

# Inertial Measurement Unit/Odometer Integrated Position and Orientation Determination Method

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**Keywords:** inertial measurement unit (IMU), odometer, position and orientation determination, dead reckoning, Kalman filter

**Abstract:** An accurate integrated position and orientation determination method for vehicles is provided in this paper, which utilises inertial measurement unit (IMU) and odometer. Firstly, inertial navigation calculation is accomplished by the outputs of IMU, and the attitude and position of the vehicle can be obtained. At the same time, the dead reckoning calculation is accomplished by the outputs of gyroscopes in IMU and odometer, and the attitude and position of the vehicle can also be obtained. The error sources of dead reckoning are analysed to establish the attitude error equations and position error equations of dead reckoning. Then the position and attitude outputs from IMU navigation and dead reckoning are taken to construct the measurements of integrated position and orientation determination. And thus the position integrated mode and the attitude/position integrated mode are designed respectively. Kalman filter is adopted to design the filtering algorithm. At last, the above two integrated mode are simulated. Simulation results show that, the attitude/position integrated mode has higher accuracy, so it is very suitable for the accurate independent position and orientation determination of a vehicle for long range.

## 1. Introduction

In order to achieve mobile operations, special military vehicles such as armoured vehicles and launchers are required to have high precision autonomous positioning and orientation ability, and they must also have strong anti-interference ability in complex electromagnetic interference environments.

At present, there are many researches on satellite-based vehicle navigation and positioning technologies at home and abroad [1-2]. As we all know, satellite navigation signals are easily disturbed or shielded, and are easily shielded in areas such as jungles and canyons. In order to solve the problem of short-term loss of satellite signals, the literature [3] studies the vehicle-mounted satellite/map matching integrated navigation technology. However, map matching navigation requires that the vehicle must travel on a known road, which is obviously unfavourable for the vehicle's motoring. In recent years, domestic scholars have studied inertial/odometer-based vehicle-mounted integrated positioning and orientation determination technology with high autonomy and strong anti-interference ability [4-5]. However, inertial/odometer combinations currently have the problem that the errors of positioning and orientation determination slowly diverge with distance.

In order to effectively overcome the above problems, this paper proposes to use the output of the gyroscope and the output of the odometer in the IMU to independently perform the dead reckoning attitude calculation and position calculation, to construct the measurements by the position and orientation information obtained from solution inertial navigation and dead reckoning, where the integrated positioning and directional filtering algorithm is designed.

## 2. Scheme of IMU/Odometer Integrated Position and Orientation Determination

The navigation frame (n-frame) is defined as east-north-up. In the IMU/odometer integrated positioning and orientation determination system, the IMU obtains the information such as the

attitude and position of the vehicle solved by navigation, at the same time, dead reckoning calculations can also obtain the information of the attitude and position by using the angular increment of the gyroscope output and the distance increase of the odometer output. Then, the IMU navigation error and dead reckoning error are regarded as the state of the system, while the measurements are constructed by the position and attitude information obtained from the IMU solution and dead reckoning. Finally, the optimal estimation of state is used to correct the IMU calculation result and the dead reckoning result respectively, which the modified navigation parameters is regarded as the outputs of the IMU/odometer integrated navigation system. Therefore, the principle diagram of IMU/odometer combination system is shown in Fig.1:

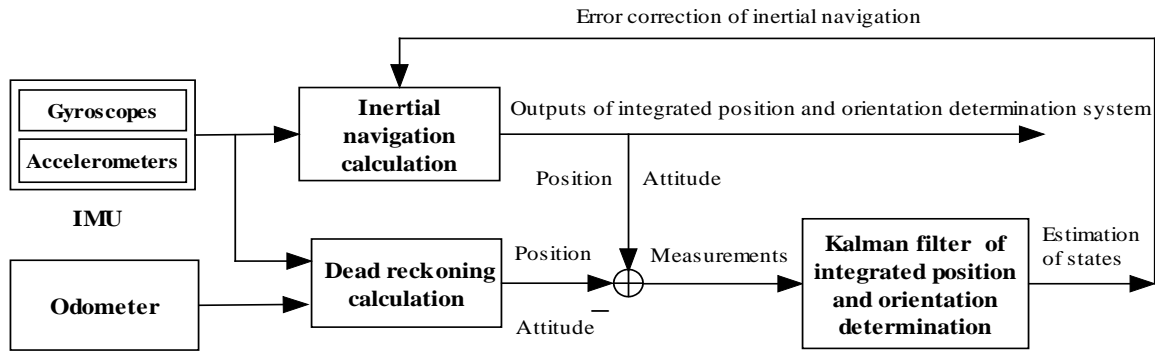


Fig. 1. The principle diagram of IMU/odometer combination system

### 3. Error Model of Integrated Positioning and Orientation Determination System

In the design of IMU/odometer integrated positioning and directional filtering algorithm, indirect method filtering will be used, which the system error is used as the state, and the optimal estimation value of the system error is obtained through filtering calculation. For this purpose, it is necessary to separately establish the error model of the navigation solution of the IMU and the error model of the gyro/odometer dead reckoning. Because the error model of the inertial navigation system is actually the error model of the strapdown inertial navigation system, it is described in detail in many literatures, so we will not describe them in detail in the article. Here, the emphasis is placed on establishing an error model for gyroscope/odometer dead reckoning.

Obviously, gyroscope error, odometer error, and initial condition error are the most important sources of error for gyroscope/odometer dead reckoning. Among them, the initial condition error can generally be solved effectively by the high precision initial alignment, so the article only considers the first two kinds of error sources.

For the gyroscope error, there are some random errors that cannot be calibrated. Usually there are two parts: the random constant part and the Gaussian white noise part. Therefore, the error of the gyroscope on the x, y, and z axes of the carrier can be expressed as:

$$\varepsilon_i = \varepsilon_{bi} + w_{gi} \quad (1)$$

where,  $w_{gi}$  is Gaussian white noise,  $\varepsilon_{bi}$  is a random constant drift that satisfies:

$$\dot{\varepsilon}_{bi} = 0, \quad (i = x, y, z) \quad (2)$$

For the odometer error, the error mainly appears as the scale factor error, which is usually related to the factors such as the tire wear condition, the inflation pressure level, the external environment temperature and the condition of the road surface. Under normal circumstances, the odometer scale coefficient error  $\delta K_D$  during each vehicle driving usually considered as a random constant as following:

$$\delta \dot{K}_D = 0 \quad (3)$$

Due to the objective errors of the above-mentioned sensor errors and initial condition errors, there are mathematical platform attitude errors, velocity errors, and position errors in the gyro/odometer heading estimation.

For attitude matrix derived from dead reckoning, its function is exactly the same as the attitude matrix of IMU. Therefore, the attitude error equations of the dead-reported math platform can be established by referring to the derivation method of strapdown inertial error equation. The process of derivation will not be repeated here. The equation for the attitude error of the mathematical platform is given directly as follows [6].

$$\dot{\phi}_D = M_1\phi_D + M_2\delta P_D + M_3\delta K_D + M_4\varepsilon_b + M_5w_g \quad (4)$$

Where  $\phi_D = [\phi_{DE}, \phi_{DN}, \phi_{DU}]^T$ ,  $\delta P_D = [\delta L_D, \delta \lambda_D, \delta h_D]^T$ ,  $\varepsilon_b = [\varepsilon_{bx}, \varepsilon_{by}, \varepsilon_{bz}]^T$ ,  $w_g = [w_{gx}, w_{gy}, w_{gz}]^T$ ,  $\phi_{DE}, \phi_{DN}, \phi_{DU}$  are the error angles of the mathematical platform along the east, north, and up directions,  $\delta L_D, \delta \lambda_D, \delta h_D$  are longitude, latitude and height errors.

Similarly, with reference to the derivation method of the strapdown inertial error equation, the position error equation for dead reckoning can be obtained as:

$$\delta \dot{P}_D = M_6\phi_D + M_7\delta P_D + M_8\delta K_D \quad (5)$$

The specific expression of the above matrix  $M_1, M_2 \dots M_8$  can be found in [6].

Due to the presence of odometer scale factor error  $\delta K_D$ , the measured value  $\hat{V}_D^b$  of the vehicle speed and the projection of the true value  $V_D^b$  under body frame meet the following relationship:

$$\hat{V}_D^b = (1 + \delta K_D)V_D^b \quad (6)$$

Supposing the attitude matrix calculated by the dead reckoning is  $\hat{C}_{Db}^n$ , the projection of the measured value of the vehicle speed under the navigation frame  $\hat{V}_D^n$  is as following:

$$\hat{V}_D^n = \hat{C}_{Db}^n \hat{V}_D^b = (I - \phi_D \times) C_b^n \hat{V}_D^b \quad (7)$$

Substituting Eq. (6) into Eq. (7), expanding and ignoring the second-order small variable of error

$$\hat{V}_D^n = V_D^n - (\phi_D \times) C_b^n V_D^b + \delta K_D C_b^n V_D^b \quad (8)$$

Due to the speed error  $\delta V_D^n = \hat{V}_D^n - V_D^n$ , the equation of speed error calculated from the dead reckoning according to formula (8) is as following:

$$\delta V_D^n = -(\phi_D \times) V_D^n + \delta K_D V_D^n \quad (9)$$

Where,  $\delta V_D^n = [\delta V_{DE}, \delta V_{DN}, \delta V_{DU}]^T$ ,  $\delta V_{DE}, \delta V_{DN}, \delta V_{DU}$  are the error of the dead reckoning in the direction of east, north and up, respectively.

## 4. Filtering algorithm of integrated positioning and orientation determination

### 4.1. System State Equation

Indirect filtering is used to design the IMU/odometer combination system filtering algorithm, which selects the system error as the filter state. Based on the systematic error model established above, the error of the IMU and the dead reckoning error are used as the system state. Where, there is a linear relationship between the dead reckoning speed error, the mathematical platform attitude error and odometer scale coefficient error according to equation (9), so the dead reckoning speed error is no longer included in the system state, that is, the system state  $X$  is

$$X = [\phi_E, \phi_N, \phi_U, \delta v_E, \delta v_N, \delta v_U, \delta L, \delta \lambda, \delta h, \varepsilon_{bx}, \varepsilon_{by}, \varepsilon_{bz}, \nabla_{bx}, \nabla_{by}, \nabla_{bz}, \phi_{DE}, \phi_{DN}, \phi_{DU}, \delta L_D, \delta \lambda_D, \delta h_D, \delta K_D]^T \quad (10)$$

Where,  $\phi_E, \phi_N, \phi_U$  are the error angles of the IMU's mathematical platform in the direction of east, north and up,  $\delta v_E, \delta v_N, \delta v_U$  are the inertia errors of the IMU in the direction of east, north and up.  $\delta L, \delta \lambda, \delta h$  are the error of latitude, longitude and height,  $\varepsilon_{bx}, \varepsilon_{by}, \varepsilon_{bz}$  are the constant drift of the gyroscope on the body frame,  $\nabla_{bx}, \nabla_{by}, \nabla_{bz}$  are the constant error of the accelerometer on the direction of  $x, y$  and  $z$ ,  $\phi_{DE}, \phi_{DN}, \phi_{DU}$  are the error angle of the mathematic platform in the direction

of the east, north and up for the dead reckoning,  $\delta L_D, \delta \lambda_D, \delta h_D$  are the error of latitude, longitude and height for the dead reckoning,  $\delta K_D$  is the odometer Scale factor error.

According to the inertial navigation error model and the dead reckoning error model, state equation of IMU/odometer integrated positioning and orientation determination can be written as:

$$\dot{X} = FX + GW \quad (11)$$

Where,  $F$  is the system state matrix,  $G$  is the system noise driver array, and  $W$  is the system noise.

## 4.2. Measurement Equation

Since the navigation of the navigation system and the dead reckoning can output information such as the position and attitude of the vehicle, there are many measurement options. Taking into account the use of positioning and orientation determination, this article focuses on the use of position information, regarding the two models of position and the combination of position/attitude information as a measurement, where establishing the corresponding measurement equation.

### 4.2.1. Measurement equation of position combination mode

Since the position combination mode is adopted, the difference of the position information of the vehicle between IMU and the dead reckoning is regarded as the measurement information:

$$Z_1 = [L_S - L_D \quad \lambda_S - \lambda_D \quad h_S - h_D]^T \quad (12)$$

Where,  $L_S, \lambda_S, h_S$  are the outputs of position by the inertial navigation.  $L_D, \lambda_D, h_D$  are the outputs of position by the dead reckoning.

Since the position outputs of the IMU navigation and the dead reckoning contain errors:

$$Z_1 = \begin{bmatrix} (L + \delta L) - (L + \delta L_D) \\ (\lambda + \delta \lambda) - (\lambda + \delta \lambda_D) \\ (h + \delta h) - (h + \delta h_D) \end{bmatrix} = \begin{bmatrix} \delta L - \delta L_D \\ \delta \lambda - \delta \lambda_D \\ \delta h - \delta h_D \end{bmatrix} \quad (13)$$

Obviously, combining with system state  $X$ ,  $Z_1$  can be expressed as following according to equation (13):

$$Z_1 = H_1 X + V_1 \quad (14)$$

Equation (14) is the measurement equation when using position combination. Where,  $H_1$  is the measurement matrix,  $V_1$  is measurement noise.

### 4.2.2. Measurement equation of attitude/position combination mode

Since the attitude/position combination is adopted, the attitude and position information of the vehicle contained in the navigation of the IMU will be subtracted from the position and position information of the dead reckoning output as measurement, so the measurement  $Z_2$  is as following:

$$Z_2 = [\psi_S - \psi_D \quad \theta_S - \theta_D \quad \gamma_S - \gamma_D \quad L_S - L_D \quad \lambda_S - \lambda_D \quad h_S - h_D]^T \quad (15)$$

where,  $\psi_S, \theta_S, \gamma_S$  are the output of vehicle attitude (heading, pitch, and roll angle) from the IMU.  $\psi_D, \theta_D, \gamma_D$  are the output of vehicle attitude (heading, pitch, and roll angle) from the dead reckoning.

Similar to the derivation method of the above formula (13), according to equation (15):

$$Z_2 = \begin{bmatrix} \delta \psi - \delta \psi_D \\ \delta \theta - \delta \theta_D \\ \delta \gamma - \delta \gamma_D \\ \delta L - \delta L_D \\ \delta \lambda - \delta \lambda_D \\ \delta h - \delta h_D \end{bmatrix} \quad (16)$$

where,  $\delta\psi, \delta\theta, \delta\gamma$  are the heading, pitch and roll angle errors solved by the IMU,  $\delta\psi_D, \delta\theta_D, \delta\gamma_D$  are the heading, pitch, and roll angle errors are derived from dead reckoning.

The carrier attitude error solved by the IMU and the inertial math platform attitude error have the following relationship [7]:

$$\delta\psi = \phi_U - \frac{T_{32}T_{22}}{T_{12}^2 + T_{22}^2} \phi_N - \frac{T_{12}T_{32}}{T_{12}^2 + T_{22}^2} \phi_E \quad (17)$$

$$\delta\theta = \frac{T_{12}}{\sqrt{1-T_{32}^2}} \phi_N - \frac{T_{22}}{\sqrt{1-T_{32}^2}} \phi_E \quad (18)$$

$$\delta\gamma = \frac{T_{21}T_{33} - T_{23}T_{31}}{T_{31}^2 + T_{33}^2} \phi_E + \frac{T_{13}T_{31} - T_{11}T_{33}}{T_{31}^2 + T_{33}^2} \phi_N \quad (19)$$

Similarly, there is a similar relationship between the attitude error of the dead reckoning carrier and the attitude error of the dead reckoning mathematical platform. The form is exactly the same as from (17) to (19), and it will not be repeated here.

Thus, substituting equations (17)-(19) into equation (16) can be obtained the following:

$$\mathbf{Z}_2 = \begin{bmatrix} -\frac{T_{12}T_{32}}{T_{12}^2 + T_{22}^2} \phi_E - \frac{T_{32}T_{22}}{T_{12}^2 + T_{22}^2} \phi_N + \phi_U + \frac{T_{12}T_{32}}{T_{12}^2 + T_{22}^2} \phi_{DE} + \frac{T_{32}T_{22}}{T_{12}^2 + T_{22}^2} \phi_{DN} - \phi_{DU} \\ -\frac{T_{22}}{\sqrt{1-T_{32}^2}} \phi_E + \frac{T_{12}}{\sqrt{1-T_{32}^2}} \phi_N + \frac{T_{22}}{\sqrt{1-T_{32}^2}} \phi_{DE} - \frac{T_{12}}{\sqrt{1-T_{32}^2}} \phi_{DN} \\ \frac{T_{21}T_{33} - T_{23}T_{31}}{T_{31}^2 + T_{33}^2} \phi_E + \frac{T_{13}T_{31} - T_{11}T_{33}}{T_{31}^2 + T_{33}^2} \phi_N - \frac{T_{21}T_{33} - T_{23}T_{31}}{T_{31}^2 + T_{33}^2} \phi_{DE} - \frac{T_{13}T_{31} - T_{11}T_{33}}{T_{31}^2 + T_{33}^2} \phi_{DN} \\ \delta L - \delta L_D \\ \delta \lambda - \delta \lambda_D \\ \delta h - \delta h_D \end{bmatrix} \quad (20)$$

Similarly, the measurement  $\mathbf{Z}_2$  can be expressed as following according to equation (20):

$$\mathbf{Z}_2 = \mathbf{H}_2 \mathbf{X} + \mathbf{V}_2 \quad (21)$$

Equation (21) is the measurement equation when using the model of attitude/position. Where,  $\mathbf{H}_2$  is the measurement matrix,  $\mathbf{V}_2$  is measurement noise.

Thus, after obtaining the state and measurement equations of system, Kalman filter equations can be used for filtering calculations to obtain the real-time estimation of the system state  $\hat{\mathbf{X}}_k$ . At this time, the estimated value  $\hat{\mathbf{X}}_k$  is used to perform error correction on the navigation result of the IMU and the dead reckoning, and the corrected IMU navigation output is used as the output of the IMU/odometer integrated system.

## 5. Simulation and conclusion

Supposing the gyro constant drift in the inertia measurement unit is  $0.03^\circ/\text{h}$ , the random walk error is  $0.003^\circ/$ , The accelerometer's constant error is 10-4g, random walk error is 10-5, the odometer's scale factor error is 0.3%D (D is the driving distance), the initial level attitude error of IMU navigation is 3', the heading error is 10', the initial position error is 10m, and the initial speed error is 0.1m/s. The initial condition of dead reckoning is the same as the navigation solution of the IMU. The filtering period is 1 s and the simulation time is 3600s.

Based on the above simulation conditions, the computer simulation is performed on the integrated positioning and orientation algorithm under the position combination mode and attitude/position combination mode studied in the paper. The simulation results are shown in Fig. 2~3. Among them, the "thin dotted line" is the result in the position mode, and the "thick solid line" is the simulation result in the attitude/position combination mode.

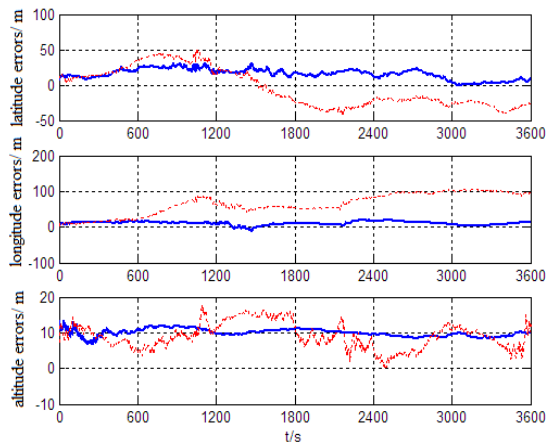


Fig. 2. Position errors

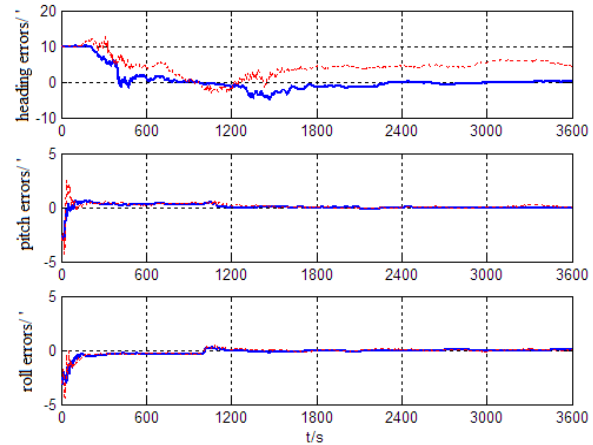


Fig. 3. Attitude errors

According to Figures 2 and 3, it can be seen that after the combination of IMU and odometer, the positioning and orientation system achieves higher accuracy, effectively suppresses the inherent deficiencies of the divergence of the purely inertial navigation error over time, and the accuracy of attitude/position combination mode is obviously better than the position combination mode: In the simulation time up to 3600s, the horizontal position accuracy of the former reaches  $\pm 30.5\text{m}$ , the altitude accuracy reaches  $\pm 12.2\text{m}$ , the heading accuracy reaches  $\pm 4.6'$ , and the horizontal attitude accuracy reaches  $\pm 0.5'$ . The latter's horizontal position accuracy is  $\pm 105.6\text{m}$ , altitude accuracy is  $\pm 17.5\text{m}$ , heading accuracy is  $\pm 6.3'$ , and horizontal attitude accuracy is  $\pm 0.6'$ . Obviously, in addition to position information, attitude information is also introduced into the measurement, which significantly improves the overall positioning and orientation accuracy of the system.

To sum up, this article studies the method of using IMU and odometer to carry out high-precision integrated positioning and orientation in on-board vehicle. It is proposed that the IMU and odometer's dead reckoning are independently performed for navigation calculation, and the use of IMU solution and dead reckoning is obtained. The position and attitude information is used to construct a combination of positioning and orientation measurements, and a Kalman filter is used to design the integrated positioning and directional filtering algorithm. The verification results show that the attitude/position combination mode has higher accuracy and is very suitable for long-term autonomous positioning and orientation determination under vehicle conditions. Therefore, this paper has good theoretical research and engineering application value.

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