

The Optimal Toll Plaza Design

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Abstract. Traffic jam at toll stations of highway has been increasing seriously, which has done harm to the function of highway. Therefore, the research in congestion of highway has been attached great importance nationwide. We formulate a vascular branches analogy model to account for the most appropriate shape and area of the toll plaza. By comparison, we come to conclusion that the best shape of the toll plaza is sector which is symmetrical about the central axis. The area is determined by angle θ which ranges from 15° to 29° . And we examine the solution through the simulation of cellular automata model.

Introduction

The traffic volume of highway reflects the transportation demand of contemporary development in economy. The traffic volume of highway can be regarded not only as a significant symbol of traffic modernization, but also a crucial measurement of the modernization of a country. However, with the constancy of improvement in traffic networks and the rapid development of the automobile industry, the demand of materials and personnel circulation has increased. And the number of highway is roaring up.

Information Value Evaluation Index System

Establishment of Model

Referring to the law of Poiseuille in hydromechanics,

$$E = w + n^{[2]}$$

E is dissipative energy of capillary.

$$w = \frac{kq^2}{r^4}; \quad n = br^a, \quad 1 \leq a \leq 2, \quad (1)$$

$$E = \left(\frac{dq^2}{r^4} + br^a\right) \cdot l + \left(\frac{dq^2}{4r_1^4} + br_1^a\right) \cdot l_1 + \left(\frac{kq^2}{4r_2^4} + br_2^a\right) \cdot l_2, \quad (2)$$

The model is analogied into meiging and diverging area to determine the optimal shape and size of the toll plaza. E is vehicle congestion factor.

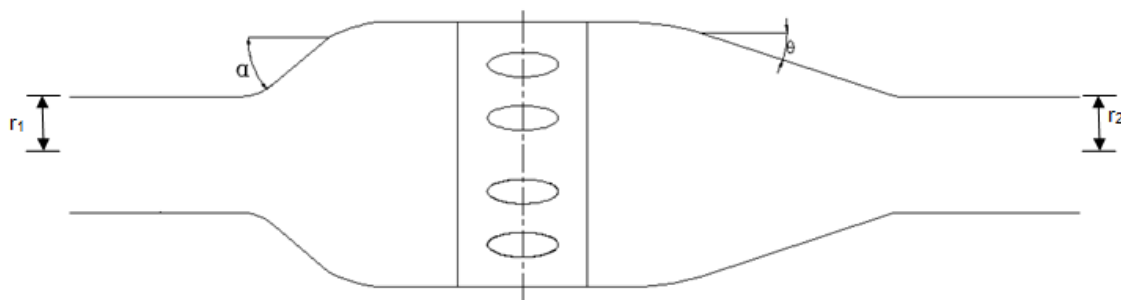


Figure 1. Plan of toll plaza

From geometrical relationship, we can figure out that

$$l = L - \frac{H}{\tan \theta_1 + \tan \theta_2}, \tag{3}$$

$$E = \left(\frac{kq^2}{r^4} + br^\alpha\right) \left(L - \frac{H}{\tan \theta_1 + \tan \theta_2}\right) + \left(\frac{kq^2}{4r_1^4} + br_1^\alpha\right) \frac{H}{(\tan \theta_1 + \tan \theta_2) \cos \theta_1} + \left(\frac{kq^2}{4r_2^4} + br_2^\alpha\right) \frac{H}{(\tan \theta_1 + \tan \theta_2) \cos \theta_2}, \tag{4}$$

According to the principle of optimality,

$$\frac{\partial E}{\partial r} = 0, \frac{dE}{dr_1} = 0, \frac{\partial E}{\partial r_2} = 0, \tag{5}$$

We get,

$$\frac{r_1}{r_2} = \left(\frac{q_1}{q_2}\right)^{\frac{2}{\alpha+4}}, \tag{6}$$

From

$$\frac{\partial E}{\partial \theta_1} = 0, \frac{\partial E}{\partial \theta_2} = 0, \tag{7}$$

We get

$$\theta_1 = \theta_2$$

$$\cos \theta = 2^{\frac{\alpha-4}{\alpha+4}} \tag{8}$$

Take $\alpha = 1$ and $\alpha = 2$, we can figure out the approximately range of θ is $15^\circ \leq \theta \leq 29^\circ$

The Verification and Simulation by Cellular Automaton

According to the reference of materials and researches, we found that the main shapes of highway toll plazas are sectors and squares. Firstly we compare and analyze the two forms to find a better solution. Under circumstances of the numbers of toll stations and lanes unchanged, it is obvious that the square area is larger and the cost is higher according to the basic geometry knowledge. From the perspective of traffic flow, a very large angle is needed if the both sides of the square lanes merge to the main lane, which will cause congestion and increase the probability of accident happening at the exit, and in this condition drivers tend to choose the middle lane causing the waste of the service ability of the border lanes, and the cost runs off.

As it indicated in the simulation diagram, severe congestion will be caused at the exit of the square plaza, the vacancy rate of the boundary toll is large and there is significant space waste of the plaza behind the toll station. Higher costs only contribute to wastes and serious congestions, it is clearly that square plaza is not the best choice for us.

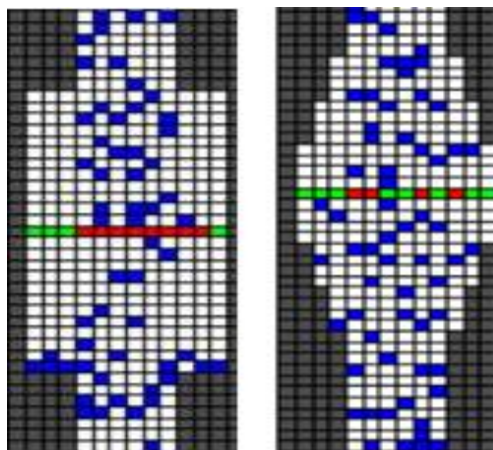


Figure 2. Comparative simulation of square and sector shape
In the toll plaza simulated by cellular automaton,

$$L_{\theta} = \tan(90^{\circ} - \theta), L_{\alpha} = \tan(90^{\circ} - \alpha)$$

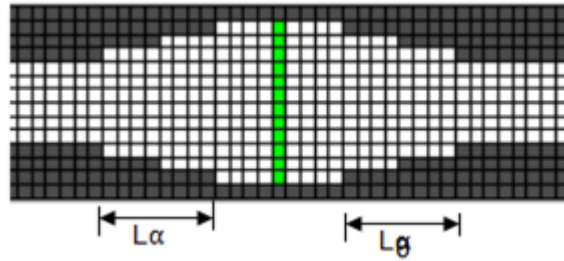


Figure 3. The toll plaza simulated by cellular automaton

Change the ratio of L_{α} and L_{θ} by giving different value to them. The optimal value of θ is verified by the throughput of the simulated vehicle in unit time (1s).

Take the 6-lane 12 toll station in the picture as an example. Take $L_{\alpha} : L_{\theta}$ as 4: 2,4: 4,2: 4 respectively. Through repeated calculation, the proportion of the throughput of three ally stable kinds of is 72/80/71. The maximum throughput is obtained around $\theta = 16^{\circ}$. The overall distribution is symmetrical. Thus, it can be proved that the optimal value of θ is in the range of the result.

In the adjustment process of the ratio is clearly shown that when the difference between θ and α is small we could get greater export throughput. It corresponds with our real life experience that the flow rate of the big export of small openings or small export large "funnel-shaped" is not as good as that of the "cylindrical" (with a similar opening and export size) object cognition.

From the analysis above, the best shape of the toll plaza can be obtained: the angle θ is in the optimum range and the value of angle α is close to the value of θ .

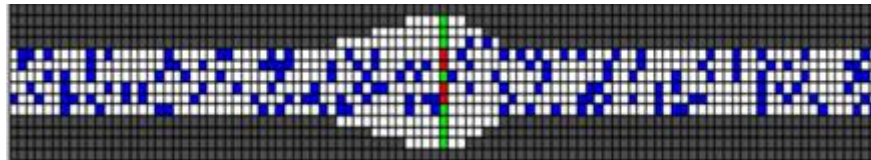


Figure 4. Simulation when $L_{\alpha} : L_{\theta}$ is 4: 2

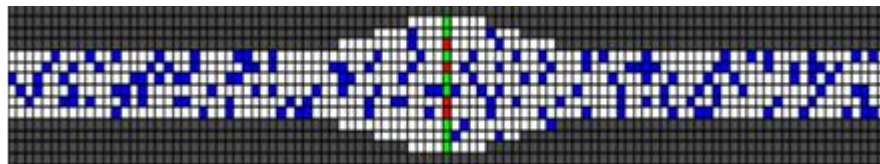


Figure 5 Simulation when $L_{\alpha} : L_{\theta}$ is 4: 4

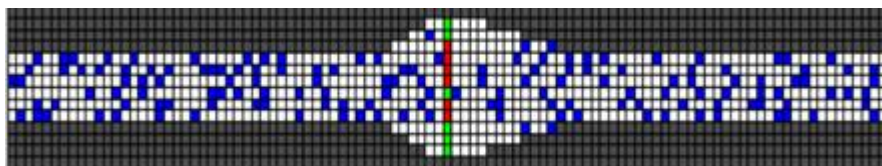


Figure 6 Simulation when $L_{\alpha} : L_{\theta}$ is 2:4

The Application of Toll Plaza Model

According to the simulation of the toll plaza model, using the price per unit area method, we estimate construction cost when taking different pattern of the toll plaza, the is relatively large.

Table 1 The imputed cost

$L\alpha : L\theta$	4:2	4:4	2:4
Imputed cost (ten thousand dollar)	192	228	192

Conclusion

In this paper, we have done optimal design for the toll station and come to conclusions. The most appropriate shape of highway toll plaza is sectors which is symmetrical about the central axis. The area of the plaza is determined by angle θ which ranges from 15° to 29° . Under the condition of what mentioned above, although the cost of building the toll station is more, larger throughput and higher service efficiency are obtained under the same circumstances, which conform to the expectation to the toll station of the public.

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