Advances in Engineering Research, volume 138
2nd International Conference on Machinery, Electronics and Control Simulation (MECS 2017)

Research on Expressway Toll Plaza

Yijin Liu
School of North China Electric Power University, Baoding 071000, China
2330413270@qq.com

Keywords: VISSIM, M/M/N system model,Highway

Abstract. Highway is an important transportation infrastructure,so it is meaningful to study it. Consider a toll highway having L lanes of travel in each direction and a barrier toll containing B tollbooths (B > L) in each direction.We establish the M/M/N system model based on the queuing theory, and get the basic relationship between the number of tollbooths B and the number of lanes L. By the relationship between B and L, I can get that B$\geq$6 is reasonable when L=3. Finally through the VISSIM we draw a simulation diagram.

1. Introduction

The road is the channel that mankind opens up or builds on the land for various activities. It develops along with the progress of human civilization and becomes an indispensable part of social economic activities. With the people on the transport speed and quality of the pursuit of a higher level, mankind has entered high-speed era. However, with the rapid development of China's highway, highway toll station and highway traffic capacity can not match the capability, likely to cause congestion at the toll station and impact on highway capacity. Therefore,it is of great significance for the research of highway.

2. Establishment of M / M / N system model

2.1 Basic assumptions in this model

Queuing theory is a mathematical theory and method for studying the random distribution of the system and the working process of the stochastic service system. Highway toll system, is a typical queuing system. The congestion of the toll station is its biggest problem. This paper discusses the design of expressway toll system from the basic theory of queuing theory.

In the process of establishing and solving the M / M / N system model[1] . I assume that the arrival process is poisson flow, the vehicle arrival interval and service time are negative exponential distribution, and the number of vehicle sources is infinite. After finding the information,T is about 40 seconds, in that way , $\mu = 90 / h$ .When a car reach the toll plaza, if no other vehicles is in front of the queue state, it can immediately pay.When there are N cars in front of the service, the vehicle which is coming after needs to be queuing in the queue waiting for services.

2.2 Establishment of M / M / N model[2]

Assuming that the system has a stable distribution, that is, the probability of the system in each state exists, as shown in Table 1.

Table 1 Probability distributions in different states

State	0	1	B-1	B	B+1
Probability	p_0	p_1	p_{B-1}	p_B	p_{B+1}

In Table 1, p_k is the probability that the system is in state $k(k=1,2,\cdots,B)$,then we can get the following relationship:

$$p_0 + p_1 + \cdots = 1 \tag{1}$$

Formula (1) is called the normalization condition, that is, the sum of the probabilities of the system in each state is 1.

X is the total traffic flow per hour passing through a toll station and it satisfies the following relations:

$$\lambda = X/L \tag{2}$$

By searching for information,we set the flow rate of 440 vehicles per hour. On this basis, we can get the following formula:

$$p_B = \frac{1}{B!}\left(\frac{\lambda}{\mu}\right)^B p_0 = \frac{1}{B!}\rho^B p_0 \tag{3}$$

According to the law of conservation of probability in steady state, write the flow balance equation for each state of Table 1.

$$\rho = \frac{\lambda}{N\mu} \tag{4}$$

Under stable conditions $\rho < 1$, according to the probability normalization condition, the sum of the system in each state is 1. The following relations can be obtained.

$$p_0 = \left[\sum_{k=0}^{n} \frac{(B\rho)^k}{K!} + \frac{(B\rho)^B}{B!} \times \frac{1}{1-\rho}\right]^{-1} \tag{5}$$

2.3 probability distribution of L_w and T_w

In the toll plaza[3], average number of drivers waiting L_w can be obtained from the probability distribution. Because the probability distribution reflects the probability of K individuals in the charging area, and when the average waiting queue length is K, there are $K+N$ people in the system (K people are waiting in queue and N people are receiving services) ,therefore L_w can be described by the following mathematical expectation.

$$L_w = \sum_{k=0}^{\infty} k\, p_{k+B} = B\rho + \rho \frac{(B\rho)^B}{B!} \times \frac{p_0}{(1-\rho)^2} \tag{6}$$

According to the empirical formula, we can get the relationship between L_w and T_w.

$$T_w = \frac{L_w}{\lambda} \tag{7}$$

3. Solving the model

The establishment of B toll booths shall be met $\frac{\rho}{B} \le 1$. Queuing system is stable, the queue will not be more and more longer. According to the preceding conditions, we can get the following equation.

$$\lambda = \frac{440}{L} \tag{8}$$

$$\mu = \frac{3600}{\lambda} / h \tag{9}$$

4. Conclusions

Through the above analysis of the problem, we basically established the required model.Then we can get the following expressions from the precondition.

$$\rho = \frac{\lambda}{\mu} = \frac{\frac{440}{L}}{\frac{3600}{\lambda}} = \frac{11\lambda}{90L} \tag{10}$$

$$\frac{\rho}{B} \leq 1 \tag{11}$$

By formula (10) and (11) we can get the following formula:

$$\frac{11\lambda}{90LB} \leq 1 \tag{12}$$

By formula (8) and (12) we can get the following formula:

$$BL^2 \leq \frac{484}{9} \approx 53.78 \tag{13}$$

Thus, we determine the relationship between the number of tollbooths B and the number of lanes L. We can draw a conclusion that when L=3, B$\geq$6 can meet the above relationship[4] . Through the VISSIM simulation software, we draw a number of simulation diagrams.

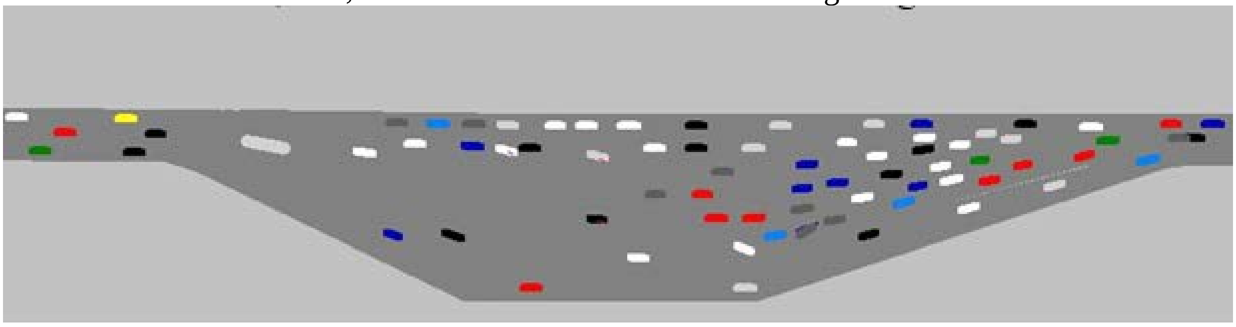

Figure 1 Simulation diagram of L=3,B=7

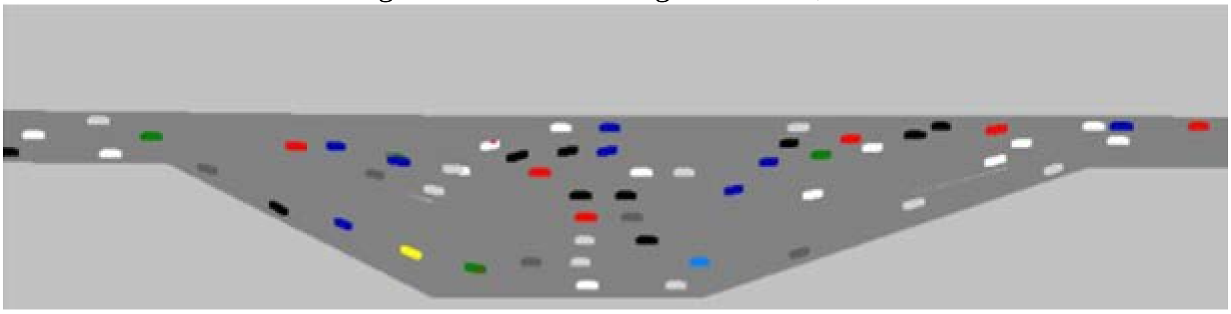

Figure 2 Simulation diagram of L=3,B=8

From the two simulation diagrams we can see that when the value of L is constant, the greater the B value, the less prone to clogging.However at the same time the cost is also greater. So in the construction[5] , we have to consider two issues, including cost and traffic congestion.

5.References

[1]. Fu Xinyu, Luo Guowang.Multi-service window wait system M / M / n queuing model [J]. Journal of Chongqing University of Technology (Natural Science), 2013, (12): 139-142.

[2]. Zou Xia. M / M / n queuing model with variable input rate and two kinds of service rates and its application [D]. Chongqing Normal University, 2013.

[3]. Su Li. Expressway toll station design research [D]. Chang'an University, 2012.

[4]. Wang Jing. Hunan Province highway toll station design research [D]. Central South University, 2006.

[5]. Ma Jun, Liu Zhiyong. The solution to the problem of highway toll station congestion [J]. highway traffic science and Technology (Application Technology Edition), 2012, (08): 372-374.