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Mathematical Modeling for the Whirling Method OF Multi-variant Screws

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***Abstract*—Multi-variant screws are a kind of complex screws with two or more variable parameters, such as the lead, diameters, groove depth and edge width. These screws are increasingly used in machinery to compress, transfer, or process materials in many industries, and their sophisticated geometry plays a crucial role in the production. With their increasing application in industry, it is in desperate need to manufacture these screws with high efficiency and low cost. This paper proposes a whirling based approach to fabricating multi-variant screws. Based on the geometric characteristics of the screws, mathematic models are established for describing multi-variant screw surfaces and so is the equivalent surfaces of the rotating whirling tool. The spatial relationship of the cutters and the design screw surfaces is discussed and the corresponding method of tool path calculation is presented.**

***Keywords-* Screws; Whirling; Variable diameter screw; Variable lead screw**

I. INTRODUCTION

Screws are a typical category of components with spiral surface which are extensively used in machinery, such as thread fasteners, worms, helical gears, etc[1]. Most screws, which have constant parameters, such as diameter, pitch, groove depth, and tooth form, are usually cut into shape on lathes, milling machines or grinding machines[2, 3].

In addition to those common screws, there are also screws or screw-type components characteristic of variable parameters, such as variable pitch screws, variable groove depth screws, taper screws, etc[4]. These screws are extensively applied in material compression, transportation and mixing in many industries[5], the sophisticated geometry of which has a profound impact on the production and therefore have been drawing more and more attention[6].

Due to complexity of the screw shape, multi-axis CNC(Computer Numerical Control) machines are often required for manufacturing this kind of components. Those methods are well-developed in screw manufacture field, but there are also some aspects can be improved

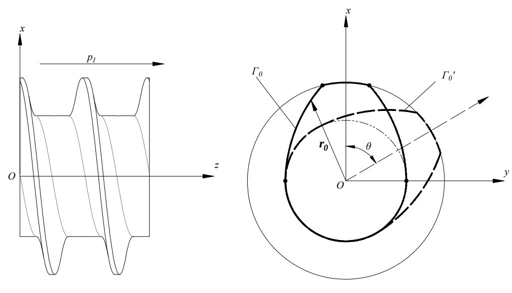
In order for higher efficiency and lower cost, a relatively new machine process, whirling[7], was exploited for cylindrical helical surfaces. Ni et al. studied the machining mechanism of internal whirling process for precise external thread[8].Han et al. presented a theoretical model[9] and a basic solution for whirling cylindrical screws using common inserts instead of profiled cutters[10]. Song et al. proposed a method of modelling and simulation of whirling basing on equivalent cutting volume[11]. Sun et al. researched the milling technique of double-thread via internal whirling[12]. This method so far is applicable only to relatively simple screws that have just one or two changing factors. Aiming to extend it for multi-variant screws, which have more than two parameters changing synchronously, this paper investigates the mathematical model, machining mechanism and toolpath planning with respect to the whirling process of multi-variant screws

II. MODELING OF THE MULTI-VARIANT SCREWS

A. Modeling of the Common Screws

Common screws are a kind of screws with constant parameters, such as diameter, pitch, groove depth, etc. The surface of a screw can be considered as the result of a moving generatrix. As shown in Figure 1, (o-*x,y,z*) is the Cartesian coordinate system(z-axis coincides with the axis of the screw) whose versors are $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$. The curve Γ_0 on *xoy* plane can be described as below(u is an independent variable parameter):

$$\vec{r}_0 = \vec{r}_0(u) = x_0(u)\vec{i} + y_0(u)\vec{j} \tag{1}$$



(a) (b)

FIGURE I. PROFILE OF THE COMMON SCREWS

When Γ_0 makes spiral movement around *z*-axis with constant pitch, the resulted helical surface can be written as:

$$\vec{R}_0 = \vec{R}_0(u,\theta) = \left(\theta\vec{k}\right) \otimes \vec{r}_0 \pm p_1\theta\vec{k} \tag{2}$$

(p_l is the parameter related to the lead; the "+" before the last item means Γ_0 do dextral motion along the *z*-axis to form a dextral surface; the "–"means Γ_0 do sinistral motion along the *z*-axis to form a sinistral surface)

The right part of Equation2 consists of two parts:

"$(\theta\boldsymbol{k})\otimes\boldsymbol{r_0}$" means Γ_0 rotate around the z-axis. As shown in Figure I b, when the original curve Γ_0 $(\boldsymbol{r_0})$ rotate θ degrees around the z-axis $(\boldsymbol{k})$, the curve moves to Γ_0' and Γ_0' can be described as "$(\theta\boldsymbol{k})\otimes\boldsymbol{r_0}$".

"$p_1\theta k$" means Γ_0's movement along the z-axis. As shown in Figure I a, the movement at z-axis (k) direction with the constant pitch p_1 can be described as "$p_1\theta k$".

B. Modeling of Variable Pitch Screws

According to the last section, the common screws can be described as a rotational motion on *xoy* plane and a spiral motion with constant pitch in *z*-axis direction. Based on this, the variable pitch screws can be described as a rotational motion on *xoy* plane and a spiral motion with variable pitch in *z*-axis direction, namely, the main difference is on the "$p_1\theta\boldsymbol{k}$" part.

Suppose $p_1 = p_1(z)$ is the equation of the changing pitch, the movement in *z*-axis direction can be written as(D is the integral region):

$$|L| = \left|\iint_D p_1(z)\, d\theta dz\right| \tag{3}$$

For variable pitch screws, the mathematic relationship between z and θ can also be expressed as $z = f_1(\theta)$:

$$|L| = \left|\int_{\theta_0}^{\theta_1}\left(p_1(f_1(\theta))\cdot f_1'(\theta)\right)d\theta\right| \tag{4}$$

Therefore, the model of the variable pitch screws can be expressed as:

$$\vec{R}_0 = \left(\theta\vec{k}\right)\otimes\vec{r}_0 \pm \left(\int_{\theta_0}^{\theta_1}\left(p_1(f_1(\theta))\cdot f_1'(\theta)\right)d\theta\right)\vec{k} \tag{5}$$

C. Modeling of the Multi-variant Screws

As pitch, diameter and groove depth are the most common factors of a screw in machinery that may change, this paper only considers those screw with variable pitch, diameter and groove depth (hereinafter referred to as multi-variant screw).

The following statements may be inferred:

(1)The screw-type components can be regarded as the combination of thread and cylinder. For example, the arc A_1B_1, B_1C_1 and D_1A_1(or $A_1'B_1'$, $B_1'C_1'$ and $D_1'A_1'$) in Figure II are the part of generatrices of the thread changing with the diameter of the screws on cross section. The arc C_1D_1 (or $C_1'D_1'$) is the part of generatrices of the cylinder whose profile is decided by the diameter, pitch and groove depth.

(2)The geometrical formation principle of the screw-type components can be explained in two ways: the profiles of the cross section on cross section scanned along the spiral line; the profiles of screw tooth on axial section scanned along the spiral line on the surface of the cylinder.

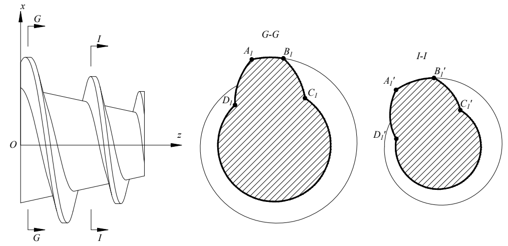


FIGURE II. THE CROSS SECTION OF THE MULTI-VARIANT SCREWS

In these views, it is possible to divide the model of the multi-variant screws into the model of thread and cylinder. Furthermore, because the complete projection of thread and cylinder on axial section (or section which forms a certain angle with axial section) are invariable, the models may be built from this point of view.

(1) *The Mathematical Model of the Cylinder Surface*

As illustrated in Figure III, it is the cylinder with variable diameter. The curve Γ_1 is a generatrix on *xoz* plane which can be expressed as(v is the independent variable parameter):

$$\Gamma_1:\ \vec{r}_1 = \vec{r}_1(v) = x_1(v)\vec{i} + z_1(v)\vec{k} \tag{6}$$

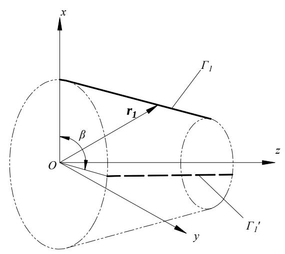


FIGURE III. THE MODEL OF THE CYLINDER SURFACE

When Γ_1 rotated to Γ_0' around *z*-axis, a conical surface is formed as(when Γ_1 rotate at the right-hand of the *z*-axis, β is positive, otherwise, β is negative):

$$\vec{R}_1 = \vec{R}_1(v,\beta) = \left(\beta\vec{k}\right)\otimes\vec{r}_1 \tag{7}$$

(2) *The mathematical model of the thread surface*

As the thread shown in Figure IV, suppose the equation of generatrix of the thread Γ_2 and the outline of the helix Γ_3 as(w is the independent variable parameter):

$$\begin{cases}\Gamma_2:\ \vec{r}_2(w) = x_2(w)\vec{i} + z_2(w)\vec{k} \\ \Gamma_3:\ \vec{r}_3(\beta) = x_3(\beta)\vec{i} + z_3(\beta)\vec{k}\end{cases} \tag{8}$$

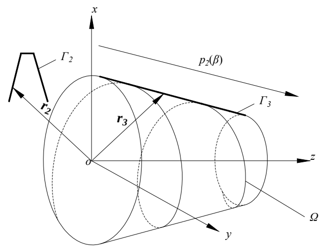


FIGURE IV. FTHE MODEL OF THE THREAD SURFACE

When the pitch and the diameter of the screw changed, the movement of Γ_2 can be described as the synthetic motion of three direction:

1) Rotation around the z-axis:

$$\vec{R}_2^{\,1}=\vec{R}_2^{\,1}(w,\beta)=\left(\beta\vec{k}\right)\otimes\vec{r}_2 \tag{9}$$

2) Axial movement with variable lead($p_2=p_2(z)$ is the equation of the changed pitch； $z=f_2(\theta)$):

$$\vec{R}_2^{\,2}(w,\beta)=\pm\left(\int_{\beta_0}^{\beta_1}\left(p_2\left(f_2(\beta)\right)\cdot f_2'(\beta)\right)d\beta\right)\vec{k} \tag{10}$$

3) Radial movement with variable diameter:

According to the equation of the outline of the helix Γ_3, the diameter at the initial position (β_0) and any other position (β) are:

$$\begin{cases} D_0=\left|x_3(\beta_0)\right| \\ D_\beta=\left|x_3(\beta)\right| \end{cases} \tag{11}$$

So the equation of radial movement can be expressed as:

$$\vec{R}_2^{\,3}(w,\beta)=\left(D_\beta-D_0\right)\cos\beta\cdot\vec{i}+\left(D_\beta-D_0\right)\sin\beta\cdot\vec{j} \tag{12}$$

4) The simultaneous equations of Equation(9), Equation(10) and Equation(12) is the mathematical model of the thread surface:

$$\vec{R}_2=\vec{R}_2^{\,1}+\vec{R}_2^{\,2}+\vec{R}_2^{\,3} \tag{13}$$

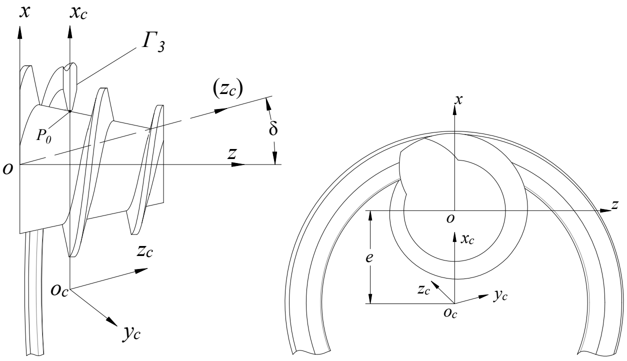
In conclusion, the surface of multi-variant screws can be divided to two parts, thread and cylinder, whose equations are Equation(7) and Equation(13) respectively. As a result, all the geometric data of the multi-variant screws can be calculated through the equation of multi-variant screws:

$$\vec{R}=\vec{R}_1(v,\beta)+\vec{R}_2(w,\beta) \tag{14}$$

III. THE WHIRLING METHOD OF MULTI-VARIANT SCREW

A. Calculation of the Cutter Position

As shown in Figure V, (*o-x,y,z*) is the Cartesian coordinate system with the z-axis coincident with the axis of the screw; (o_c-x_c,y_c,z_c) is the coordinate of the cutter of the whirling machine whose axis is the z_c-axis. The *x*-axis and x_c-axis are parallel. In the practical whirling processing, there is an eccentricity e between the screw and cutter, and the cutter is tilted an angle δ to avoid interference between the cutter and the workpiece.



(a) (b)

FIGURE V. THE POSITION OF THE CUTTER AND SCREW

Suppose the cutter moves z_0 distance in the direction of *z*-axis, and the tilted angle and eccentricity are δ_0 and e_0($z_0\geq 0$; $e_0\geq 0$;the direction of δ_0 is clockwise around x_c-axis). The transformation equation between coordinate system (o_c-x_c,y_c,z_c) and (*o-x,y,z*) can be described as:

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 & e_0 \\ 0 & \cos(-\delta_0) & -\sin(-\delta_0) & 0 \\ 0 & \sin(-\delta_0) & \cos(-\delta_0) & -z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} \tag{15}$$

As for the tool, the rotative surface of the tool is involved in cutting. Curve Γ_3 in Figure V a is the generatrix of the cutter. On the basis of the screw modeling method**,** it is easy to build the equation of the surface and convert to (*o-x,y,z*) as(g,γ are independent variable parameters):

$$\vec{R}_3(g,\gamma)=R_{3x}(g,\gamma)\vec{i}+R_{3y}(g,\gamma)\vec{j}+R_{3z}(g,\gamma)\vec{k} \tag{16}$$

Correspondingly, the normal vector of the surface can be express as:

$$\vec{N}_3(g,\gamma)=N_{3x}(g,\gamma)\vec{i}+N_{3y}(g,\gamma)\vec{j}+N_{3z}(g,\gamma)\vec{k} \tag{17}$$

$$\begin{cases} N_{3x}(g,\gamma)=\begin{vmatrix} \partial R_{3y}/\partial g & \partial R_{3z}/\partial g \\ \partial R_{3y}/\partial\gamma & \partial R_{3z}/\partial\gamma \end{vmatrix} \\ N_{3y}(g,\gamma)=\begin{vmatrix} \partial R_{3z}/\partial g & \partial R_{3x}/\partial g \\ \partial R_{3z}/\partial\gamma & \partial R_{3x}/\partial\gamma \end{vmatrix} \\ N_{3z}(g,\gamma)=\begin{vmatrix} \partial R_{3x}/\partial g & \partial R_{3y}/\partial g \\ \partial R_{3x}/\partial\gamma & \partial R_{3y}/\partial\gamma \end{vmatrix} \end{cases} \tag{18}$$

Based on this method, the normal vector of multi-variant screws can be also solved as:

$$\vec{N}(v,w,\beta)=N_x\vec{i}+N_y\vec{j}+N_z\vec{k} \tag{19}$$

Because the point is either on the thread or cylinder, v and w won't exist at same time. Actually, there are only two variable parameters in the equations above.

In whirling processing, there are always a contact line between the spiral surface and rotative surface of the cutter. An arbitrary point P on contact line follows these properties below:

Property 1: All the points on contact line are common points of both surfaces.

Property 2: Two surfaces are tangent at this point, moreover, the normal vector of this point is orthogonal to the tangent lines on any direction.

In terms of the whirling method of multi-variant screws, we get:

$$\begin{cases} \vec{R}(v,w,\beta)=\vec{R}_3(g,\gamma) \\ \vec{N}(v,w,\beta)\times\vec{N}_3(g,\gamma)=0 \end{cases} \tag{20}$$

In whirling process, the coordinate value of the cutter center (o_c) is the control data of the cutter. Through the Equation(20), the corresponding coordinate of o_c at any processing point can be calculated.

B. The Method of Tool Path Planning

By means of calculation method as stated above, the corresponding control point(the center of the tool nose arc) of each processing point on a multi-variant screw can be obtained by solving Equation(20). During the whirling process, the cutter moved along the z-axis, so the control point can be regard as on a current cross section of the screw at each moment. Furthermore, the workpiece can be divided to several cross section being cut at each moment. The feeding modes between each section can affect the processing results directly. Suppose there are several processing points having been chosen, two feeding modes are discussed below:

(1) Intermittent feeding:

First of all, it should be clarified that the tool path planning is about the path of the control points, not the processing points. The cutter is supposed to feed along one direction to assure the smoothness of acceleration and deceleration process and avoid the error caused by the gap of transmission system of machine tools. However, sometimes when processing points are on the same cross section, control points are not which distribute in both sides of the section unevenly, namely, the cutter will vibrate at some position.

Actually, there are always some control points are coplanar on each section perpendicular to the main feeding direction of cutter. Due to this, the whole screw can be divided to a number of control section as (Σ_1, $\Sigma_2 \dots \Sigma_n$) in Figure VI which are distributed several control points.

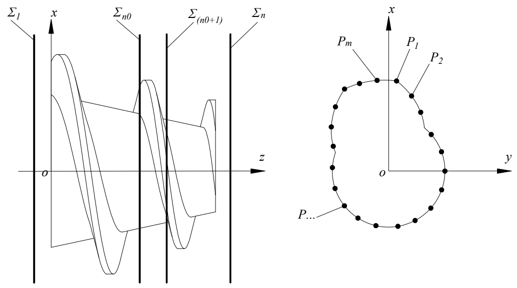


FIGURE VI. INTERMITTENT FEEDING

Suppose Σ_{n0} and $\Sigma_{(n0+1)}$ are two adjacent control sections, control points(P_1,$P2 \dots P_m$) are distributed on section Σ_{n0},.During the whirling process, the cutter move to$\Sigma_{(n0+1)}$ until (P_1,$P2 \dots P_m$) on Σ_{n0} had been machined, so that the feeding process is intermittent.

This method satisfy the principle of cutter moving along the main feeding direction, and the multi-variant screw can be machined through this way. However, because of the intermittent feeding mode, there are always an impact force when the cutter finished the processing points on a section and move to another which caused obvious vibration and sometimes tipping in the machining experiments.

(2) Continuous feeding:

To solve the issues existed in intermittent feeding, a continuous feeding method is proposed. On the basis of intermittent feeding, some additional control sections were fetched between the adjacent control sections, and there is only one control point distributed on each section. As m points fetched on Σ_{n0} in Figure VII, (m-1) additional control sections(Σ_2, Σ_3, …, Σ_m) are added between these two sections. Meanwhile, (m-1) corresponding control points are distributed on each section, such as P_1' onΣ_{n0} (Σ_1), P_2' on Σ_2, …, P_{m0}' on$\Sigma_{(m0+1)}$, …, $P_{(m-1)}'$ on Σ_m During the whirling process, each control point isn't coplanar with others, so that the cutter feed continuously.

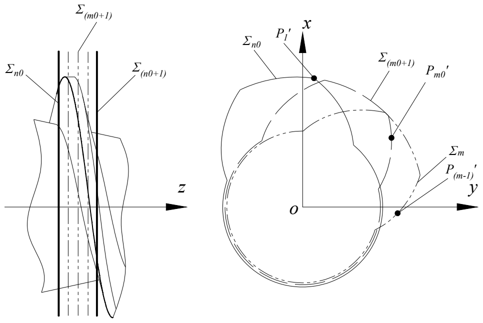


FIGURE VII. CONTINUOUS FEEDING

Compare to the intermittent feeding, the cutter moves along a direction continuously which causes the force imposed on cutter nose changing smoothly during the whirling process and reduces the vibration. Furthermore, the interval of cutting track is relatively small, so that the machine quality is better than intermittent feeding under the same condition. However, because the small interval requires higher precision of the machine tool, the application of these two modes should depend on the actual condition.

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