

Research on Evaluation Method of Rail Transit Train Operation Scheme Based on DEA Method

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Abstract. In this paper, we established an evaluation index system and then calculated different efficiencies of the two kinds of efficient decision-making units and the non-effective ones to evaluate and rank them synthetically. At the same time, the inherent requirements and characteristics of the Data Envelopment Analysis is also considered. The comparison between our method and the existing research verifies the reasonability and practicability of this paper. The results of the case study illustrate that the method proposed in this paper will play a guiding role in train planning comparison and selection practice.

1. Introduction

The passenger train operation scheme is the basic of railway organization work, which can decide passenger rail transportation service level, including train type, routes and numbers. In the meanwhile, train operation scheme evaluation can guarantee the optimal comprehensive benefits of the rail transit system and the basis of the comparison of different train operation schemes. It is a systematic work involving many factors such as economy, technology and service, which is supposed to meet the passenger demand and social responsibility and to obtain the maximum economic benefits.

Many researchers have conducted in-depth studies on the evaluation of train operation scheme. Deng [1] combines Cluster Analysis and Principal Component Analysis to reduce the number of evaluation indexes and reduce the influence of correlation among the indicators. Tao [2] uses Matrix Analysis to study the influencing factors of economic benefits of the train operation scheme. Aside, A. [3] and Pu [4] apply data envelopment analysis method to analyze the production efficiency based on the classification of evaluation indicators for the train operation scheme. Kulak et.al [5] compare different indicators by applying fuzzy comprehensive evaluation to train operation scheme evaluation. Xu and Cheng [6] proposed a weighted sum evaluation model and a minimum weighted relative deviation distance evaluation model; they also discuss and prove the opposite result of the two evaluation methods on the same train operation scheme, giving the application condition.

Based on the existing research results, this paper proposes a comprehensive benefit ranking evaluation model based on DEA (data envelopment analysis) method, which can solve the problem that the traditional model cannot rank all the DMUs (decision making units) and take the essential application characteristics of the DEA into consideration, which were usually ignored.

2. Evaluation Index System Establishment

As the basis of the comprehensive train operation scheme evaluation, the content of evaluation index system determines the rationality of the evaluation method, and it's supposed to reflect the attributes of the different aspects and the overall efficacy of the rail transit system (Qu and Xu [7]). We can study the train operation evaluation system from three aspects: economy, technology, and service. The evaluation index system of this paper is conducted from the above three aspects on the base of existing researches, giving consideration to the purpose and systematic.

Table 1. Evaluation system of train operation scheme.

Economy	Technology	Service
Total cost (10 thousand yuan)	Average train travel distance(km)	Average passenger travel time(hour)
	Passenger transport volume(pkm)	
Total income (10 thousand yuan)	Train marshalling	Comfort level (Fatigue recovery time)
	Average stop times(time)	Average waiting time(minute)
	Number of the running trains	
Average attendance rate(%)	Average travel speed(km/h)	Average direct ratio(%)
	Railway line capacity utilization(%)	Average transfer times(time)
	Total train travel time(hour)	

In practice, suitable indicators can be selected for different evaluation environments. For example, passenger waiting time is an essential factor in the evaluation of urban rail transit train operation scheme; in comparison, waiting time is such an unimportant element that can be neglected.

3. Evaluation Indexes Pretreatment

3.1 Input and Output Index Classification.

Since DEA method applies to the multi-input and multi-output situations, all the indexes should be classified according to the principle that the behavioral indicator that influences the DMU production is the input index and the output is generated by the DMUs, as shown in Table 2. At the same time, some unsuitable indexes can be excluded when the method is used in a certain circumstance.

Table 2. Evaluation indices input/output classification.

category	Name and symbol	category	Name and symbol
Input	Total cost	Output	Total income
	Average train travel distance		Average attendance rate
	Train marshalling		Passenger transport volume
	Average stop times		Average travel speed
	Number of the running trains		Railway line capacity utilization
			Total train travel time
			Average passenger travel time
			Comfort level
			Average waiting time
			Average direct ratio
			Average transfer times

3.2 Evaluation Indexes Positive or Negative Rationalization.

The input indexes are supposed to be negative, the smaller value the better; the output indexes are positive and the bigger value, the better. When the actual situation is inconsistent with this premise, the original indexes, such as travel time, waiting time and transfer times in this paper, are required to be transformed into the expected ones by using linear transformation or in some another method. The

method below is applied in this paper; the original value of the rah indicator is $y_{r,j}$ and the transformed value is $y'_{r,j}$, as shown in Eq. 1.

$$y'_{r,j} = \sum_{j=1}^n y_{r,j} - y_{r,j} \quad (1)$$

Where: $j=1, 2, n$ and n are the number of all the DMUs. It should be noted that the transformed index will be used directly to represent the original one for the convenience of description and the symbol won't be changed, that is, use the value $y'_{r,j}$ in the place of that of $y_{r,j}$.

3.3 Number of Indexes Reduction.

Experience shows that when the number of DMUs is 2-3 times the total number of indexes, the evaluation result is accurate and reasonable enough, which is a requirement of DEA called the degree of freedom. Therefore, when this cannot be satisfied in reality, some method should be applied in the first place to reduce the number of the indexes such as selecting main indicators or weighting methods, on the premise of guarantee the positive and negative property of the indexes. This paper uses the Variation Coefficient Method to integrate the same type of indexes by weighting them, which can eliminate the impact of different dimensions and the subjective factors caused by human participation. Let n indicates the total number of the indexes of any DMU, and X_{ij} indicates the jet one in group i , and $j = 1, 2, \dots, k, i = 1, \dots, n$, here's the detailed calculating steps:

Step1: calculate the average \bar{X}_j and its standard deviation $\bar{\sigma}_j$;

Step2: calculate the coefficient of variation of every index, $V_j = \frac{\bar{\sigma}_j}{\bar{X}_j}$;

Step3: calculate the weight of every index, $W_j = \frac{V_j}{\sum_{j=1}^k V_j}$.

4. Train Operation Evaluation Model Based on DEA Method

DEA has the advantage that the function relation between output and input is not considered, so it has lower requirements on the original data and stronger evaluation objectivity. But traditional DEA models cannot compare and rank all the efficient DMUs. This paper can solve this problem through the following procedure: firstly, the traditional model is applied to distinguish strong efficient, common efficient and non-efficient DMUs and rank the non-efficient ones at the same time; secondly, calculate a measurement to rank the strong and common efficient DMUs; finally, the three kind of DMUs will be ranked according to the principle that strong efficient DMUs are better than common efficient ones and the latter are better than non-efficient DMUs.

4.1 Non-efficient DMUs Ranking Model.

The traditional BCC model (input-oriented) shown in Equation 2 was used to select and rank non-efficient DMUs.

$$\theta_o^* = \text{Min } \theta_o \quad (2)$$

$$s. t. \left\{ \begin{array}{l} \sum_{j=1}^n \lambda_j x_{ij} + s_{io}^- = \theta_o x_{io}, i = 1, 2, \dots, m \\ \sum_{j=1}^n \lambda_j y_{rj} - s_{ro}^+ = y_{ro}, r = 1, 2, \dots, s \\ \sum_{j=1, j \neq b}^n \lambda_j = 1 \\ s_{io}^-, s_{ro}^+ \geq 0 \\ s_{io}^-, s_{ro}^+, \lambda_j \geq 0, j = 1, 2, \dots, n \end{array} \right.$$

Where: $s_{i_0}^-$ and $s_{r_0}^+$ are the slack variable of the input and output of DMU_o. As for DMU_o, the technique-scale efficiency θ_o^* can be obtained from the result of Eq. 2, and it's easy to know that $\theta_o^* \in [0, 1]$. If $\theta_o^*=1$, the DMU_o can be judged as efficient. Particularly, efficient DMUs can also be divided into two sorts: strong and common efficient. The DMU_o will be judged as strong efficient only if the value of every slack variable is 0, otherwise it is a common efficient DMU. So, if $\theta_o^*<1$, the DMU_o is non-efficient, and the non-efficient DMUs can be compared and ranked by θ_o^* , the bigger its value the better the DMU_o.

4.2 Efficient DMUs Ranking Model.

4.2.1 Strong Efficient DMU Ranking Model.

Let J_n indicates the set of all the DMU. J_b is any strong efficient DMU and J_a is a DMU from the set of non-efficient and common efficient DMUs, that is, $J_n = \sum J_a + \sum J_b$.

If a J_b is removed from J_n , any J_a will improve towards the more DEA-efficient situation, which is the reason why this paper only studies how an J_b influences a J_a . This paper uses the idea of literature[8] to calculate the ρ_a^* -efficiency (shown in Eq. 3) of J_a and its new $\rho_{a,b}^*$ -efficiency (shown in Eq. 4) after J_b is removed; the difference between $\rho_{a,b}^*$ and ρ_a^* , that is, $\rho_{a,b}^* - \rho_a^*$, as a measure of the influence a J_b have on a J_a , the bigger the value the more efficient the J_b is; and the influence of a J_b on all J_a , Ω_b (shown in Eq. 5), is regarded as a comprehensive evaluation index, which is used as the ranking basis later.

$$\rho_a^* = \text{Min } \alpha - \frac{\sum_{i=1}^m \frac{s_{ia}^-}{R_i^-}}{m} - \frac{\sum_{r=1}^s \frac{s_{ra}^+}{R_r^+}}{s} \quad (3)$$

$$s. t. \left\{ \begin{array}{l} -\sum_{j=1} x_{ij} \lambda_j + \alpha x_{ia} - s_{ia}^- = 0, i = 1, \dots, m \\ \sum_{j=1} y_{rj} \lambda_j - y_{ra} - s_{ra}^+ = 0, r = 1, \dots, s \\ \sum_{j=1} \lambda_j = 1 \\ \lambda_j, s_{ia}^-, s_{ra}^+, \alpha \geq 0 \end{array} \right.$$

$$\rho_{a,b}^* = \text{Min } \beta - \frac{\sum_{i=1}^m \frac{s_{ia}^-}{R_i^-}}{m} - \frac{\sum_{r=1}^s \frac{s_{ra}^+}{R_r^+}}{s} \quad (4)$$

$$s. t. \left\{ \begin{array}{l} -\sum_{j \neq b} x_{ij} \lambda_j + \beta x_{ia} - s_{ia}^- = 0, i = 1, \dots, m \\ \sum_{j \neq b} y_{rj} \lambda_j - y_{ra} - s_{ra}^+ = 0, r = 1, \dots, s \\ \sum_{j \neq b} \lambda_j = 1 \\ \lambda_j, s_{ia}^-, s_{ra}^+, \beta \geq 0, j \neq b \end{array} \right.$$

$$\Omega_b = \sum_a (\rho_{a,b}^* - \rho_a^*) \quad (5)$$

4.2.2 Common Efficient DMU Ranking Model.

The method shown in Eq. 3, Eq. 4 and Eq. 5 are only useful for a strong efficient DMUs, so another method is applied here to evaluate common efficient ones. This paper uses the results of $s_{i_0}^-$ and $s_{r_0}^+$

from Eq. 2 direct to calculate a new measure index δ_o as the comparing and ranking basis (Jahanshahloo G R et.al [9]), as shown as bellow.

$$\delta_o = \frac{2 - \frac{1}{m} \sum_{i=1}^m \frac{s_{io}^-}{x_{io}}}{1 + \frac{1}{s} \sum_{r=1}^s \frac{s_{ro}^+}{x_{ro}}} \quad (6)$$

It's easy to know that smaller s_{io}^- and s_{ro}^+ means a better DMU. On the base of the original model, this paper adds 1 to the numerator, so the value of δ_o Satisfies the condition of $\delta_o \in (1, 2)$, as a result, δ_o can be compared direct with θ_o^* .

5. Case Analysis

5.1 The Original Data of the Train Operation Schemes and its Processing.

This paper researches on 10 train operation scheme of Beijing-Shanghai railway and compare the result with the method in literature [10]. DMU₁, DMU₇ and DMU₁₀ aim at minimizing travel costs; DMU₂ focuses on improving economic benefit and service; DMU₃ is designed mainly from the operator's point of view; DMU₄ take the benefits of the operator and travelers into account; DMU₆ aims to reduce stop times; DMU₅ and DMU₈ take into consideration of economic, technology and service and DMU₉ is a train operation scheme used in reality.

Since the case in this paper is aimed at a single railway line, not all the indexes are used here, that is, the waiting time, direct ratio, and transfer times are not taken into account. Because the requirement of degree of freedom is not satisfied here, original data dimension reduction is necessary in the pretreatment process. This paper integrates the two types of indicators according to the three attributes in the index system. As a result, the input or output indexes are transformed to a combination of a comprehensive economic index, a comprehensive technical index and a comprehensive service index. The result after pretreatment is shown in Table 3, which is applied in the case analysis afterwards.

Table 3. Preprocessed value of the DMUs.

DMU	Input			Output		
	Economy X1	Technology X2	Economy Y1	Technology Y2	Service Y3	
1	940.53	150.26	2097.296	28016567.81	803.577	
2	2438.28	130.48	1820.326	16932428.22	788.593	
3	1580.025	129.66	2167.022	28016560.61	735.831	
4	1403.905	177.77	3744.302	101808141	855.544	
5	2027.84	140.01	2854.964	39277138.81	826.605	
6	2101.14	142.64	2854.079	39277157.66	823.775	
7	2636.93	132.85	333.653	49648404.61	772.805	
8	3682.802	235.53	9907.603	139396982.6	820.362	
9	3286.36	240.16	110.510	117985303.4	801.096	
10	925.45	150.42	1422.253	18430556.76	543.407	

5.2 Calculation Results and Analysis.

The results of θ_o^* and δ_o from Eel 2 and Eel 6 are shown in Table 4 and Table 5.

Table 4. Θ -efficiency of the DMUs.

DMU	1	2	3	4	5	6	7	8	9	10
S_{1o}^-	0	0	0	0	0	49.2861	0	0	388.0714	0
S_{2o}^-	0	0	0	0	0	0	0	0	0	0
S_{1o}^+	0	0	0	0	0	0	0	0	6286.3013	0
S_{2o}^+	1.054	0.0107	0.0055	0	0	183326.3707	0	0	0.1906	0
S_{3o}^+	0	0	0	0	0	0	0	0	39.3064	0
θ_o^*	1	1	1	1	1	0.982	1	1	0.844	1
property	C-E	C-E	C-E	S-E	S-E	N-E	S-E	S-E	N-E	S-E

Note: C-E indicates common efficient, S-E indicates strong efficient and N-E indicates non-efficient.

Table 5. Δ -efficiency of the common efficient DMUs.

DMU	1	2	3
δ_o	1.99999996433	1.99999999954	1.99999999982

Table 4 and 5 tell us that $DMU_6 > DMU_9$ are non-efficient DMUs that shouldn't be accepted (" $>$ " means better than) and can be optimized in reality. Given the truth that DMU_6 and DMU_9 are designed from the view of minimum stop times, and other solutions are all the optimal design from different perspectives, the calculation result is rational. $DMU_3 > DMU_2 > DMU_1$ Are all common-efficient DMUs which shouldn't be accepted in principle? $DMU_4, DMU_5, DMU_7, DMU_8$ and DMU_{10} are strong efficient ones that can be accepted if conditions permit; so, their relative merits should be further studied. The detailed ranking result from formula (3.2) - (3.4) is shown in Table 6.

Table 6. Ω_b -efficiency of the strong efficient DMUs.

DMU	4	5	7	8	10
Ω_b	0.092866	0.07757	0.04128	0.489643	0.041478

As shown in Table 4.4, the ranking of the five strong efficient DMUs is $DMU_8 > DMU_4 > DMU_5 > DMU_{10} > DMU_7$, and so we can know the ranking of all the DMUs is $DMU_8 > DMU_4 > DMU_5 > DMU_{10} > DMU_7 > DMU_3 > DMU_2 > DMU_1 > DMU_6 > DMU_9$. Considering economy, technology and service, the 8th train operation scheme is the best one that maximizes the comprehensive benefits and should be adopted at first; this is also compatible with the essence of DMU_8 taking into account multiple benefits, that is, DMU_8 is more rational. From the value of Ω_b , it is easy to know that there is a significant difference between DMU_8 and the other 4 strong efficient DMUs while they differ little from each other. So, we can see DMU_8 has absolute advantage over the other four DMUs. This is the influence of the objects of these train operation schemes, different optimization goals will lead to different results. Since all the five ones are all strong efficient, DMU_4, DMU_5, DMU_7 and DMU_{10} can be adopted successively when the reality not permit.

5.3 Method Comparison.

The results in this paper will be compared with that of the method in literature [10]. The method in [10] classifies strong and weak effective DMUs into one category and calculating their impact metrics on effective and non-efficient DMUs respectively; after integrating the two indicators, the final ranking is as shown in Table 7 below:

Table 7. The ranking results of the contrastive method.

ranking	1	2	3	4	5	6	7	8	9	10
DMU	DMU_8	DMU_4	DMU_1	DMU_5	DMU_3	DMU_2	DMU_7	DMU_{10}	DMU_6	DMU_9

The key of applying DEA method is effective DMUs analysis, so DMU_6 and DMU_9 can be excluded from consideration. Comparing the two sets of results, we can find that the two methods have the same judgment on efficient and non-efficient DMU classification but the ranking of the efficient ones is obviously different. By contrast, this paper researches the strong efficient, common efficient and non-efficient DMUs and integrate the results finally. Therefore, the results of the method in this paper is consistent with the principle of strong efficient better than common efficient better than non-efficient DMUs, which is not reflected in literature [10]. In literature [10], some of the common efficient ones are judged better than some strong efficient DMUs, and this can obviously prove the scientific, the ranking of the 5 strong efficient DMUs in the contrastive method is roughly consistent with that in this paper, so that can also prove that the method proposed in this paper is effective for the strong and effective DMU sorting method.

6. Summary

There have been many researches on the application of DEA method in recent years, and the related models are being mature gradually. Since train operation scheme is a problem of multiple inputs and outputs, DEA is suitable for the evaluation and decision-making behaviors about it. Based on the existing researches, this paper proposes an evaluation and ranking method that includes all the strong efficient, common efficient and non-efficient DMUs, considering the requirements of the DEA method on the positive and negative indexes of the original data and the degree of freedom. In the meantime, it can also improve its application universality by selecting appropriate indexes according to the reality. Finally, the rationality of this model is verified through case analysis and method comparison. Therefore, the DEA-based model of train operation scheme evaluation proposed in this paper has practical guiding significance.

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