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Tests for Spatial Correlation of Dependent Variables in Spatial Dynamic Panel Data Models

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***Abstract*—This paper considers a spatial dynamic panel data regression model with fixed effects, spatial correlation of dependent variables and error serial among cross sectional units. When the number of individuals n , the number of time periods T are large, and T is asymptotically large relative to n, the paper derives various Lagrange multiplier tests and likelihood ratio test statistics for this panel data regression model including tests for spatial correlation of dependent variables in jointly, marginally or conditionally (one -dimensional and two –dimensional). Limiting null distributions of the tests are derived.**

Keywords—dynamic spatial panel data; spatial correlation; lagrange multiplier tests; likelihood ratio tests

I INTRODUCTION

Spatial panel models are becoming popular in econometric applications. To capture correlation among cross-sectional units, the spatial autoregressive (SAR) model by Cliff and Ord (1973) has received the most attention in economics. The texting for SAR models, Baltagi and Li (1995) addressed the problem of jointly testing for error serials correlation and individual effects for the panel data model with no spatial effects; Anselin (1988, 2001) and Anselin and Bera (1998), have studied the problem of testing for spatial dependence; Baltagi et al. (2003) considered the problem of jointly testing for random region effects in the panel as well as spatial correlation of error serials across these regions. however, the study did not consider the added problem of serial correlation in the remainder error term. Baltagi et al. (2007) consider the testing of spatial and serial dependence in an extended model, where serial correlation on each spatial unit over time, in addition to spatial dependence across spatial units are allowed in the disturbances; Myoung Jin Jang and Dong Wan Shin(2014) consider the texting for time effects and spatial effects by Lagrange multiplier and likelihood ratio texts about panel model with spatial autocorrelation and heterogeneity of error serials across time, they derived the limiting null distributions of test statistics jointly, marginally or conditionally. These panel models do not incorporate time lagged dependent variables as dynamic structures in the regression equation, this paper shall generalizes the previous studies by deriving joint , conditional and marginal tests of LM and LR, that consider the texting of the lag effects on dependent variables, spatial and serial dependence in the extended dynamic panel model with spatial correlation of dependent variables and error serial among cross sectional units, and studies their small sample properties using Monte Carlo experiments, our Monte Carlo evidence shows their good size properties and power properties except for a few cases where there is significant size distortion and the lower power.

The remainder of the paper is organized as follows. Section 2 presents a spatial dynamic panel data regression model with fixed effects and spatial correlation of dependent variables cross sectional units. Section 3 provides the LM and LR test statistics as well as their limiting null distributions.

II THE MODEL AND THE LIKELIHOOD FUNCTION

A. The Model

The regression model Considered in this paper is

$$Y_{nt} = \lambda_0 W_n Y_{nt} + \gamma_0 Y_{n,t-1} + X_{nt}\beta_0 + C_{n0} + U_{nt}$$

$$U_{nt} = \rho_0 M_n U_{nt} + V_{nt}$$

$$t = 1,2,...T \tag{1}$$

where $Y_{nt} = (y_{1t}, y_{2t}, ...y_{nt})$ and $V_{nt} = (v_{1t}, v_{2t}, ...v_{nt})$ are $n \times 1$ column vector and v_{it} is $i.i.d$ across i and d with zero mean and variance σ_0^2. W_n is an $n \times n$ nonstochastic spatial weights matrix that generates the spatial dependence on y_{it} between cross sectional units, X_{it} is the p-dimensional vector of covariates, and C_{n0} is an $n \times 1$ column vector of fixed effection. M_n is an $n \times n$ nonstochastic spatial weights matrix for the disturbance.

Therefore, the parameters in this model is $(\lambda, \gamma, \beta', \rho, \sigma^2)$. if unnecessary, will make the inference complicated and even inefficient when T or N is fixed.

Define $S(\lambda) = I_n - \lambda W_n$ and $R(\rho) = I_n - \rho M_n$ for any λ and ρ .at the true parameter Assuming the infinite sums are well-defined, by continuous substitution,

$$Y_{nt} = \sum_{h=1}^{\infty} (S_n^{-1})^h \gamma^{h-1} (C_{n0} + X_{n,t-h}\beta_0 + (R_n^{-1})^{h-1} V_{n,t-h})$$

$$= A_n + \mathrm{X}_{nt}\beta_0 + E_n$$

Where

$A_n = \sum_{h=1}^{\infty}(S_n^{-1})^h \gamma^{h-1} C_{n0}$, $\mathrm{X}_{nt} = \sum_{h=1}^{\infty}(S_n^{-1})^h \gamma^{h-1} X_{n,t-h}$,and $E_n = \sum_{h=1}^{\infty}(S_n^{-1})^h (\gamma R_n^{-1})^{h-1} V_{n,t-h}$.(2)

B. The Concentrated Likelihood Function

Denote $\theta = (\lambda, \gamma, \rho, \sigma^2)$, θ_0 indicates the true value of parameters. For notational purpose,we define $\tilde{Y}_{nt} = Y_{nt} - \frac{1}{T}\sum_{t=1}^{T} Y_{nt}$, $\bar{Y}_{nT,-1} = \frac{1}{T}\sum_{t=1}^{T} Y_{n,t-1}$, $S = S_n(\lambda)$, $\hat{S} = S_n(\hat{\lambda})$, $\breve{S} = S_n(\breve{\lambda})$. $\tilde{Y} = (\tilde{Y}'_{n1}, ..., \tilde{Y}'_{nT})'$, $\tilde{Y}_{-1} = (\tilde{Y}'_{n0}, ..., \tilde{Y}'_{nT-1})'$.Where $\hat{\lambda}$ is unrestricted QMLES, $\breve{\lambda}$ is restricted QMLES, Similarry, we define $\tilde{X}_{nt}, \tilde{V}_{nt}, R, \hat{R}, \tilde{R}, \tilde{X}, \tilde{X}_{-1}, \tilde{X}, \tilde{X}_{-1}$.

By the transformation approach we get the likelihood function is

$$L_{nT}(\theta) = -\frac{n(T-1)}{2}\ln 2\pi - \frac{n(T-1)}{2}\ln\sigma^2 + (T-1)(\ln|S| + \ln|R|) - \frac{1}{2\sigma^2}\sum_{t=1}^{T}\tilde{V}'_{nt}\tilde{V}_{nt} \quad (3)$$

Where $\tilde{V}_{nt} = R[S\tilde{Y}_{nt} - \gamma\tilde{Y}_{n,t-1} - \tilde{X}_{nt}\beta]$ (4)

III TEST STATISTICS

In this section, we derive LM tests and LR tests as well as their limiting null distributions. The LM statistic for testing a specific null hypothesis H_0 against H_1 is given by

$$LM = \left(\frac{\partial L_{nT}}{\partial\theta_*}\bigg|_{\theta_*=\tilde{\theta}_*}\right)'\left[E\left(\frac{\partial^2 L_{nT}(\theta,\beta)}{\partial\theta_*\partial\theta_*'}\right)\bigg|_{\theta_*=\tilde{\theta}_*}\right]^{-1}\left(\frac{\partial L_{nT}}{\partial\theta_*}\bigg|_{\theta_*=\tilde{\theta}_*}\right) \quad (5)$$

Where $\theta_* = (\lambda, \gamma, \rho)'$, $\tilde{\theta}_*$ is the restricted QMLES under H_0 ,and $\frac{\partial L_{nT}(\theta,\beta)}{\partial\theta_*}\bigg|_{\theta_*=\tilde{\theta}}$ and $\frac{\partial^2 L_{nT}(\theta,\beta)}{\partial\theta_*\partial\theta_*'}\bigg|_{\theta_*=\tilde{\theta}}$ are the partial derivatives with respect to θ , evaluated at $\tilde{\theta}$ .The LR statistic is given by $LR = L(\hat{\theta}) - L(\tilde{\theta})$, where $\hat{\theta}$ is the unrestricted MLE. For simplicity of notation, let $I(\theta) = E(\partial^2 L_{nT}(\theta,\beta)/\partial\theta\partial\theta')$, $L_U = L(\hat{\theta})$ and

$$L_R = L(\tilde{\theta})$$

IV TESTS FOR SPATIAL CORRELATION ON y_{it} BETWEEN CROSS SECTIONAL UNITS $H_0: \lambda = 0$

We derive marginal and conditional tests for s for spatial correlation on y_{it} between cross sectional units.

A. One -dimensional Conditional Tests for Spatial Correlation $H_0: \lambda = 0$ Assuming Admitting Non-zero γ and ρ

Under H_0

$$\tilde{Y}_{nt} = \sum_{h=0}^{\infty}\gamma^h(\tilde{X}_{n,t-h}\beta_0 + R_n^{-1}\tilde{V}_{n,t-h})$$

$$= \tilde{X}_{nt}\beta + R_n^{-1}E_{nt}$$

Where, $\tilde{X}_{nt} = \sum_{h=0}^{\infty}\gamma^h \tilde{X}_{n,t-h}$, $E_{nt} = \sum_{h=0}^{\infty}\gamma^h \tilde{V}_{n,t-h}$

Under H_0, we have

$$\frac{\partial L_{nT}}{\partial\theta_*}\bigg|^{c}_{\theta_*=\hat{\theta}_*} = \begin{pmatrix} D(\hat{\lambda}) \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\hat{\sigma}^2}\sum_{t=1}^{T}(\hat{R}W\tilde{Y}_{nt})'\tilde{V}_{nt} \\ 0 \\ 0 \end{pmatrix} \quad (14)$$

the elements of the information matrix $I(\hat{\theta}_*)\Big|^{c}_{H_0} = E\left(\frac{\partial^2 L_{nT}(\theta,\beta)}{\partial\theta_*\partial\theta_*'}\right)\bigg|^{c}_{\theta_*=\tilde{\theta}}$ are given by

$$I_{11}\Big|^{d}_{H_0} = (T-1)trW^2 + \hat{A}_r\, tr(\hat{R}^{-1}W'\hat{R}'\hat{R}W\hat{R}^{-1}) + \frac{1}{\hat{\sigma}^2}\hat{\beta}'\sum_{t=1}^{T}\tilde{X}'_{nt}W'\hat{R}'\hat{R}W\tilde{X}_{nt}\hat{\beta}$$

$$I_{12}\Big|^{d}_{H_0} = \frac{1}{\hat{\sigma}^2}\hat{\beta}'\sum_{t=1}^{T}\tilde{X}'_{nt}W'\hat{R}'\hat{R}W\tilde{X}_{nt-1}\hat{\beta}$$

$$I_{13}\Big|^{d}_{H_0} = \hat{B}_\lambda\left[T - 1 - \frac{(\hat{\gamma} - \hat{\gamma}^T)}{(1-\hat{\gamma})(1-\hat{\gamma}^2)} + \frac{\sum_{i=1}^{T-1} i\hat{\gamma}^i}{(1-\hat{\gamma}^2)T}\right]$$

$$I_{22}\Big|_{H_0}^{d} = \frac{1}{\hat{\sigma}^2}\hat{\beta}'\sum_{t=1}^{T}\hat{X}'_{nt-1}\hat{R}'\hat{R}\tilde{X}_{nt-1}\hat{\beta} + A_\lambda n$$

$$I_{23}\Big|_{H_0}^{d} = -2\left[\frac{(T-1)-(T-1)(\hat{\gamma}-\hat{\gamma}^{T-1})}{T(1-\hat{\gamma})} - \frac{\sum_{i=1}^{T-2} i\hat{\gamma}^{i}}{T}\right]tr(M\hat{R}^{-1})$$

$$I_{33}\Big|_{H_0}^{c} = (T-1)tr[\hat{H}_n'\hat{H}_n + \hat{H}_n^2]$$

Where $\hat{A}_\lambda = \left[\frac{T-1}{(1-\hat{\gamma}^2)} - \frac{2(1-\hat{\gamma}^T)}{(1-\hat{\gamma})(1-\hat{\gamma}^2)} + \frac{2\sum_{i=1}^{T-1} i\hat{\gamma}^{i}}{T(1-\hat{\gamma}^2)}\right]$

$$\hat{B}_\lambda = tr[\hat{R}^{-1}WM + (\hat{R}^{-1})^2 W\hat{R}M]$$

$$\hat{\beta} = (\tilde{X}_n'\hat{R}'\hat{R}\tilde{X}_n)^{-1}\tilde{X}_n'\hat{R}'\hat{R}(\tilde{Y}_{nt} - \hat{\gamma}\tilde{Y}_{nt-1}),$$

$$\hat{\tilde{V}}_{nt} = \hat{R}(\tilde{Y}_{nt} - \hat{\gamma}\tilde{Y}_{n,t-1} - \tilde{X}_{nt}\hat{\beta}), \hat{\sigma}^2 = \sum_{t=1}^{T}\hat{\tilde{V}}_{nt}'\hat{\tilde{V}}_{nt} / n(T-1) \quad (15)$$

So, by equation (5) the resulting conditional LM statistic is given

$$LM_\lambda = D(\hat{\lambda})^2 / \left\{(T-1)trW^2 + A_\lambda tr(\hat{R}^{-1}W'\hat{R}'\hat{R}W\hat{R}^{-1}) + \frac{1}{\sigma^2}\hat{\beta}'\sum_{t=1}^{T}\hat{X}'_{nt-1}W'\hat{R}'\hat{R}W\hat{X}_{nt-1}\hat{\beta}\right\} \quad (16)$$

The derivation of this LM test statistic is given in Appendix A.3. This LM statistic should be asymptotically distributed as χ_1^2 under H_0 as $T, N \to \infty$ which will be formally proved in Theorem 3 in Section 3.6.

The conditional LR test t for $H_0: \lambda = 0$ vs $H_1: \lambda \neq 0$ admitting non-zero λ and ρ is given by $LR_\lambda = 2(L_U - L_R)$

Where

$$L_U = -\frac{n(T-1)}{2}(\ln 2\pi\hat{\sigma}^2 + 1) + (T-1)(\ln|\hat{S}| + \ln|\hat{R}|),$$

$$L_R = -\frac{n(T-1)}{2}(\ln 2\pi\hat{\sigma}^2 + 1) + (T-1)\ln|\hat{R}|$$

Where the unrestricted MLEs are same as equation (9).

Under H_0 the LR_λ statistic is asymptotically distributed as χ_1^2

B. One -dimensional Marginal Tests for Spatial Error Correlation $H_0: \lambda = 0$ *Assuming* $\gamma = \rho = 0$

This is a one-dimensional marginal test for spatial dependence on y_{it} no ignoring the presence of lag effect and spatial error correlation cross sectional units. when $\gamma = \rho = 0$, R reduce to unit matrix , under the null hypothesis, $\hat{\beta} = (\tilde{X}_n'\tilde{X}_n)^{-1}\tilde{X}_n'\tilde{Y}_n$, $\hat{\tilde{V}}_{nt} = \tilde{Y}_{nt} - \tilde{X}_{nt}\hat{\beta}$, $\hat{\sigma}^2 = \sum_{t=1}^{T}\hat{\tilde{V}}_{nt}'\hat{\tilde{V}}_{nt}/n(T-1)$

$D(\hat{\lambda}) = \frac{1}{\hat{\sigma}^2}\sum_{t=1}^{T}(W\tilde{Y}_{nt})'\hat{\tilde{V}}_{nt}$, $I_{11}\Big|_{H_0}^{c} 2(T-1)trW^2 + \frac{1}{\sigma^2}\hat{\beta}'\sum_{t=1}^{T}\tilde{X}_{nt}'W'W\tilde{X}_{nt}\hat{\beta}$

the LM test statistic for testing H_0 ,call it $LM_{\lambda/\gamma\rho}$

$$LM_{\lambda/\gamma\rho} = D(\hat{\gamma})^2 / (2(T-1)trW^2 + \frac{1}{\sigma^2}\hat{\beta}\sum_{t=1}^{T}X_{nt}'W'W\tilde{X}_{nt}\hat{\beta})\hat{\tilde{W}}_{nt} / n(T-1) \quad (17)$$

This LM statistic should be asymptotically distributed as χ_1^2 under H_0 as as $T, n \to \infty$ which will be formally proved in Theorem 2 in Section 3.5.

The Marginal LR test for $H_0: \gamma = 0$ vs $H_1: \gamma \neq 0$ assuming $\lambda = \rho = 0$ is given by $LR_{\lambda/\gamma\rho} = 2(L_U - L_R)$

$$L_U = -\frac{n(T-1)}{2}(\ln 2\pi\hat{\sigma}^2 + 1), \quad L_R = -\frac{n(T-1)}{2}(\ln 2\pi\hat{\sigma}^2 + 1)$$

Where $\hat{\beta} = (\tilde{X}_n'\tilde{X}_n)^{-1}\tilde{X}_n'(\tilde{Y}_{nt} - \hat{\gamma}\tilde{Y}_{nt-1})$,

$$\hat{\tilde{V}}_{nt} = \tilde{Y}_{nt} - \hat{\gamma}\tilde{Y}_{nt-1} - \tilde{X}_{nt-1}\hat{\beta}, \hat{\sigma}^2 = \sum_{t=1}^{T}\hat{\tilde{V}}_{nt}'\hat{\tilde{V}}_{nt} / n(T-1),$$

Under H_0 the $LR_{\lambda/\gamma\rho}$ statistic is asymptotically distributed as χ_1^2

V SUMMARY

This paper generalized the model in Baltagi and Li (1995) paper, Baltagi et al. (2003) paper and Baltagi et al. (2007) paper, and considers a spatial dynamic panel data regression model with fixed effects, spatial correlation of dependent variables and error serial among cross sectional units. When the number of individuals n, the number of time periods T are large, and T is asymptotically large relative to n, the paper derives various Lagrange multiplier tests and likelihood ratio test statistics for this panel data regression model including tests for spatial correlation of dependent variables. Limiting null distributions of the tests are derived.

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