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Parameter Evaluation of 3-parameter Weibull Distribution based on Adaptive Genetic Algorithm

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Abstract: In the determination of a reliability life model in 3-parameter Weibull distribution, there was large error and inefficient problem in parameter evaluation. Firstly, the maximum likelihood equation was made. Then the deficiency, lied in steps of using traditional genetic algorithm to solve the maximum likelihood equations, was optimized. An adaptive genetic algorithm was obtained that can be used to solve the maximum likelihood equations. Finally, compared the simulation results in MATLAB between adaptive genetic algorithm and traditional genetic algorithm, it can be concluded that the adaptive genetic algorithm was more efficiency and adaptable than the traditional genetic algorithm. This method also provided a reference to solving the similar problem in parameter evaluation.

Introduction

The estimation of the parameter is the key to the life model,whether it can predict the life of product accurately in the reliability engineering,but it is difficult to estimate the parameteIn this paper, a system of maximum likelihood function equations with three parameters of Weibull distribution is established first, and then improve the adaptive genetic algorithm to solve the equations, which not only can not overcome the limitation of the traditional numerical methods.The efficiency is low, but also for similar scenes under the parameter evaluation problem provides a train of thought[1].

Three - parameter Weibull distribution model

The probability density function of the three-parameter Weibull distribution is

$$f\left(t;\beta,\eta,\gamma\right)=\frac{\beta}{\eta}\left(\frac{t-\gamma}{\eta}\right)^{\beta-1}\exp\left(-\left(\frac{t-\gamma}{\eta}\right)^{\beta}\right)\quad\left(t\geq\gamma;\eta>0;\beta>0\right)$$

Shape parameters, scale parameters, and positional parameters.

The log-likelihood function can be obtained from the above formula.

$$l=\ln L=n\ln\beta+\left(\beta-1\right)\sum_{i=1}^{n}\ln\left(t_i-\gamma\right)-n\beta\ln\eta-\frac{\sum_{i=1}^{n}\left(t_i-\gamma\right)^{\beta}}{\eta^{\beta}}$$

Logarithmic Maximum Likelihood Function Equations

Using l respectively, the three parameters β, η, γ partial derivative, get it logarithmic Maximum Likelihood Function Equations

$$\frac{\partial l}{\partial \beta} = \frac{n}{\beta} + \sum_{i=1}^{n} \ln\left(t_i - \gamma\right) - n\ln\eta - \sum_{i=1}^{n}\left[\left(\frac{t_i - \gamma}{\eta}\right)^{\beta} \ln\left(\frac{t_i - \gamma}{\eta}\right)\right] = 0 \qquad (1)$$

$$\frac{\partial l}{\partial \eta} = -n\beta\frac{1}{\eta} + \frac{\beta\sum_{i=1}^{n}\left(t_i - \gamma\right)^{\beta}}{\eta^{\beta+1}} = 0 \qquad (2)$$

$$\frac{\partial l}{\partial \gamma} = \left(1-\beta\right)\sum_{i=1}^{n}\frac{1}{t_i - \gamma} + \frac{\beta}{\eta^{\beta}}\sum_{i=1}^{n}\left(t_i - \gamma\right)^{\beta-1} = 0 \qquad (3)$$

The above three equations, logarithmic maximum likelihood function equations, where the position parameter γ on the three parameters Weibull distribution of the probability density function plays only to adjust its position in the role of solving the equations[2], can temporarily make it 0, then it can simplify the logarithmic maximum likelihood function equation into

$$y = \frac{n}{\beta} + \sum_{i=1}^{n}\ln t_i - \frac{n}{\beta}\ln\left(\frac{\sum_{i=1}^{n} t_i^{\beta}}{n}\right) - \sum_{i=1}^{n}\left[\frac{n t_i^{\beta}}{\sum_{i=1}^{n} t_i^{\beta}}\ln\left(\frac{t_i}{\sqrt[\beta]{\frac{1}{n}\sum_{i=1}^{n} t_i^{\beta}}}\right)\right] = 0 \qquad (4)$$

$$\eta = \sqrt[\beta]{\frac{\sum_{i=1}^{n} t_i^{\beta}}{n}} \qquad (5)$$

In the above equations, y is the objective function of genetic algorithm.

GA for Solving Logarithm Maximum Likelihood Equations

Genetic Algorithm (GA) is an intelligent algorithm based on the genetic evolution process of biomimetic natural population to solve the global optimal solution.Because of the advantages of global optimization, strong extensibility and good robustness[3], this paper combines traditional genetic algorithm with three parameter Weibull distribution The maximum likelihood function equations are combined, and the steps of solving are as follows：

1. Pm, Pc and maximal total population MaxGeneration, the initial population data are randomly generated and distributed evenly in the search domain [a, b]
2. Determine the individual fitness value, calculate and record the total group fitness value
3. Select and copy, the group of individuals with high fitness to choose, copy

operation

4. Crossover, the cross-group of individuals in the previous step
5. Mutation, variation of all individuals in the population
6. Go back to the third step and repeat until the maximum total population is reached, otherwise the test continues
7. Parameter evaluation, based on the objective function value to calculate the three parameters

In the above operation, the selection of coding mode, the selection of fitness function, mutation, setting of crossover probability Pm, Pc, setting of genetic exit condition will affect the final result. Aiming at the above process, an adaptive genetic algorithm is proposed, which can improve the efficiency and accuracy of the algorithm by optimizing every link.

Solving Log-Likelihood Maximum Likelihood Equations Based on Adaptive GA

Considering the influence of various factors on the solution of the maximum likelihood function equations of the three parameters Weibull distribution, the following steps are optimized for each step in the solving process[4].

Using binary encoding

In the process of solving, the genetic algorithm has binary, decimal and real coding.Under the real number coding, the total population is divided into 0 ~ 1 binary control gene sequence and real gene sequence, but the real coding method needs to add in the decoding way Decimal encodings can store more information than binary encodings on the same number of bits, but binary encodings can be compensated for by increasing the number of bits, and decimal encodings are used in subsequent selection, duplication, crossover, and Variation and other operations in the error precision control is different from the binary coding.A comprehensive consideration, the final use of binary coding.

The selection of fitness function

In the process of genetic process, the more outstanding individuals, the greater the fitness function value of the selected equation (4), y is the objective function value, and the optimal value that can make y from the nearest zero value, it can make:

$$T = \frac{1}{|y|}$$

As the fitness function, when the value of y is closer to zero, the greater the value of T, the greater the fitness value of the individual, in the genetic process to retain the probability of the greater.

Pm

In the genetic process, it is not the case that the fitness value is large (ie, the value of Pm and the value of Pc is the same), and the probability of mutation is fixed. Individuals), the corresponding mutation probability should be smaller than the fitness of small individuals, so as to continuously optimize the entire population, making the whole population moving towards the global optimal direction, so as not to fall into the local population optimal solution.

When the individual fitness value Fitvalue is no less than the population mean fitness , the mutation probability.

$$P_{m=}\begin{cases} 0.09-\dfrac{Fitvalue-f_{mean}}{f_{\max}}*0.09 \\ 0.09 \end{cases}$$

Improvement of termination condition of genetic algorithm

In the traditional genetic algorithm, the stopping condition of the genetic algebra is the maximal hereditary algebra. However, in practical solving process, it is almost impossible to find the solution satisfying the objective function value 0, while the too large genetic algebra is very early for the convergent population. Difficult to improve the entire population. In the actual numerical calculation process, as long as the numerical solution within the error range, can be identified as a global optimal solution, so the exit condition of genetic algorithm to

1. The objective function value is less than the set precision eps
2. The genetic algebra reaches the maximum hereditary number MaxGeneration

   The above two conditions can satisfy any of the conditions to exit the calculation, but the priority to meet the conditions 1

The Solving Process Based on Adaptive Genetic Algorithm

Combined with the optimization of the above parts, the adaptive genetic algorithm is used to solve the global optimal solution of the system[5].

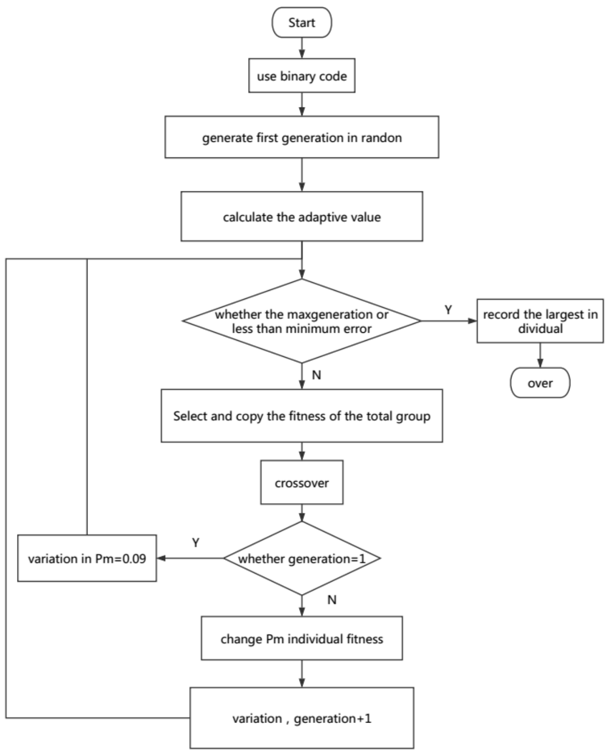

Fig. 1 Flowchart of Adaptive GA

The initial population size is 50, Pc = 0.9, the Pm value is set to 4.3, the objective function is y, and the fitness function is T, And the maximum hereditary algebra Gmax = 100

Simulation and result analysis

In order to verify the performance of the adaptive genetic algorithm is better than the traditional genetic algorithm, the random sampling method from the simulation software to get each of the four groups, each group of 50 obey Weibull distribution of the sample data[6,7,8], Algorithm and adaptive genetic algorithm for its multiple simulation, simulation results in Tables:

Table 1 Sample data in β=2.18, η=227.42, γ=1.58

	Traditional GA	Adaptive GA
β	2.1898	2.1896
η	227.4265	227.4218
γ	1.5800	1.5832
y	-0.0025	0.0006
l	-293.874	-293.873

Table 2 Sample data in β=1.94, η=221.80, γ=1.58

	Traditional GA	Adaptive GA
β	1.9849	1.9462
η	222.6587	221.7505
γ	1.5832	1.5845
y	-0.7176	-0.0004
l	-299.784	-299.725

Table 3 Sample data in β=3.10, η=248.00, γ=1.58

	Traditional GA	Adaptive GA
β	3.2500	3.1092
η	251.0502	248.0042
γ	1.5854	1.5854
y	-1.02492	0.1240
l	-287.758	-287.515

After comparing and analyzing the simulation results under random parameters, it is found that the accuracy and accuracy of the adaptive genetic algorithm for solving such problems are obviously superior to the traditional genetic algorithms

Summary

Based on the study of parameter estimation of Weibull distribution and the maximum likelihood estimation, the paper establishes a system of maximum likelihood function equations and compares the advantages and disadvantages of adaptive genetic algorithm and traditional genetic algorithm in solving such problems. By comparing the simulation results, it is concluded that the adaptive genetic algorithm can better avoid the premature convergence and filter the local optimal solution when solving the problem of parameter evaluation, and get a more accurate reliability life model

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