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An Am-Supermagic Decomposition Of The Cartesian Product of A Sun Graph And A Path

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Abstract—In this paper, we defined m be a positive integer, graph Am is a graph where $V(A_m) = \{v_i, v_i', v_i''|i = 0,1, \dots, m\}$ and $E(A_m) = \{v_i v_{i+1}, v_i' v_{i+1}'|i = 0,1, \dots, m-1\} \cup \{v_i v_i', v_i v_i''|i = 0,1, \dots, m\}$. Let n be an integer at least 3, a sun graph Sn is a graph where $V(S_n) = \{v_j, v_j'|j = 0,1, \dots, n-1\}$ and $E(S_n) = \{v_j v_{j+1(mod\,n)}, v_j v_j'|j = 0,1, \dots, n-1\}$. A path P_m is a graph whose vertices can be labeled $v_0, v_1, \dots, v_m$ such that $E(P_m) = \{v_0 v_1, v_1 v_2, \dots, v_{m-1} v_m\}$. In this paper we show that the Cartesian product of a sun graph and a path, has an Am- supermagic decomposition.

Keywords- Cartesian product, decompositions, path, supermagic decompositions, sun graph

I. INTRODUCTION

All graphs considered here are finite, simple and undirected. The Cartesian product of G and H, denoted by G□H, is a graph whose vertex set is $V(G)XV(H)$; two vertices (g,h) and (g',h') are adhacent, if g=g' and hh'ϵ E(H) or gg'ϵ E(G)and h=h'. The concept of an H-Supermagic decomposition of G arises from the marriage between graph labelings and graph decompositions. A family L = { H1, H2 , ... , Hn} of subgraphs of G is said an H-decomposition of G, if all subgraphs are isomorphic to a graph H, E (Hi) ∩E (Hj) = $\emptyset$ for $i \neq j$ and $\cup_{i=1}^{n}$ E (Hi) = E (G). We say that G has an H-magic decomposition, if G has an H-decomposition Land there is a total labeling $f: V \cup E \rightarrow \{1,2, \dots, |V| + |E|\}$ such that for each subgraph Hi= (Vi, Ei) ∈ L, the sum $\sum_{v \in V_i} f(v) + \sum_{e \in E_i} f(e)$ is constant, such constant is called the magic constant of f. Additionaly when $f(V) = \{1,2, \dots, |V|\}$, G has an H-superrmagic decomposition.

In [1] show that the complete graph K2m+1admits T-magic decompositions by any graceful tree with m edges. In [4], Salman and Baskoro gave Ph- supermagiclabelings for certain shackles and amalgamations of connected graph. In [5] gave conditions for the existence of C2k- supermagic decomposition of the complete n-partite graph.

Let n be an integer at least 3, a sun graph $S_n = (V(S_n), E(S_n))$ is a graph where $V(S_n) = \{v_j, v_j'|j = 0,1, \dots, n-1\}$ and $E(S_n) = \{v_j v_{j+1(mod\,n)}, v_j v_j'|j = 0,1, \dots, n-1\}$. A path Pm is a graph whose vertices can be labeled $v_0, v_1, \dots, v_m$ such that $E(P_m) = \{v_0 v_1, v_1 v_2, \dots, v_{m-1} v_m\}$. We defined that a graph $A_m = (V(A_m), E(A_m))$ is a graph where $V(A_m) = \{v_i, v_i', v_i''|i = 0,1, \dots, m\}$ and $E(A_m) = \{v_i v_{i+1}, v_i' v_{i+1}'|i = 0,1, \dots, m-1\} \cup \{v_i v_i', v_i v_i''|i = 0,1, \dots, m\}$. In this paper we investigate an Am-Supermagic decomposition of the Cartesian product of a path and a sun graph. To prove our main results we use concept of a k balanced multiset [4]. We use the notation [a,b] to mean $\{x \in N | a \leq x \leq b\}$. Let $k \in N$ and Y be a multiset that contains positive integers. Y is said to be k-balanced, if there exist k subset of Y, say Y1,Y2,Y3,…Yk, such that for every $i \in [1,k], |Y_i| = \frac{|Y|}{k}$, $\sum Y_i = \frac{\sum Y}{k} \in N$, and $\uplus_{i=1}^{k} Y_i = Y$. In this case, for every $i \in [1,k]$, Y_i is called a balanced subset of Y.

Lemma 1.1. [3] Let x, y, and k be integers such that $1 \leq x \leq y$ and k >1. If X= [x,y] and is a multiple of 2k, then X is k-balanced.

Lemma 1.2. Let n be non-negative integers. If Z=[n+1, n+3k] for odd k ≥ 3, then Z is k-balanced.

Proof. For each $i \in [1,k]$, define $Z_i = \{a_i, a'_i, a''_i\}$ where:

$$a_i = n + 1;$$

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$$a'_i = \begin{cases} n + \left\lceil \frac{3k}{2} \right\rceil + i, for\ i \in \left[1, \left\lfloor \frac{k}{2} \right\rfloor\right]; \\ n + \left\lceil \frac{k}{2} \right\rceil + i, for\ i \in \left[\left\lceil \frac{k}{2} \right\rceil, k\right]; \end{cases}$$

$$a''_i = \begin{cases} n + 3k + 1 - 2i, for\ i \in \left[1, \left\lfloor \frac{k}{2} \right\rfloor\right]; \\ n + 3k + 2\left\lceil \frac{k}{2} \right\rceil - 2i, for\ i \in \left[\left\lceil \frac{k}{2} \right\rceil, k\right]; \end{cases}$$

Then we define:

$A = \{a_i | i \in [1,k]\};$

$B = \{a'_i | i \in [1,k]\}$

$C = \{a''_i | i \in [1,k]\}$

We obtain that:

$A = [n+1, n+k];$

$B = [n+k+1, n+2k]$

$C = [n+2k+1, n+3k]$

So, $A \uplus B \uplus C = Z$ and $\uplus_{i=1}^{k} Z_i = Z.$ For each $i \in [1,k]$, $|Z_i| = 3$ and $\sum Z_i = 3(n+1) + \frac{3}{2}(3k-1)$. Hence for odd k ≥ 3, Z is k-balanced.

By using Lemma 1.1 and Lemma 1.2, we have the next corollary,

Corollary 1.3. Let n be non-negative integers, let c and k be two odd integers at least 3. If Z=[n+1, n+ck], then Z is k-balanced.

2. Main Results

Theorem 2.1. Let m and n be positive integers such that n ≥ 3. Let a graph G= (V(G), E(G)) be the Cartesian product of Pm and Sn , then there exist an Am- decomposition of G.

Proof. For j $\in$ [0, n-1] we define $A_m^j = \left(V(A_m^j), E(A_m^j)\right).$ Let $\mathcal{L} = \{A_m^j \,|\, j \in [0, n-1]\}$ be family of subgraphs of G where $V(A_m^j) = \{v_{ij}, v'_{ij}, v_{ij+1} \,|\, i \in [0,m]\}$ and $E(A_m^j) = \{v_{ij}\ v_{ij+1\ (mod\ n)}, v_{ij}\ v'_{ij}, | \ i \in [0,m]\} \cup \{v_{ij}\ v_{i+1j}, v'_{ij}\ v'_{i+1j}, | \ i \in [0,m]\}$. It is clear that all subgrah in $\mathcal{L}$ isomorphic to a graph Am , $\forall j, l \in \{1, \dots, n\},$ if $j \neq l, E(A_m^j) \cap E(A_m^l) = \emptyset,$ and $\cup_{j=0}^{n-1} E(A_m^j) = E(G).$ So there exist an an Am-decomposition of G.□

Theorem 2.2. Let m and n be positive integers, m ≥ 1, n ≥ 3, such that m is odd or m is even and n is odd. Let G be the Cartesian product of Pm and Sn , then G has Am-supermagic decomposition. Let f be Am-supermagic decomposition labeling and c be the constant of Am, then $(m+1(n(m+1)+1))/2 + (3m+2)(2n(3m+2)+1) \leq c \leq (m+1(3n(m+1)+1))/2 + (3m+2)(2n(3m+2)+1).$

Proof. By Theorem 2.1, G can be decomposed into n copy of Am, named $A_{m-k}, k = j+1$. Defined T as the vertex set of G which is use 2 time $T = \{v_{ij} \,| i = 0,1, \dots, m, j = 0,1, \dots, n\}$ and $T_k = \{v_{ij} \,| i = 0,1, \dots, m\}$ as subset of T (see figure 2). Let Q set of other vertex of G and $Q_k = \{v'_{ij} \,| i = 0,1, \dots, m\}$ as subset of Q. $|V(G) = 2n(m+1)|, |\mathrm{E(G)}| = 2\mathrm{n}(2\mathrm{m}+1),$ we divide proof into two cases.

Case 1: m is odd

Let $Z = [1, 2n(3m+2)]$. Partition Z into 3- set namely X, X', and Y. $X = [1, n(m+1)], X' = [n(m+1)+1, 2n(m+1)],$ and $Y = [2n(m+1)+1], 2n(3m+2)].$ Then $|X|, |X'|,$ and $|Y|,$ multiple of 2n. By lemma 1.1., X, X', and Y n-balanced. Let Xk , X'k, and Yk. be a balanced sub multiset of X, X', and Y. Since X, X', and Y is set, then Xk , X'k, and Yk is a set.

For every $k \in [1, n]$, in order to get minimum c is clear that we should use smallest label for Tk. Define total labeling f of G as follows:

Use the elements of Xk to label the vertices of Tk.

Use the elements of X'k to label the vertices of Qk.

Use the elements of Yk to label all remaining edges of Am-k.

We obtained that $|V(A_{m-k}) \cup E(A_{m-k})| = |X_k| + |X'_k| + |Y_k|.$

Then

$f(A_{m-k}) = 2\sum X_k + \sum X'_k + \sum Y_k = \frac{m+1(n(m+1)+1)}{2} + (3m+2)(2n(3m+2)+1)$

. Hence f is Am -supermagic labeling of G. □

For getting the maximum c is clear that we should use smallest label for Qk. Define total labeling f of G as follows:

Use the elements of X'k to label the vertices of Tk.

Use the elements of Xk to label the vertices of Qk.

Use the elements of Yk to label all remaining edges of Am-k.

We obtained that $|V(A_{m-k}) \cup E(A_{m-k})| = |X_k| + |X'_k| + |Y_k|$.

Then

$f(A_{m-k}) = 2\sum X_k + \sum X'_k + \sum Y_k = \frac{m+1(3n(m+1)+1)}{2} + (3m+2)(2n(3m+2)+1)$

. Hence f is Am -supermagic labeling of G. □

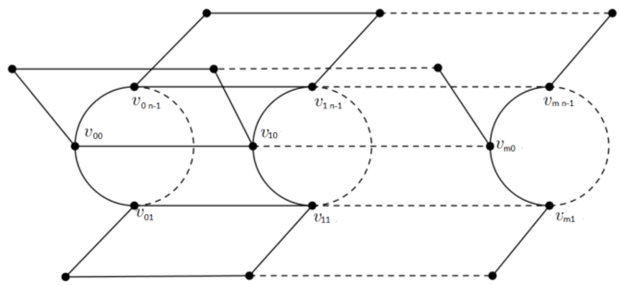

Figure 1. Graph G with T= $\{v_{00}, \dots, v_{0n-1}, v_{10}, \dots, v_{1n-1}, \dots, v_{mn-1}\}$

Case 2: m is even and n is odd

Let $Z = [1, 2n(3m+2)]$. Partition Z into 3- set namely X, X', and Y. $X = [1, n(m+1)], X' = [n(m+1)+1, 2n(m+1)]$, and $Y = [2n(m+1)+1], 2n(3m+2)]$. By Corollary 1.3, $|X|, |X'|$ are n-balance, because Y multiple of 2n, by Lemma 1.2, Y n-balanced.

For every $k \in [1, n]$, we can see that to get minimum we use smallest label for Tk. Define total labeling f of G as follows:

Use the elements of Xk to label the vertices of Tk.

Use the elements of X'k to label the vertices of Qk.

Use the elements of Yk to label all remaining edges of Am-k.

We obtained that $|V(A_{m-k}) \cup E(A_{m-k})| = |X_k| + |X'_k| + |Y_k|$.

Then

$f(A_{m-k}) = 2\sum X_k + \sum X'_k + \sum Y_k = \frac{m+1(n(m+1)+1)}{2} + (3m+2)(2n(3m+2)+1)$

. Hence f is Am -supermagic labeling of G. □

For getting the maximum c, we use smallest label for Qk. Define total labeling f of G as follows:

Use the elements of X'k to label the vertices of Tk.

Use the elements of Xk to label the vertices of Qk.

Use the elements of Yk to label all remaining edges of Am-k.

We obtained that $|V(A_{m-k}) \cup E(A_{m-k})| = |X_k| + |X'_k| + |Y_k|$.

Then

$f(A_{m-k}) = 2\sum X_k + \sum X'_k + \sum Y_k = \frac{m+1(3n(m+1)+1)}{2} + (3m+2)(2n(3m+2)+1)$

. Hence f is Am -supermagic labeling of G. □

For illustration, please see an A2 -supermagic labeling of $P_2 \square S_3$ in figure 2.

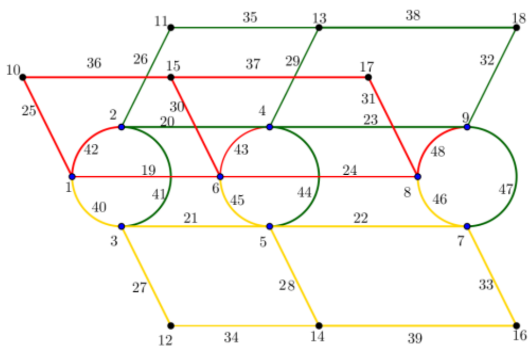

Figure 2. A2 -supermagic labeling of $P_2 \square S_3$

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