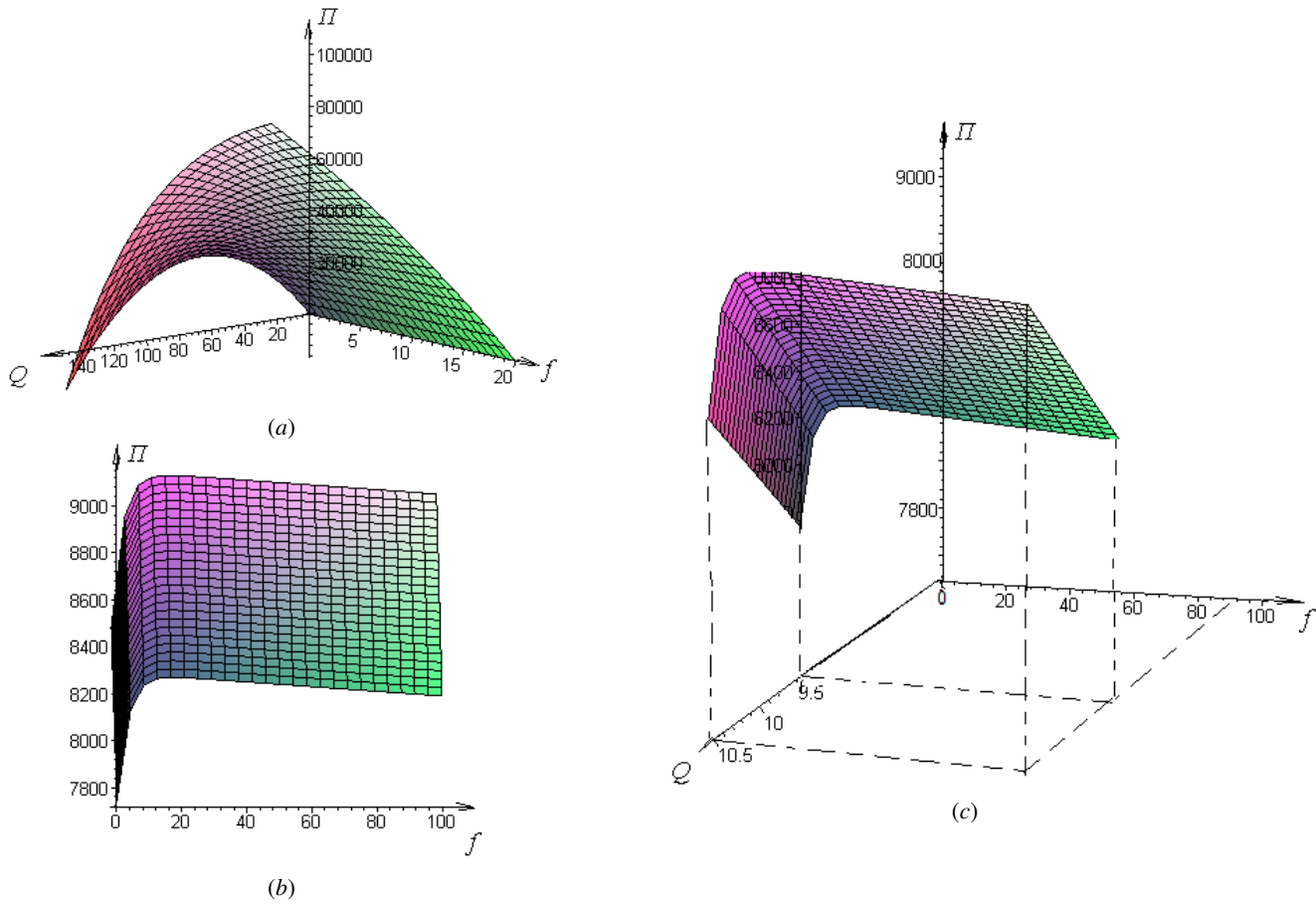


Figure 1. Graph of the profit function: (a) the whole graph, (b) part of the graph, (c) part of the graph, view from the  $Q$  axis

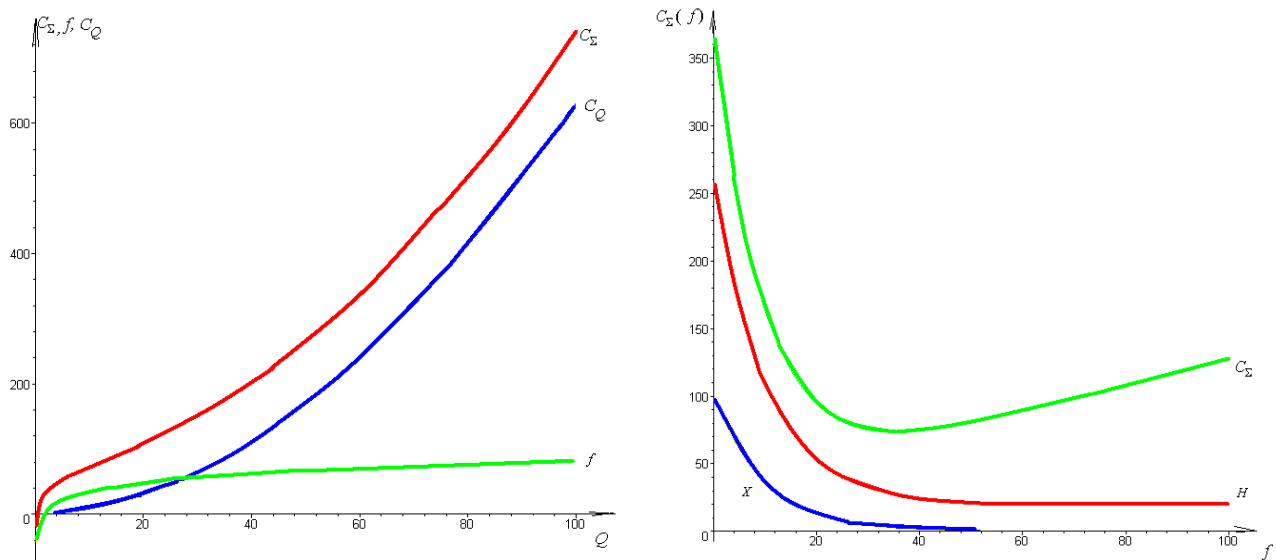


Figure 2. Graph of the costs dependences on  $Q$  and  $f$

### V. CONCLUSION

We proposed a subdivision of the firm's costs into the internal damage, the external damage, and the VRC. This

division enables to solve the firm's costs minimizing problem (1) and the profit maximizing problem (3), considering the costs of the following accidents: the ecology accidents, the

penalties for damage to the health and the property of other firms and individuals and the compensations paid for the non-fulfillment of contracts. The VRC function (11) depends on the production volume, and it takes into account the possible damage and the penalties caused by the firm's activities. The VRC function minimizes the total costs of the firm on the base of parameters of the accidents damage.

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