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Simulation of VaR Based on Monte Carlo-Copula - GARCH Model

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Abstract. In this paper, Monte Carlo method is used to establish the model of China's CITIC Securities and Huatai Securities for three years, and to test the accuracy of the model. Considering the non-normality of the distribution of the return rate of China's securities market, a GARCH model which can describe the spike and tail feature of variance and response rate distribution is used. The Monte Carlo algorithm is also improved by combining Copula function to compare.

Introduction

With the deepening of economic globalization and internationalization of capital flows, the global financial market has shown a trend of vigorous development. But at the same time, the ever-changing financial market has brought economic, social and political risk factors Significantly increased, leading to various types of financial institutions are facing increasingly serious systemic and non-systemic risk.

Monte Carlo method is often used in the calculation of VaR in financial risk measurement, but the Monte Carlo method also has the disadvantage such as too slow convergence. In this paper, the Monte Carlo method is compared with the Monte Carlo method combined with Copula-GARCH by estimating the VaR of the portfolio respectively.

Basic statistical analysis of data

The sample is based on the original data of CITIC Securities and Huatai Securities's closing price. The sample period is May 7, 2012 to May 20, 2016. CITIC Securities has a total of 985 closing price data. Huatai Securities has a total of 977 Closing price data. The above data comes from the RESSET database. The modeling and analysis methods are completed using Eviews 6.0 and Matlab.

First, we use Eviews6.0 for data statistics and description.

Data Yield Price Trends.

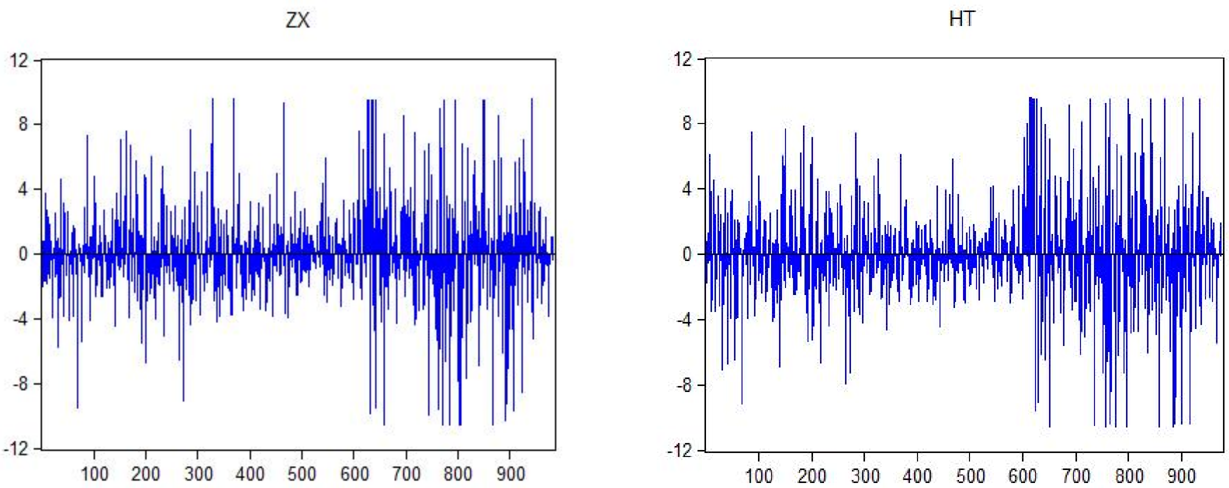
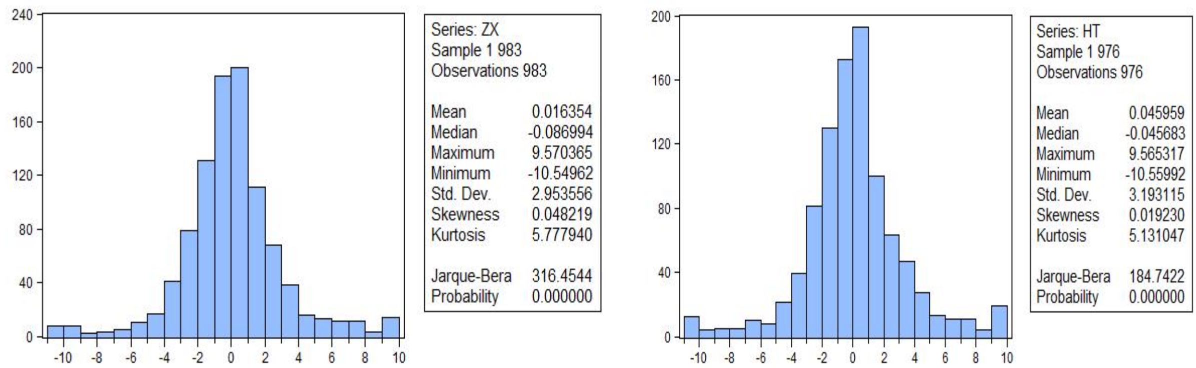


Fig. 1 Fluctuation of logistic yield series of CITIC Securities and Huatai Securities

From Fig.1, the expected rate of return is zero, reflecting a certain degree of volatility and sequence fluctuations in the conditions of heterosexual signs which is not a random swimming process.

The positive test of the yield sequence. With further data analysis, we can obtain the mean, median, maximum, minimum, standard deviation, skewness, kurtosis, Jarque-Bera statistic and companion probability of CITIC Securities and Huatai Securites, as shown in Fig.2.

Fig.2 Data statistical analysis of CITIC Securities and Huatai Securities



We assume that the random error W is subject to the normal distribution. The common normality test is the Jarque-Bera test, referred to as the JB test.

It can be seen from Fig. 2 that the JB statistic of the two stock returns are 316.4544 and 184.7422. The skewness are 0.048219 and 0.019230. The kurtosis are 5.777940 and 5.131047. The Jarqne-Bera test has a p-value of 0.000000, which rejects the original hypothesis at a confidence level of 99%, which is not subject to a normal distribution.

Data stability test. In the actual research process, we usually use ADF (Augmented DF test) test. The following results were obtained with ADF test with Eviews, as shown in Fig.3.

Fig.3 Data stability test of CITIC Securities and Huatai Securities

Augmented Dickey-Fuller Unit Root Test on ZX

Null Hypothesis: ZX has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic based on SIC, MAXLAG=21)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-29.27053	0.0000
Test critical values:	1% level	-3.436789	
	5% level	-2.864272	
	10% level	-2.568277	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(ZX)
Method: Least Squares
Date: 06/14/16 Time: 19:39
Sample (adjusted): 2 983
Included observations: 982 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
ZX(-1)	-0.932718	0.031865	-29.27053	0.0000
C	0.017418	0.094112	0.185081	0.8532

R-squared	0.466453	Mean dependent var	0.003123
Adjusted R-squared	0.465909	S.D. dependent var	4.035404
S.E. of regression	2.949136	Akaike info criterion	5.002936
Sum squared resid	8523.456	Schwarz criterion	5.012895
Log likelihood	-2454.442	Hannan-Quinn criter.	5.006724
F-statistic	856.7639	Durbin-Watson stat	2.000358
Prob(F-statistic)	0.000000		

Augmented Dickey-Fuller Unit Root Test on HT

Null Hypothesis: HT has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic based on SIC, MAXLAG=21)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-27.51046	0.0000
Test critical values:	1% level	-3.436837	
	5% level	-2.864293	
	10% level	-2.568288	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(HT)
Method: Least Squares
Date: 06/15/16 Time: 08:34
Sample (adjusted): 2 976
Included observations: 975 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
HT(-1)	-0.875047	0.031808	-27.51046	0.0000
C	0.042300	0.101557	0.416518	0.6771

R-squared	0.437516	Mean dependent var	0.003773
Adjusted R-squared	0.436937	S.D. dependent var	4.225660
S.E. of regression	3.170829	Akaike info criterion	5.147912
Sum squared resid	9782.695	Schwarz criterion	5.157928
Log likelihood	-2507.607	Hannan-Quinn criter.	5.151724
F-statistic	756.8255	Durbin-Watson stat	1.994720
Prob(F-statistic)	0.000000		

As can be seen in Fig.3, the ADF values of the yield series are -29.27053, -27.51046, less than 1%, 5% and 10% confidence intervals. The probability is almost zero, that is, there is no unit root, which indicates that the sample index yield sequence is very smooth.

Data autocorrelation test. If there is a correlation between the time series data among the residuals, this will make the variable inactive feedback. The LQ method is used to test autocorrelation and partial correlation.

Correlogram of ZX

Date: 06/14/16 Time: 19:40
Sample: 1 983
Included observations: 983

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.067	0.067	4.4624	0.035
		2	0.005	0.000	4.4842	0.106
		3	-0.020	-0.021	4.8878	0.180
		4	0.032	0.035	5.8887	0.208
		5	0.031	0.027	6.8357	0.233
		6	-0.044	-0.049	8.7784	0.186
		7	0.045	0.053	10.825	0.146
		8	0.069	0.064	15.596	0.049
		9	0.098	0.086	25.164	0.003
		10	-0.030	-0.039	26.035	0.004
		11	0.006	0.013	26.070	0.006
		12	0.006	0.000	26.104	0.010
		13	0.046	0.040	28.243	0.008
		14	-0.041	-0.047	29.885	0.008
		15	-0.013	-0.003	30.058	0.012
		16	0.050	0.038	32.606	0.008
		17	0.043	0.026	34.419	0.007
		18	0.060	0.050	37.994	0.004
		19	-0.009	-0.003	38.077	0.006
		20	0.030	0.020	38.975	0.007
		21	0.061	0.056	42.701	0.003
		22	0.002	-0.009	42.705	0.005
		23	-0.087	-0.081	50.252	0.001
		24	0.028	0.037	51.064	0.001
		25	0.071	0.049	56.175	0.000
		26	-0.018	-0.043	56.495	0.000
		27	-0.024	-0.018	57.074	0.001
		28	0.056	0.065	60.287	0.000
		29	0.004	-0.034	60.301	0.001
		30	-0.053	-0.056	63.198	0.000
		31	-0.080	-0.055	69.683	0.000
		32	-0.058	-0.045	73.066	0.000
		33	0.002	-0.018	73.068	0.000
		34	-0.006	-0.010	73.102	0.000
		35	-0.014	-0.004	73.316	0.000
		36	-0.030	-0.030	74.219	0.000

Correlogram of HT

Date: 06/15/16 Time: 08:35
Sample: 1 976
Included observations: 976

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.125	0.125	15.275	0.000
		2	-0.005	-0.021	15.298	0.000
		3	-0.016	-0.013	15.548	0.001
		4	0.041	0.046	17.227	0.002
		5	0.027	0.016	17.959	0.003
		6	-0.040	-0.046	19.528	0.003
		7	0.048	0.062	21.758	0.003
		8	0.021	0.005	22.173	0.005
		9	0.019	0.014	22.538	0.007
		10	-0.049	-0.049	24.925	0.005
		11	0.012	0.024	25.071	0.009
		12	0.049	0.039	27.461	0.007
		13	0.108	0.100	38.925	0.000
		14	-0.018	-0.043	39.244	0.000
		15	-0.017	-0.005	39.545	0.001
		16	-0.006	-0.012	39.585	0.001
		17	-0.010	-0.013	39.683	0.001
		18	0.028	0.031	40.472	0.002
		19	-0.005	-0.003	40.494	0.003
		20	0.063	0.049	44.473	0.001
		21	0.050	0.040	46.989	0.001
		22	-0.015	-0.028	47.226	0.001
		23	-0.045	-0.030	49.289	0.001
		24	-0.003	0.001	49.296	0.002
		25	0.049	0.032	51.715	0.001
		26	0.010	-0.005	51.819	0.002
		27	-0.005	0.003	51.844	0.003
		28	0.063	0.068	55.842	0.001
		29	-0.013	-0.038	56.010	0.002
		30	-0.039	-0.028	57.573	0.002
		31	-0.074	-0.067	63.141	0.001
		32	-0.037	-0.033	64.490	0.001
		33	0.003	-0.009	64.502	0.001
		34	-0.021	-0.017	64.946	0.001
		35	0.001	0.019	64.948	0.002
		36	-0.036	-0.029	66.289	0.002

Fig.4 Data autocorrelation test of CITIC Securities and Huatai Securities

From the Fig.4, we can see that the data are mostly in the confidence interval. Q statistic is not significant, until the value of the lag 36 order is less than the critical value of the significance level, indicating that the yield sequence exists autocorrelation and ARCH effect .

VaR calculation based on Monte Carlo method

Using the above selected data, the VaR value of the next trading day (ie May 23, 2016) is calculated by Monte Carlo method. The holding period is 1 day. Assuming that the confidence level is 95%, we use the geometric Brownian motion as a stochastic model for the change of the price of CITIC securities. The holding period of the day is divided into 20 equal periods. Each mean and standard deviation of the time period are $\frac{\mu}{20}$, $\frac{\sigma}{\sqrt{20}}$.

The followings are specific steps of calculation of CITIC Securities VaR on May 23, 2020 using general Monte Carlo simulation method.

Estimate the mean and standard deviation. We estimate the μ and the standard deviation σ between CITIC Securities and Huatai Securities on May 20, 2012 to May 20, 2016. Then we calculate the mean return of CITIC securities in each period of time $\frac{\mu}{20}$ and the standard deviation $\frac{\sigma}{\sqrt{20}}$.

Generate random numbers. Generate 20 random numbers that obey the standard normal distribution $\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_{20}$.

Simulate a possible path to a change in the price of a security. The gain of t+1 is obtained by substituting S_t (yield on May 20, 2016), $\frac{\mu}{20}$, $\frac{\sigma}{\sqrt{20}}$ and ε_i into the formula and we get

$$S_{t+1} = S_t + S_t(\frac{\mu}{20}\Delta t + \frac{\sigma}{\sqrt{20}}\varepsilon_1\sqrt{\Delta t}) \quad (1)$$

And so on we have

$$S_{t+2} = S_{t+1} + S_{t+1}(\frac{\mu}{20}\Delta t + \frac{\sigma}{\sqrt{20}}\varepsilon_2\sqrt{\Delta t}) \quad (2)$$

……

$$S_{t+20} = S_{t+19} + S_{t+19}\left(\frac{\mu}{20}\Delta t + \frac{\sigma}{\sqrt{20}}\varepsilon_{20}\sqrt{\Delta t}\right) = S_T \quad (3)$$

$S_{t+1}, S_{t+2}, \cdots, S_{t+20}$ is a possible path for the change of the two kinds of securities, and S_T is the possible rate of return on May 23, 2016.

Simulation of the May 20, 2016 two kinds of securities 100 possible rate of return. Repeat the step 2 and 3 for 100 times so that we get 100 possible yield of the two kinds of securities as S_T^1,S_T^2,……,S_T^{100}. We array the time T in ascending order. Given the confidence level (1-α), we find the VaR of the confidence level (1-α) and the quantile S_T^* (that is, the kα-th simulation price) of 100α%.

$$\text{VaR}=S_t\text{-}S_T^* \quad (4)$$

The total investment rate of the portfolio $R_P = \sum_{n=1}^{2} w_n S_{nt+1}, w_1 = w_2 = \frac{1}{2}$. Given the confidence level, VaR can be obtained. Using Matlab to program the above steps, you can calculate the VaR of the portfolio for the next trading day (May 23, 2016) as shown in Table 1:

Table 1 Calculation results of VaR based on MC method

Significance level	95%	99%
VaR	2.5724	1.8793

Test the VaR model with the failure frequency test method given by Kupiec. In the failure frequency test, it is assumed that the confidence degree of calculating VaR is c, the actual inspection number of days is T and the number of failed days is N, the failure frequency is: p = N / T.

On the basis of the previous part, we use Matlab to calculate 100 times to get 100 days' daily trading VaR using 95% and 99% confidence levels respectively. The test results are shown in Table 2.

Table 2 VaR model test results based on MC method

Significance level	95%	99%
Non-refusal interval	37<N<65	4<N<17
Actual days	59	21

The results in Table 2 show that the model is acceptable at the 95% confidence level. But there are 21 failed days at the 99% confidence level that exceeds the maximum value of the non-rejected area, indicating that the model is underestimated Actual loss value.

VaR calculation based on copula-GARCH-MC method

Parameter estimation of the GARCH model. The general expression of the GARCH (1,1) model can be written as:

$$r_t = x\beta' + \varepsilon_t \quad (5)$$

$$h_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}^2 \quad (6)$$

$$\varepsilon_t = v_t h_t \tag{7}$$

r_t is the conditional mean and h_t^2 is the conditional variance. A is the return coefficient, β is the hysteresis coefficient, and the size of the parameters α and β determines the shape of the volatility sequence. v_t is an independently distributed random variable, h_t and v_t are independent of each other. It is often assumed that v_t is a standard normal distribution.

The parameters of the GARCH (1,1) model are estimated from the CITIC Securities and Huatai Securities yield data for the entire time period from May 7, 2012 to May 20, 2016. The parameters of the GARCH (1,1) model are as Fig.5:

Dependent Variable: ZX
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 06/14/16 Time: 20:56
Sample: 1 983
Included observations: 983
Convergence achieved after 11 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000189	0.000862	0.219917	0.8259
Variance Equation				
C	1.38E-05	3.36E-06	4.114497	0.0000
RESID(-1)^2	0.057058	0.008968	6.362310	0.0000
GARCH(-1)	0.927430	0.009853	94.12914	0.0000

R-squared	-0.000001	Mean dependent var	0.000164
Adjusted R-squared	-0.000001	S.D. dependent var	0.029536
S.E. of regression	0.029536	Akaike info criterion	-4.359772
Sum squared resid	0.856648	Schwarz criterion	-4.339871
Log likelihood	2146.828	Hannan-Quinn criter.	-4.352202
Durbin-Watson stat	1.864837		

Dependent Variable: HT
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 06/15/16 Time: 08:40
Sample: 1 976
Included observations: 976
Convergence achieved after 8 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000320	0.000836	0.383351	0.7015
Variance Equation				
C	6.59E-06	2.70E-06	2.437757	0.0148
RESID(-1)^2	0.064552	0.010313	6.259037	0.0000
GARCH(-1)	0.930408	0.009639	96.52697	0.0000

R-squared	-0.000019	Mean dependent var	0.000460
Adjusted R-squared	-0.000019	S.D. dependent var	0.031931
S.E. of regression	0.031931	Akaike info criterion	-4.280776
Sum squared resid	0.994127	Schwarz criterion	-4.260762
Log likelihood	2093.019	Hannan-Quinn criter.	-4.273160
Durbin-Watson stat	1.749469		

Fig.5 GARCH (1,1) model parameters of CITIC Securities and Huatai Securities

The result of Fig.5 is expressed by the equations:

$$r_t = 0.000189 + \varepsilon_t \tag{8}$$

$$h_t^2 = 1.38 * 10^{-5} + 0.057058\varepsilon_{t-1}^2 + 0.927430h_{t-1}^2 \tag{9}$$

$$r_t = 0.000320 + \varepsilon_t \tag{10}$$

$$h_t^2 = 6.59 * 10^{-6} + 0.064552\varepsilon_{t-1}^2 + 0.930408h_{t-1}^2 \tag{11}$$

Calculation and testing of VaR. The Monte Carlo simulation is used to simulate the future earnings of the two stocks selected in this paper. The specific methods are as follows.

The GARCH (1,1) -t model is used to fit each asset yield sequence. ξ_t obey the distribution of $t_v(0,1)$, and then converted into $t_v(\xi_t)\sim U(0,1)$. The residual sequence $t_v(\xi_t)\sim U(0,1)$ is obtained.

Estimation of the parameter θ of Copula function with maximal likelihood. Monte Carlo simulations are used to generate random numbers (u_1, u_2) with t-Copula function distributions. With the inverse probability integral we get $(\xi_1, \xi_2) = (t_1^{-1}(u_1), t_2^{-1}(u_2)$**.** Replace (ξ_1, ξ_2) into the volatility equation to obtain the next model of the analog yield.

Repeat the simulation 100 times. Get 100 groups of distribution $F(\xi_1, \xi_2)$, the entire portfolio simulation yield $R_P = \sum_{n=1}^{2} w_n S_{nt+1}, w_1 = w_2 = \frac{1}{2}$. Given the confidence level, VaR can be obtained.

Calculate the VaR of the portfolio for the next trading day (May 23, 2016) as shown in Table 3:

Table 3 VaR calculation results based on copula-GARCH (1,1) method

Significance level	95%	99%
VaR	2.5029	1.9981

Ibid, using the failure frequency test given by the upiec to verify the accuracy of the VaR model.

Table 4 Test results of VaR model based on copula-GARCH (1,1) method

Significance level	95%	99%
Non-refusal interval	37<N<65	4<N<17
Actual days	52	16

The results in Table 4 show that at both 95% and 99% of the confidence level did not exceed the non-rejected area, indicating that the combination of the Copula-GARCH model with the Monte Carlo method is more accurate than merely using the Monte Carlo method.

Conclusions

Through the above analysis, we can find that CITIC Securities and Huatai Securities have high peak tail non-normality, fluctuation clustering and ARCH effect. Through the empirical comparison of MC simulation method and Copula-GARCH-MC simulation method, we find that the latter is more effective and more conducive to investors to determine the risk of investment.

Copula-GARCH is a good multivariable financial time series model. In the empirical application research, different models can be used to construct some new Copula functions such as EGARCH, TGARCH and GARCH-M to describe the marginal distribution.

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