

# Design for Permanent Magnet Linear Servo Controller Based on $H_\infty$ Disturbance Compensator

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**Keywords:** PMLSM.  $H_\infty$  control. sliding mode control.

**Abstract.** In order to meet high precision and high reliability required for the permanent magnet linear motor servo system, a method that combined sliding tracking mode controller and the  $H_\infty$  disturbance compensator is proposed. The weighted integral gain sliding mode controller effectively solves the problem of switching gain increase on non sliding mode phase. The  $H_\infty$  disturbance compensator effectively suppresses the effect of chattering sliding mode control in the linear motor servo control system and inhibited a variety of disturbances within the closed-loop system. The results of simulation show that the proposed method can make permanent magnet linear motor servo system have good dynamic performance, strong robustness and the insensitive to system parameter variations.

## Introduction

Compared with permanent magnet rotating machine, the permanent magnet linear motor has the advantages of high positioning accuracy. For all these reasons, the linear motor will be widely used in NC servo system. In order to improve the servo performance of direct drive control system, it is necessary to use effective methods to suppress or compensate for various disturbances<sup>[1-3]</sup>. This paper proposed a composite control strategy of weighted integral gain  $H_\infty$  robust sliding mode controller and disturbance compensator combination according to the requirements of dynamic characteristics and robustness of high precision permanent magnet linear motor servo system. In order to make the linear servo system has a fast dynamic response tracking, put forward the integral switch sliding mode variable structure controller, the negative direction into the integral term in the weights of  $K_f$  can effectively avoid the non sliding mode switching gain some problems.

## Design of weighted integral gain sliding mode controller

In order to achieve high precision, high stability and good dynamic tracking performance of the direct drive system, this paper adopts the method of weighted integral switching gain sliding mode control to design the tracking controller. In this section, a sliding mode control method is proposed to improve the integral gain, which is called  $K_f$ , which is used to solve the problem. The vector control motion equation of permanent magnet linear synchronous motor in d-q coordinate system is<sup>[3]</sup>:

$$M\dot{\mathbf{v}} + B\mathbf{v} = K_f i_q - F_d \quad (1)$$

$$\dot{\mathbf{d}} = \mathbf{v}$$

Attention:

$$F_d = F_L + F_{ef} \quad (2)$$

$F_d$  shows externals total disturbance;  $F_L$  shows load disturbance;  $F_{ef}$  shows periodic fluctuation thrust caused by end effect of linear motor. Mathematical model :

$$F_f = F_{fm} \cos\left(\frac{s}{t} 2\pi + q_0\right) \quad (3)$$

$F_{fm}$  shows the amplitude of fluctuating thrust;  $q_0$  shows initial phase angle;  $t$  shows polar distance;  $s$  shows the displacement distance of motor mover.

If the tracking error is  $e_s = (s_{ref} - s)$ , state variable is  $x = [e_s \ \dot{e}_s]^T$ , the equation can be rewritten as:

$$\dot{x}(t) = A_c x(t) + B_c u(t) + D_c F_d \tag{4}$$

$$y(t) = \Gamma_c x(t) \tag{5}$$

Attention:

$u(t) = i_q(t)$  is the control input signal.

$$A_c = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{B_v}{M} \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ \frac{K_F}{M} \end{bmatrix}, \quad D_c = \begin{bmatrix} 0 \\ -\frac{1}{M} \end{bmatrix}, \quad \Gamma_c = [1 \ 0]$$

The chattering of the sliding mode controller is the key point of the design. In this paper, the sliding mode control law is presented:

$$u = u_{eq} + u_d \tag{6}$$

Attention:

$$u_{eq} = -(CB)^{-1} CAx$$

$$u_d = -K_w |r| \text{sgn}(s), \quad K_w > 0$$

$$r = \int_0^t (K_f r + s) dt, \quad K_f < 0 \tag{7}$$

$s(t) = Cx$ ,  $C$  satisfies the stability condition of sliding mode and  $CB > 0$ . In the integral expression above  $r$ , when  $r > 0, K_f r < 0$ , on the other hand, when  $r < 0, K_f r > 0$ . Due to the addition of negative weights  $K_f$  in formula (7).

### Design of disturbance compensator on $H_\infty$

$x = [x_1 \ x_2]^T$  is state variable,  $x_1$  shows the difference between the reference input and the actual output;  $x_2$  is integral for  $x_1$ ;  $u$  is Control input signal;  $\omega$  is total external disturbance; The augmented object  $G$  includes weighting function and object state equation:  $K = [K_1 \ K_2]$  shows the disturbance compensator of  $K_1(s)$  and  $K_2(s)$ .  $C$  and  $D$  are Weighting matrix.  $z$  is disturbance compensation evaluation signal.  $\omega$  shows the sum of external disturbances.

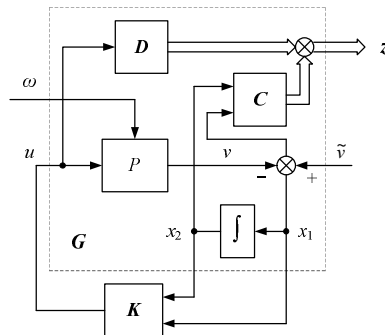


Fig.1: The structure of disturbance compensator on  $H_\infty$

The uncertainty caused by changes in the internal parameters of the motor design parameters  $a$  and  $d$  perturbation respectively  $\delta_1(t)$ ,  $\delta_2(t)$ , by equation (4) by permanent magnet linear motor augmented object state equation:

$$\left\{ \begin{array}{l} \dot{\mathbf{x}} = \begin{bmatrix} -a(1+d_1(t)) & 0 \\ 1 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} -d(1+d_2(t)) \\ 0 \end{bmatrix} \omega \\ \quad + \begin{bmatrix} b(1+d_2(t)) \\ 0 \end{bmatrix} u \\ \mathbf{z} = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \\ 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \end{array} \right. \quad (8)$$

The coefficient of the external perturbation is usually taken as the maximum value of the perturbation. The weighting coefficient  $q_1 > 0$ ,  $q_2 > 0$  is used to adjust the allowable range of error. By the formula (4) to determine the  $H_\infty$  infinity with standard internal parameter perturbation and external disturbance of the controlled object:

$$\left\{ \begin{array}{l} \dot{\mathbf{x}} = (\mathbf{A} + \Delta\mathbf{A})\mathbf{x} + \mathbf{B}_1\omega + (\mathbf{B}_2 + \Delta\mathbf{B})u \\ \mathbf{z} = \mathbf{C}\mathbf{x} + \mathbf{D}u \end{array} \right. \quad (9)$$

Attention:  $\mathbf{A}, \mathbf{B}_1, \mathbf{B}_2, \mathbf{C}, \mathbf{D}$  is Known quantity,  $\Delta\mathbf{A}$  and  $\Delta\mathbf{B}$  are parameter perturbation matrix.  $\Delta\mathbf{A} - \Delta\mathbf{B}$ :

$$[\Delta\mathbf{A} \quad \Delta\mathbf{B}] = \mathbf{E}\mathbf{Q}[\mathbf{F}_a \quad \mathbf{F}_b] \quad (10)$$

Attention:  $\mathbf{E}, \mathbf{F}_a, \mathbf{F}_b$  is Known constant matrix,  $\mathbf{Q}$  is Unknown matrix and

$$\begin{aligned} \Omega &= \{ \mathbf{Q}(t) | \mathbf{Q}^T(t)\mathbf{Q}(t) \leq \mathbf{I}, \forall t \}, \\ \mathbf{Q} &\in \Omega, \mathbf{I} \text{ is unit matrix.} \end{aligned}$$

For a given object augmented state equation (9), available for state feedback controller:

$$u = \mathbf{K}\mathbf{x} \quad (11)$$

The closed-loop system satisfies the following two standard  $H_\infty$  control standards:

- 1  $\mathbf{Q} \in \Omega$  stability of closed loop system
- 1  $\mathbf{Q} \in \Omega$  the closed loop transfer function from  $\omega$  to  $\mathbf{z}$  is satisfied:

$$\| \mathbf{T}_{zw}(s) \|_\infty < 1 \quad (12)$$

Built up  $\mathbf{D}^T[\mathbf{D} \quad \mathbf{C}] = [\mathbf{I} \quad \mathbf{0}]$ , For the controlled object equation (9), the state feedback controller  $\mathbf{K}$  and positive definite matrix  $\mathbf{X}$ . The sufficient and necessary condition for the closed-loop control system to satisfy the criterion and the second is that the scalar lambda  $\lambda > 0$  makes the Riccati inequality:

$$\begin{aligned} &\mathbf{A}^T\mathbf{X} + \mathbf{X}\mathbf{A} + \mathbf{X}(\mathbf{B}_1\mathbf{B}_1^T + I^2\mathbf{E}\mathbf{E}^T)\mathbf{X} + \mathbf{C}^T\mathbf{C} + \frac{1}{I^2}\mathbf{F}_a^T\mathbf{F}_a \\ &- (\mathbf{X}\mathbf{B}_2 + \frac{1}{I^2}\mathbf{F}_a^T\mathbf{F}_b)\mathbf{R}^{-2}(\mathbf{B}_2^T\mathbf{X} + \frac{1}{I^2}\mathbf{F}_b^T\mathbf{F}_a) < 0 \end{aligned} \quad (13)$$

There are definite solutions, which if type (12) has a solution, then meet the standard  $H_\infty$  robust performance criterion of the state feedback controller of is as follows:

$$\mathbf{K} = -\mathbf{R}^{-2}(\mathbf{B}_2^T\mathbf{X} + I^{-2}\mathbf{F}_b^T\mathbf{F}_a) \quad (14)$$

If the maximum value of the perturbation in formula (8) is considered.  $|d_2(t)| \leq s_2$ ,  $|d_1(t)| \leq s_1$ ,  $s_1 > 0$ ,  $s_2 > 0$  are Known quantity.

### Simulation results and analysis

The parameters of the controller are  $K_w=46, \mathbf{C}=[30 \ 1], K_f=-0.6$ . The maximum amplitude of the parameter variation is 10%, and  $|d_1(t)| < 25\%$ ,  $|d_2(t)| < 12.5\%$ ,  $s_1 = 25\%$ ,  $s_2 = 12.5\%$ . when  $q_1 = 50; q_2 = 2500; I = 3.1$ , the  $H_\infty$  disturbance compensator parameters is  $\mathbf{K} = [70.70 \ 2511.1]$  The

simulation was carried out on the sudden load of permanent magnet linear motor servo system under the condition of varying load parameters.

Condition 1: the weighted integral gain sliding mode controller is simulated.

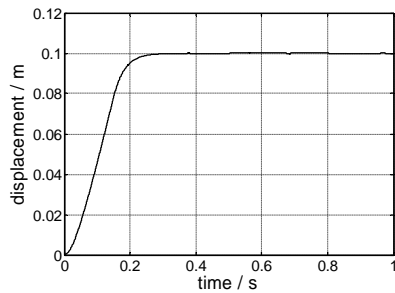


Fig. 2: Displacement step response curve

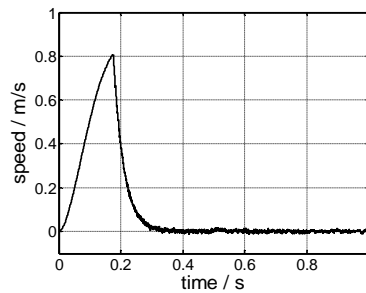


Fig. 3: Speed response curves

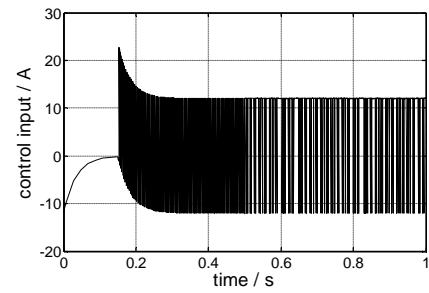


Fig. 4: Control input signal curve

Condition 2: the weighted integral type sliding mode control combined  $H_{\infty}$  control method for load disturbance compensation.

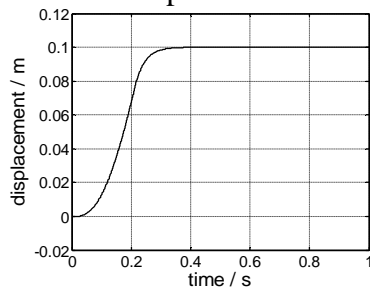


Fig. 5: Displacement step response curve

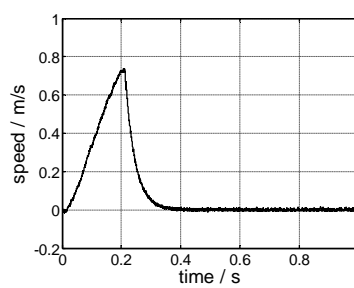


Fig. 6: Speed response curves

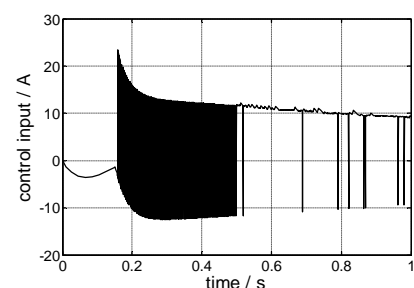


Fig. 7: Control input signal curve

Comparison of Fig 3 and Fig 7 shows that there is a certain chattering phenomenon in displacement step response curve, and the steady-state position error in condition 1. The chattering phenomenon is eliminated effectively, and the steady-state displacement response has no static error in condition 2. Comparison of Fig 4 and Fig 7 can be seen: Condition 1 is not only faster than the simulation 2 of the output of a large overshoot and steady-state output error is also large.

## Conclusions

This paper proposed a method of weighted integral gain compensation control of sliding mode control and  $H_{\infty}$  control based on disturbance. In order to make the direct drive servo system have good dynamic tracking performance and strong robustness, this paper presents an integral switching sliding mode variable structure controller. In order to solve the chattering problem in sliding mode near the line, using the standard  $H_{\infty}$  control theory to design a disturbance compensator.

## References

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