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Analysis of Institution Mechanism on Cable-strut-beam

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***Abstract*—The cable-strut-beam belongs to the spatial developable structure, and prestress is an important source of structural stiffness. How to apply prestress depends on the institution's self-stress mode. Firstly, the balance equation of cable-strut-beam is established. Then the self - stress mode of the cable-strut-beam is solved by using the column principal element gaussian elimination. From analysis, the prestress importing way is defined and the initial bending moment on structural formfinding was also indicated.**

***Keywords*—institution mechanism; cable-strut-beam; prestress**

I. THE CABLE-STRUT-BEAM BALANCE EQUATION

A. *Balance Equation of Space Beam Element* [1]

It is assumed that the space beam element is a straight beam with two nodes according to the assumption of flat section. The section warping and local buckling of beam element were ignored (See Figure I-II).

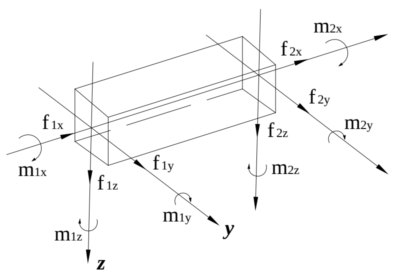
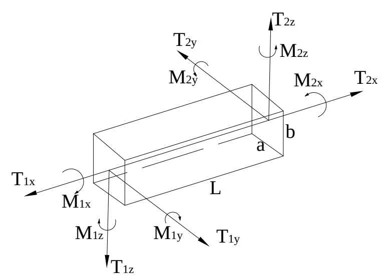


FIGURE I. SPACE BEAM ELEMENT ROD END FORCE VECTOR



(L, A AND B ARE THE LENGTH OF THREE SIDES ON THE BEAM ELEMENT)

FIGURE II. SPATIAL BEAM ELEMENT INTERNAL FORCE

In the local coordinate system, the external force vector and generalized stress vector matrix of the spatial beam element were written as:

$$f^e = \left(f_{1x}\ f_{1y}\ f_{1z}\ m_{1x}\ m_{1y}\ m_{1z}\ f_{2x}\ f_{2y}\ f_{2z}\ m_{2x}\ m_{2y}\ m_{2z} \right)^T \quad (1)$$

$$t^e = \left(T_{1x}\ T_{1y}\ T_{1z}\ M_{1x}\ M_{1y}\ M_{1z}\ T_{2x}\ T_{2y}\ T_{2z}\ M_{2x}\ M_{2y}\ M_{2z} \right)^T \quad (2)$$

According to the principle of equilibrium, it can be showed that:

$$\begin{aligned} T_{1x} &= T_{2x} \\ T_{1y} &= -T_{2y} = \frac{1}{L}\left(M_{1z} - M_{2z}\right) \\ T_{1z} &= -T_{2z} = \frac{1}{L}\left(M_{1y} - M_{2y}\right) \\ M_{1x} &= M_{2x} \end{aligned} \quad (3)$$

The reduced equilibrium equation can be expressed as:

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{L} & 0 & -\frac{1}{L} \\ 0 & 0 & \frac{1}{L} & 0 & -\frac{1}{L} & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{L} & 0 & \frac{1}{L} \\ 0 & 0 & -\frac{1}{L} & 0 & \frac{1}{L} & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} T_{1x} \\ M_{1x} \\ M_{1y} \\ M_{1z} \\ M_{2y} \\ M_{2z} \end{bmatrix} = \begin{bmatrix} f_{1x} \\ f_{1y} \\ f_{1z} \\ m_{1x} \\ m_{1y} \\ m_{1z} \\ f_{2x} \\ f_{2y} \\ f_{2z} \\ m_{2x} \\ m_{2y} \\ m_{2z} \end{bmatrix} \quad (4)$$

Namely: $A^{e*}t^{e*} = f^{e}$ (5)

Where, A^{e*} is the reduced balance matrix, t^{e*} is the generalized stress vector after reduction, f^{e} is the rod end force vector in the local coordinate system. In the local coordinate system, the spatial beam element after reduction had 6 independent internal force vectors: 1 axial force, 1 torque and 4 bending moments. According to the matrix transformation principle $A = TA^{e}$, the equation (5) was transformed into the equilibrium matrix under the whole coordinate system. designations.

B. Balance Equation of the Cable Strut Element

The equilibrium equation of cable-strut element[2] was established according to the node coordinate relationship shown in Figure III.

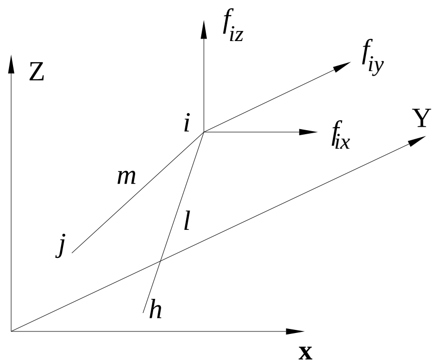


FIGURE III. THE NODAL EQUILIBRIUM OF THE CABLE- STRUT ASSEMBLY

$$\begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \frac{(x_i - x_h)}{L_l} & \cdot & \cdot & \frac{(x_i - x_j)}{L_m} & \cdot & \cdot \\ \cdot & \cdot & \frac{(y_i - y_h)}{L_l} & \cdot & \cdot & \frac{(y_i - y_j)}{L_m} & \cdot & \cdot \\ \cdot & \cdot & \frac{(z_i - z_h)}{L_l} & \cdot & \cdot & \frac{(z_i - z_j)}{L_m} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix}_{3N-k} \times \begin{Bmatrix} t_1 \\ \cdot \\ \cdot \\ t_l \\ \cdot \\ \cdot \\ t_m \\ \cdot \\ \cdot \\ t_b \end{Bmatrix} = \begin{Bmatrix} \cdot \\ \cdot \\ \cdot \\ f_{ix} \\ f_{iy} \\ f_{iz} \\ \cdot \\ \cdot \\ \cdot \end{Bmatrix} \quad (6)$$

Namely: $At = f$ (7)

Where, A is the balance matrix, t is the rod internal force vector of b dimension, f is the nodal force vector, is the total number of bar, is total number of nodes, is the total number of constraints.

II. SOLVING OF SELF-STRESS MODE

According to the equilibrium equation, the singular value decomposition method [3] can be used to calculate the self-stress mode number of the structure. To the cable-strut-beam, the balance equation was analyzed by the column principal element gaussian elimination using the augmented matrix. And the results are verified by singular value decomposition method. Because of different freedom degrees on the beam element and cable element, the balance is divided into two parts: the cable-strut is A_{sg} and the beam element is A_l . (See Figure IV)

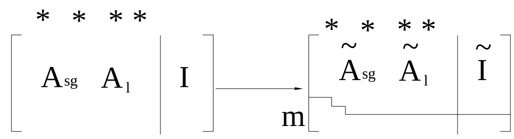


FIGURE IV. THE COLUMN PRINCIPAL ELEMENT GAUSSIAN ELIMINATION

The columns with * are the columns of the pivot entries in the elimination process [4-6], and the total number of these columns was s. These columns are linearly dependent columns, and these columns correspond to extra internal force. For the cable strut, these columns corresponded the axial force; to the beam elements, these columns represented as the axial force or bending moment. The gaussian elimination is divided into two parts: A1 is the matrix of non-redundant internal forces, A2 is the matrix of redundant internal forces. The corresponding element internal force vectors were T_1 and T_2 . Then, the self-stress mode of i was:

$$T_{1i} = -A_1^{-1} A_2 T_2 \quad (8)$$

Assuming that the internal force of the redundant bars corresponding to unit force, the remaining excess internal forces are 0, then

$$T_{2i} = \pm E \quad (9)$$

The self-stress mode of i was expressed as:

$$T_i = \begin{Bmatrix} T_{1i} \\ T_{2i} \end{Bmatrix} \quad (10)$$

III. PRESTRESS IMPORTING EFFICIENCY

For the multi-self-stress modal institution $s = n_c - r_A$. The prestress distribution of the institution is a linear combination of these independent self-stress modes, which can be expressed as:

$$\boldsymbol{T}=\boldsymbol{T}_1\beta_1+\boldsymbol{T}_2\beta_2+\cdots+\boldsymbol{T}_s\beta_s \tag{11}$$

Where, $\beta_1, \beta_2, \cdots, \beta_s$ are combination of factors and random real constants.

Given the prestress of any redundant member, the same rod has the same prestress force, so the remaining unknown bar prestress can be calculated. Then, the inner product[7] of the self-stress mode $\boldsymbol{T}_i$ can be expressed as:

$$\langle \boldsymbol{T}_i, \boldsymbol{T}_i \rangle = \boldsymbol{T}_{1i}^2+\boldsymbol{T}_{2i}^2+\cdots+\boldsymbol{T}_{bi}^2 \tag{12}$$

Comparing the inner product size of each order self-stress mode, the efficiency of this prestress method can be obtained. Furthermore, the most effective prestress tension method on overall structure were determined also.

IV. THE SELF-STRESS MODE ON THE CABLE-STRUT-BEAM INSTITUTION

A. This Institution had 20 Nodes, 8 Fixed Nodes and 12 Beam Elements, 12 Cables.

The balance equation is a matrix 60×68 order and the self-stress modes is 20. The 16th order is provided by the bending moment of the beam element, and the fourth order is generated by the unit axial force (See Figure V).

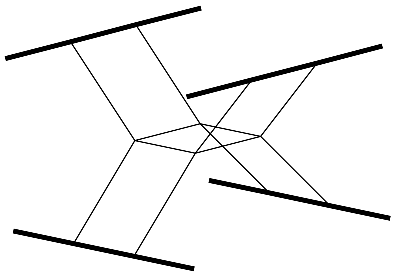


FIGURE V. THE INSTITUTION OF CABLE-STRUT-BEAM

B. This Institution is Non-fully Symmetry Geiger Cable Dome and it had 72 Nodes, 24 Fixed Nodes and 36 Beam Elements, 72 Cables and Struts.

The balance equation is a matrix 60×68 order and the self-stress modes is 80. The 20 order is provided by the bending moment of the beam element, and the 60 order is generated by the unit axial force. It can be seen that the initial bending moment and axial force of beam element in the beam structure can be applying prestress on structural formfinding (See Figure VI).

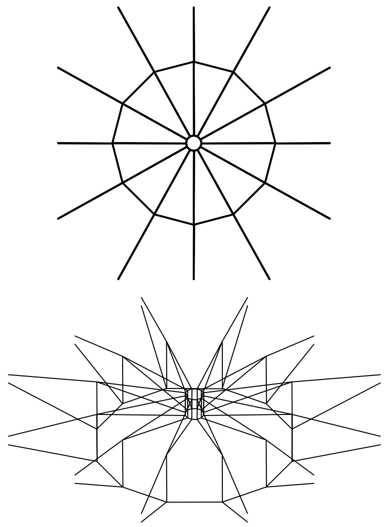


FIGURE VI. NON-FULLY SYMMETRY GEIGER CABLE DOME

V. SUMMARY

In this paper, the balance equation of the beam mechanism is established. Then the equilibrium matrix is analyzed by the column principal element gaussian elimination method and a singular value decomposition method is used to check the mode number. To determine the stress modal structure prestressed import way of efficiency, this paper also put forward that the stress modal inner product used to judge the import prestressed efficiency, and its effectiveness and feasibility is verified by numerical analysis.

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