Optical reconstruction of digital hologram using cascaded liquid crystal spatial light modulators

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Abstract

A dynamic optical reconstructed system for digital hologram is developed using two commercially available twisted-nematic-liquid-crystal spatial light modulators (TNLC-SLMs), which are operating in the amplitude- and phase-mostly modulation modes, respectively. The complex modulated properties of this cascaded two TNLC-SLMs can be analyzed by the Jones matrix method and also be measured in experiment. According to the characteristics of our cascaded SLM module, the dynamic optical reconstruction of the digital hologram with the complex-values can be demonstrated. Both analytic and experimental results are presented and discussed.

Keywords: digital holography, twisted nematic liquid crystal spatial light modulators, three-dimensional reconstruction.

1. Introduction

Recently, digital holography[1] has been increasingly studied and become a viable technique among several 3D display technologies[2] with the progress of mega-pixel CCD sensors and SLM display panels, which provide sufficient dynamic range and high resolution in each pixel.

In general, the digital hologram is a complex field, which is created using the phase-shift digital holography[3]. However, for commercially available display devices such as LC-SLMs, they only can be separated in either phase or amplitude modulation mode[2]. The complex digital hologram can not be displayed using only one commercially LC-SLM[4,5]. Therefore, in this contribution we propose a cascaded LC-SLM module with two TN-LCDs to display the complex digital hologram for the optical reconstruction of 3D object. The principle of digital hologram with Fresnel diffracted propagation are described and analyzed. The complex modulated characteristics of our cascaded module are presented and the 3D object are optically reconstructed from the complex digital hologram in real-time.

2. Principle

The schematic diagram of digital holography for recording and reconstructing procedures are shown in Fig. 1. The hologram plane is placed at the Fresnel diffraction region from the object plane. Hence, the distribution of object wave propagated to the hologram plane H(x,y) can be written as a function of the Fresnel transformation of the object wave $U(\xi,\eta)$ at the object plane as follows[6]

$$H(x,y)=\frac{e^{jkz_o}}{j\lambda z_o}e^{j\frac{k}{2z_o}(x^2+y^2)}\int\!\!\int_{-\infty}^{\infty}\left[U(\xi,\eta)\,e^{j\frac{k}{2z_o}(\xi^2+\eta^2)}\right]e^{-j\frac{2\pi}{\lambda z_o}(x\xi+y\eta)}d\xi d\eta \tag{1}$$

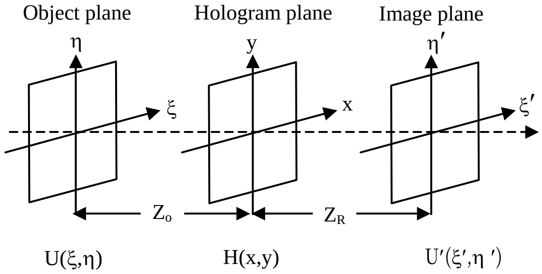


Fig. 1 Coordinate system for image

This object wave is interference with a plane wave $H_r(x,y)e^{i\Phi_r}$ as reference beam at the hologram plane. Φ_r is a constant which represent the phase difference between the object and reference waves. The intensity distribution of the interference pattern at the hologram plane can be given by :

$$I_H(x,y;\Phi_r) = \left|H(x,y) + H_r(x,y)e^{i\Phi_r}\right|^2 \\ = |H|^2 + |H_r|^2 + 2\,\mathrm{Re}\left[H^* H_r e^{i\Phi_r}\right] \qquad (2)$$

where * denotes the complex conjugate. Here, we use the phase shift technique to remove the DC terms and create the digital hologram. Four holograms are recorded by four reference waves with phase shift is $\pi/2$, and are written as :

$$I_n = I_H\left(x,y;\Phi_r = \frac{\pi}{2}n\right),\ n = 1,2,3,4 \qquad (3)$$

According to the phase shifting digital holographic technique, the amplitude A(x,y) and phase Φ (x,y) distribution of the digital hologram can be expressed as :

$$A(x,y) = \frac{\sqrt{(I_4 - I_2)^2 - (I_1 - I_3)^2}}{2} \qquad (4)$$

$$\Phi(x,y) = \tan^{-1}\left(\frac{I_4 - I_2}{I_1 - I_3}\right) \qquad (5)$$

The digital hologram can be calculated by Eqs. (4) and (5), and thus written as

$$H_d(x,y) = A(x,y)e^{j\Phi(x,y)} \qquad (6)$$

For optical reconstruction, we use a plane wave as the readout wave to illuminate the digital hologram. Therefore, the diffracted wave can be derived as a function of the Fresnel transformation of the digital hologram as follows

$$U'(\xi',\eta') = \frac{e^{jkz_R}}{j\lambda z_R} e^{j\frac{k}{2z_R}(\xi'^2+\eta'^2)} \iint_{-\infty}^{\infty} \left[H_d(x,y) \, e^{j\frac{k}{2z_R}(x^2+y^2)}\right] e^{-j\frac{2\pi}{\lambda z_R}(x\xi'+y\eta')} dxdy \qquad (7)$$

When we use the conjugate reference wave to reconstruct the digital hologram, the reconstructed object should be located at the plane of $Z_R = -Z_o$. According to the Eqs. (4)-(7), the complex distribution of the digital hologram and the reconstructed image can be calculated. Figure 2 shows the computer simulated results. We use the distances of Z_o and Z_R are 400mm.

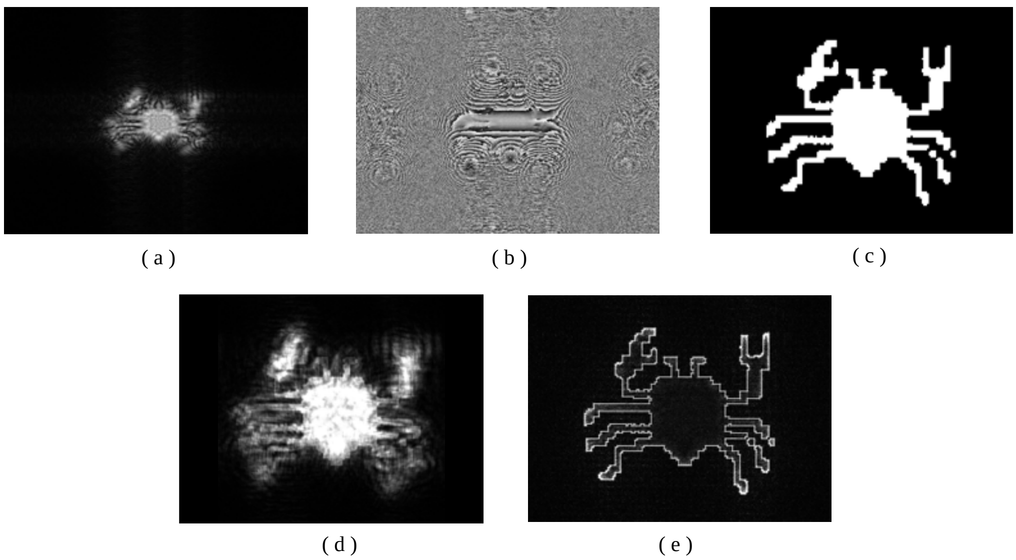


Fig. 2 Simulation results of the digital hologram. (a) amplitude distribution of the hologram, (b) phase distribution of the hologram, (c) reconstruction of the complex hologram, (d) reconstruction of amplitude- only hologram, and (e) reconstruction of phase-only hologram. The image sizes of (a) and (b) are 18.4mm ☓ 13.8 mm, and (c), (d), and (e) are 9.36mm ☓ 7.02 mm.

The amplitude and phase distribution of digital hologram in the hologram plane are shown in Fig. 2(a) and 2(b), respectively. Figure 2(c) shows the reconstruction of the complex digital hologram in the image plane. It is seen that the reconstruction of the complex hologram is as same as the object image. In addition, we also calculate the reconstruction of the phase-only and amplitude-only holograms, and the simulated results are shown in Fig. 2(d) and 2(e), respectively. In Fig. 2(d), we can see that the reconstructed image becomes smear because the phase information of the hologram is lost. On the other hand, as we can see in Fig. 2(e), the edge of reconstructed image is enhanced significantly due to the amplitude information of the hologram is lost. Therefore, the computer simulated results show that the complex hologram can obtain the best reconstructed image. However, the commercial TNLC-SLM can not be operated the phase and amplitude modulation independently. It represents that we can not obtain the complex hologram using a TNLC-SLM. Hence, we propose a cascaded LC-SLM module with two TN-LCDs. One of TN-LCDs is operated in the phase modulated mode, and the other is operated in the amplitude modulated mode. The complex modulation can be achieved by our module. In the following, the analytic and experimental results are described.

3.Experimental results and Discussion

Figure 3 shows the architecture of cascaded two TNLC-SLMs. To combine the modulated properties of these two TNLC-SLMs, we arrange a 4-F telescopic system to align two TN-LCDs by pixel to pixel. The aperture size of the TN-LCD panel with 800 ╳ 600 pixels is 18.4mm ╳ 13.8mm, and the size of each pixel is approximately 23μm ╳ 23μm. The molecular director of TN-LCD in the front plane is 45° with x-axis and the twist angle is 90°. The wavelength of laser is 532nm. The modulated properties of TNLC-SLM can be calculated by Jones matrix method with the different angles of polarizer and analyzer. Also, the modulated characteristic of TNLC-SLM can be measured by the optical experimental system. For the phase-mostly modulated mode, θ_1 and θ_1' are 63° and 111°, respectively. For amplitude-mostly modulated mode, θ_2 and θ_2' are 45° and 135°, respectively. Figure 4 shows that the modulated properties of TNLC-SLM are the function of the input gray level for the phase-mostly mode and amplitude-mostly modes. For ideal phase modulated mode, the phase shift should be operated from 0 to 2π, and the transmission should be a constant for any input gray level. However, Fig. 4(a) shows that the maximum phase modulation of the TNLC-SLM is about 353°, and the variation of the transmission is 80% for phase-mostly mode. On the other hand, the curves in Fig. 4(b) show that the transmission variation is approximately 99.2%, and the maximum of phase shift is approximately 93° for amplitude-mostly mode. The experimental results of the modulation properties in Fig. 4 represent that the commercial TNLC-SLM can not be operated either in the ideal phase and amplitude modulated mode. Some information of the complex hologram will get lost if we use this TNLC-SLM to display the complex hologram. Therefore, we also calculate the reconstruction of the complex hologram with the modulated properties of the commercial TNLC-SLM. The computer simulated results are shown in Fig. 5. Figure 5(a) and (b) represent the phase-mostly

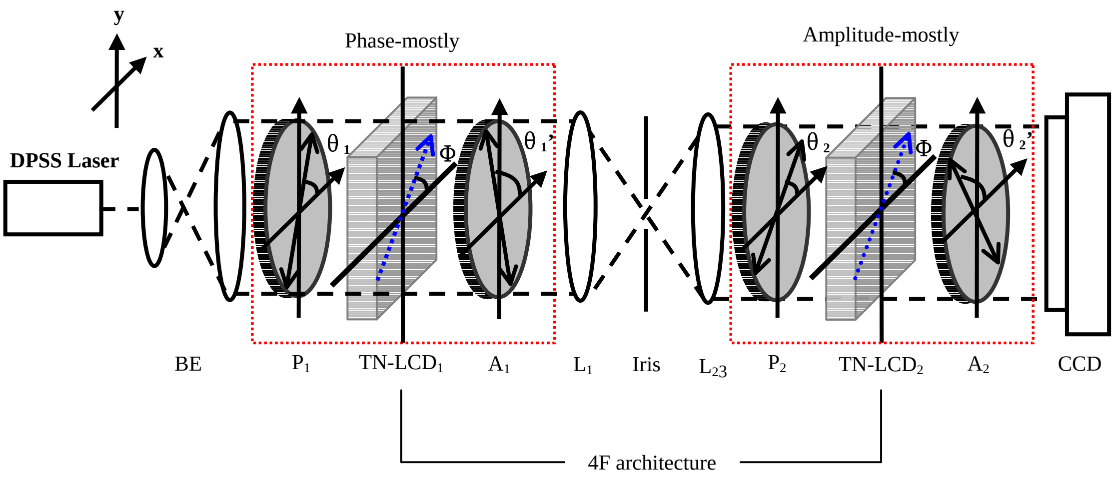


Fig. 3 Schematic diagram of the cascade TNLC-SLMs.

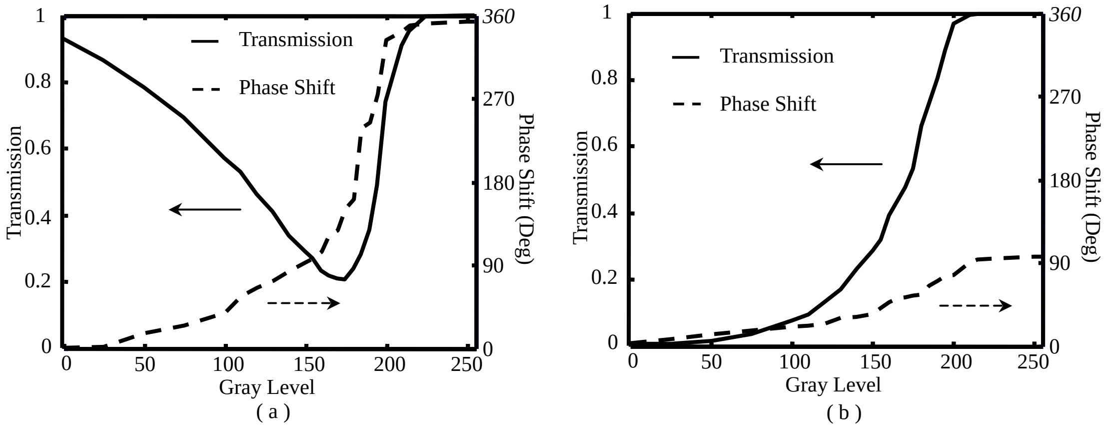
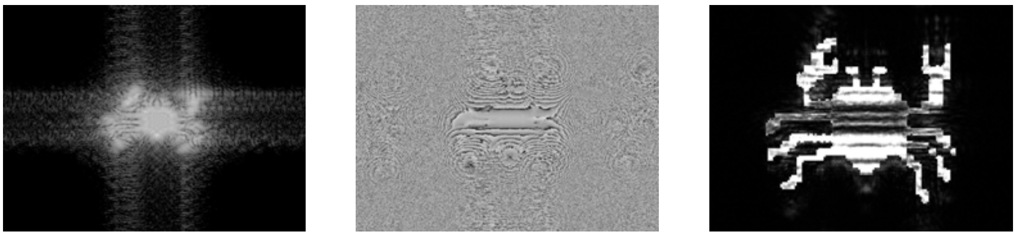


Fig. 4 Modulation properties of the TNLC-SLM: (a) phase-mostly modulation as a function of gray level at $\theta_1 = 63°$ and $\theta_1' = 111°$; and (b) amplitude-mostly modulation as a function of gray level at $\theta_2 = 45°$ and $\theta_2' = 135°$



(a) (b) (c)

Fig.5 Hologram and reconstructed images with the modulation properties of TNLC-SLM.
(a) hologram in amplitude-mostly mode, (b) hologram in phase-mostly mode, and (c) the reconstruction image. The image sizes of (a) and (b) are 18.4mm × 13.8 mm, and (c) is 9.36mm × 7.02 mm.

and the amplitude-mostly mode of digital hologram in the commercial TNLC-SLM, respectively. Compared with Fig. 2(a) and (b), it is seen clearly the difference between Fig. 5(a), (b) and Fig. 2(a), (b). The reconstruction of the complex hologram will provide some noise using the commercial TNLC-SLM, and the simulated result is shown in Fig. 5(c). The mean square error of reconstruction image is approximately 0.0475.

4. Conclusions

We have proposed a cascaded TNLC-SLMs module to reconstruct the complex digital hologram. Both phase-mostly and amplitude-mostly modulation modes are found in experiment and theoretic analysis. Using the commercial TNLC-SLMs, the complex hologram can be reconstructed in real time.

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