Adaptive Sliding Model Control for Passive Electro-Hydraulic Servo Loading System with Backstepping

Geqiang Li[1,2,*], Yongsheng Gu[1] and Yuesong Li[1]
[1]Henan University of Science and Technology, Jianxi Borough 471003, Luoyang China
[2]Collaborative Innovation Center of Machinery Equipment Advanced Manufacturing of Henan Province, Henan University of Science and Technology, Luoyang 471003, Henan, China
[*]Corresponding author

***Abstract*—This paper studies the problem of loading torque control for passive electro-hydraulic servo loading system with the external disturbance and parameter uncertainties. The state space model is established based on the nonlinear mathematical model of the system. Based on backstepping technique, an adaptive sliding mode backstepping control method is proposed for the system. By defining the sliding manifold and selecting a proper Lyapunov function, the final design of adaptive backstepping sliding mode controller is conducted and the stability of the controller is testified by the Lyapunov stability theory. The function of the controller was analyzed by comparative simulation study, which proves the feasibility and effectiveness of the proposed control strategy.**

Keywords-backstepping; passive electro-hydraulic servo loading system; adaptive sliding mode control; parameter uncertainties; external disturbance

I. INTRODUCTION

Passive electro-hydraulic servo loading system is hardware-in-the-loop simulation equipment on the ground and is used to simulate the aerodynamic load of an aircraft during flight[1]. Because of the passive electro-hydraulic servo loading system not only bearing the load disturbance of the steering gear but also existing the influences of nonlinear characteristics and parameter uncertainties, it is difficult to achieve better control performance. In view of the problems above, scholars at home and abroad have put forward various methods to improve the loading performance of the system[2]. Parameter optimal feed forward compensation control is adopted[3]; A combined control strategy based on the auto disturbance rejection control technique and proportional integral syn-chronization error feedback correction is designed in [4]; a new adaptive back-stepping control method based on command filtering is prop-osed, which to improve the performance of the system[5].

In this paper, the mathematical model of the passive electro-hydraulic servo loading system is reduced order processing by using the backstepping control theory, and combined with the sliding mode control. Finally, the adaptive backstepping sliding mode controller is designed. Compared with the tradional backstepping adaptive controller[6], the nesting problem caused by the mutual inclusion of parameter adaptive law and control input was effectively solved. The simulation results illustrate the effectiveness of the control method.

II. MATHEMATICAL MODEL OF THE SYSTEM

The stiffness of connecting shaft should be similar to the torque sensor, the loading system belongs to a double degree of freedom driving force control system, and the output torque of the system is $T_L=G(\theta_j-\theta_d)$, The θ_j is the input angular displacement of the loading system, and the θ_d is the external disturbance input angular displacement. The state variable is defined as $\boldsymbol{\theta_j}=[\theta_j,\dot{\theta}_j,\ddot{\theta}_j]^T$, where $\theta_{j1}=\theta_j$, $\theta_{j2}=\dot{\theta}_j$, $\theta_{j3}=\ddot{\theta}_j$, the nonlinear state space model of passive electro-hydraulic servo loading system

$$\begin{cases}\dot{\theta}_{j1}=\theta_{j2}, \\ \dot{\theta}_{j2}=\theta_{j3}, \\ \dot{\theta}_{j3}=a_1\theta_{j1}+a_2\theta_{j2}+a_3\theta_{j3}+(1/a_4)g(x_v)u_m+a_5, \\ y=T_L\end{cases} \tag{1}$$

where, u_m is system input; y is system output; $a_1=-4\beta_e C_{tm}G/(JV_m)$; $a_2=-G/J-(4\beta_e D_m^2+4\beta_e C_{tm}B_c)/(JV_m)$;

$a_3=-B_c/J-4\beta_e C_{tm}/V_m$; $a_4=JV_m\sqrt{\rho}/(4D_m\beta_e C_d wK_a G_{sv})$;

$a_5=G\dot{\theta}_d/J+4\beta_e C_{tm}G\theta_d/(JV_m)$; $g(x_v)=\sqrt{P_s-P_L\,\mathrm{sgn}(x_v)}$.

where, C_d is valve port flow coefficient; w is area gradient for servo valve; x_v is opening degree of servo valve eleent; P_s is source pressure for oil; P_L is loading pressure; ρ is hydraulic oil density; β_e is elasticity modulus of effective volume; J is rotational inertia of motor; B_c is viscous damping coefficient of motor; T_L is system output torque; G is torsional stiffness of the connecting shaft; θ_d is angular displacement of load; K_a is servo amplifier gain; G_{sv} is servo valve gain; D_m is theoretical displacement of motor; θ_j is angular displacement of motor; C_{tm} is overall leakage coefficient of motor; V_m is total volume of motor chamber and pipeline.

III. DESIGN OF CONTROLLER

The final output error of the system is defined as $e_0 = T_L - T_{Lin} = G(\theta_{j1} - \theta_{jd1})$ .The load torque output accuracy in the system is determined by θ_{j1} and θ_{jd1} , all errors are defined.

$$e_1 = \theta_{j1} - \theta_{jd1} \tag{2}$$

$$e_2 = \theta_{j2} - \theta_{jd2} \tag{3}$$

$$e_3 = \theta_{j3} - \theta_{jd3} \tag{4}$$

where, T_{Lin} is the system command input; θ_{ji}（$i=1,2,3$） is the actual value and $\theta_{jdi}(i=1,2,3)$ is the expected value of the system state variables. The following is the derivation of the virtual control of each subsystem, $f_i(i=1,2,3)$ is the controller adjustment coefficient.

Step 1: The derivative of (2) with (3) gives

$$\dot{e}_1 = \dot{\theta}_{j1} - \dot{\theta}_{jd1} = e_2 + \theta_{jd2} - \dot{\theta}_{jd1} \tag{5}$$

The Lyapunov function is chosen as

$$V_1 = 1/2e_1^2 \ge 0 \tag{6}$$

The virtual control is as follows

$$\theta_{jd2} = \dot{\theta}_{jd1} - f_1 e_1 \tag{7}$$

The derivative of (6) with (7) gives

$$\dot{V}_1 = -f_1 e_1^2 + e_1 e_2 \tag{8}$$

Step 2: Substituting (7) into (3) gives

$$e_2 = \theta_{j2} - \dot{\theta}_{jd1} + f_1 e_1 \tag{9}$$

The Lyapunov function is chosen as

$$V_2 = 1/2e_1^2 + 1/2e_2^2 \ge 0 \tag{10}$$

The virtual control is as follows

$$\theta_{jd3} = -e_1 + \dot{\theta}_{jd2} - f_2 e_2 \tag{11}$$

The derivative of (10) with (11)gives

$$\dot{V}_2 = -f_1 e_1^2 - f_2 e_2^2 + e_2 e_3 \tag{12}$$

Step 3: Sliding mode control is used and the sliding manifold is defined as

$$s = c_1 e_1 + c_2 e_2 + e_3 \tag{13}$$

where, c_1 , c_2 is normal number, differentiating (13) gives

$$\begin{aligned}\dot{s} = {} & c_1(\theta_{j2} - \dot{\theta}_{jd1}) + c_2(\theta_{j3} - \dot{\theta}_{jd2}) + a_1\theta_{j1} + \\ & a_2\theta_{j2} + a_3\theta_{j3} + (1/a_4)g(x_v)u_m + a_5 - \dot{\theta}_{jd3}\end{aligned} \tag{14}$$

To avoid the design of adaptive law, $\hat{a}_4$ contains u_m , the Lyapunov function is chosen as

$$V_3 = V_2 + 1/2a_4 s^2 \ge 0 \tag{15}$$

The derivative of (15) gives

$$\begin{aligned}\dot{V}_3 = {} & -f_1 e_1^2 - f_2 e_2^2 + e_2 e_3 + s[c_1 a_4(\theta_{j2} - \\ & \dot{\theta}_{jd1}) + c_2 a_4(\theta_{j3} - \dot{\theta}_{jd2}) + \tau_1\theta_{j1} + \\ & \tau_2\theta_{j2} + \tau_3\theta_{j3} + g(x_v)u_m + \tau_4 - a_4\dot{\theta}_{jd3}]\end{aligned} \tag{16}$$

where, $\tau_1 = a_1 a_4$, $\tau_2 = a_2 a_4$, $\tau_3 = a_3 a_4$, $\tau_4 = a_4 a_5$, $\boldsymbol{\tau} = [\tau_1, \tau_2, \tau_3, \tau_4]^T$.

Define $\tilde{\boldsymbol{\tau}} = \boldsymbol{\tau} - \hat{\boldsymbol{\tau}}$, $\tilde{a}_4 = a_4 - \hat{a}_4$,the estimated values of $\boldsymbol{\tau}$, a_4 are $\hat{\boldsymbol{\tau}}$, $\hat{a}_4$.

The Lyapunov function of system is chosen as

$$\begin{aligned}V = {} & V_3 + \frac{1}{2}\lambda_1\tilde{\tau}_1^2 + \frac{1}{2}\lambda_2\tilde{\tau}_2^2 + \\ & \frac{1}{2}\lambda_3\tilde{\tau}_3^2 + \frac{1}{2}\lambda_4\tilde{\tau}_4^2 + \frac{1}{2}\lambda_5\tilde{a}_4^2 \ge 0\end{aligned} \tag{17}$$

where, $\lambda_i > 0(i=1,2,3,4,5)$ is the parameter adaptive law adjustment coefficient.

$$\begin{aligned}\dot{V} = {} & -f_1 e_1^2 - f_2 e_2^2 + e_2 e_3 + s[c_1 a_4(\theta_{j2} - \dot{\theta}_{jd1}) + \\ & c_2 a_4(\theta_{j3} - \dot{\theta}_{jd2}) + \tau_1\theta_{j1} + \tau_2\theta_{j2} + \tau_3\theta_{j3} + \\ & g(x_v)u_m + \tau_4 - a_4\dot{\theta}_{jd3}] + \lambda_1\tilde{\tau}_1(-\dot{\hat{\tau}}_1) + \lambda_2\tilde{\tau}_2(-\dot{\hat{\tau}}_2) + \\ & \lambda_3\tilde{\tau}_3(-\dot{\hat{\tau}}_3) + \lambda_4\tilde{\tau}_4(-\dot{\hat{\tau}}_4) + \lambda_5\tilde{a}_4(-\dot{\hat{a}}_4)\end{aligned} \tag{18}$$

Finally, the adaptive backstepping sliding mode controller is designed as follows.

$$u_m = \frac{1}{g(x_v)}[-c_1\hat{a}_4(\theta_{j2}-\dot{\theta}_{jd1}) - c_2\hat{a}_4(\theta_{j3}-\dot{\theta}_{jd2}) - \hat{\tau}_1\theta_{j1} - \hat{\tau}_2\theta_{j2} - \hat{\tau}_3\theta_{j3} - \hat{\tau}_4 + \hat{a}_4\dot{\theta}_{jd3} - f_3 s] \quad (19)$$

The adaptive law of parameter variation is

$$\begin{cases} \dot{\hat{\tau}}_1 = \frac{1}{\lambda_1}s\theta_{j1}, \quad \dot{\hat{\tau}}_2 = \frac{1}{\lambda_2}s\theta_{j2}, \quad \dot{\hat{\tau}}_3 = \frac{1}{\lambda_3}s\theta_{j3}, \quad \dot{\hat{\tau}}_4 = \frac{1}{\lambda_4}|s| \\ \dot{\hat{a}}_4 = \frac{1}{\lambda_5}[c_1 s(\theta_{j2}-\dot{\theta}_{jd1}) + c_2 s(\theta_{j3}-\dot{\theta}_{jd2}) - s\dot{\theta}_{jd3}] \end{cases} \quad (20)$$

The stability condition of the system can be obtained by the analysis of the Lyapunov function. Substituting(19) into (18) gives

$$\dot{V} \le -f_1 e_1^2 - f_2 e_2^2 + e_2 e_3 - f_3 s^2 = -\boldsymbol{E}^T\boldsymbol{QE} \quad (21)$$

where, $\boldsymbol{E} = [e_1, e_2, e_3]^T$

$$\boldsymbol{Q} = \begin{bmatrix} f_1 + f_3 c_1^2 & c_1 c_2 f_3 & c_1 f_3 \\ c_1 c_2 f_3 & f_2 + f_3 c_2^2 & c_2 f_3 - \frac{1}{2} \\ c_1 f_3 & c_2 f_3 - \frac{1}{2} & f_3 \end{bmatrix}$$

To ensure that the system is asymptotically stable, the parameters c_1, c_2, f_1, f_2, and f_3 in the ABSC controller must satisfy the inequality

$$\begin{cases} f_1 > 0, f_2 > 0, f_3 > 0, c_1 > 0, c_2 > 0 \\ f_1 f_2 + f_1 f_3 c_2^2 + f_2 f_3 c_1^2 > 0 \\ f_1 f_2 f_3 + f_1 f_3 c_2 - (f_1 + f_3 c_1^2)/4 > 0 \end{cases} \quad (22)$$

Define $W = \boldsymbol{E}^T\boldsymbol{QE}$. By formula (21), $\dot{V} \le -W$ because $e_1, e_2, e_3, \tilde{\tau}_1, \tilde{\tau}_2, \tilde{\tau}_3, \tilde{\tau}_4$, and $\tilde{a}_4$ are bounded, so V is bounded. According to the Barbalat lemma, when $t \to \infty$, $e_i (i = 1,2,3) \to 0$. That is, the entire system is asymptotically stable.

IV. Controller Simulation Analysis

According to the derivation of the ABSC controller, the nonlinear control system block diagram, as shown in Figure 1, can be obtained. The parameters of the controller are selected as f_1=43, f_2=17, f_3=32, c_1=0.1, c_2=0.2, λ_1=4×10^{-10}, λ_2=2×10^{-12}, λ_3=1×10^{-9}, λ_4=3×10^{-6}, λ_5=1.5×10^{-3}. See Table 1 for the main parameters of the system.

TABLE I. SIMULATION PARAMETER TABLE

Parameter	Name	Value
J	Equivalent load inertia of motor	1.01×10^{-3} kg·m^2
D_m	Theoretical displacement of motor	4.40×10^{-5} m^3·rad^{-1}
V_m	Effective volume of the system	4.00×10^{-4} m^3
C_{tm}	leakage coefficient	4.00×10^{-12} m^5·(N·s)$^{-1}$
β_e	Elasticity modulus of effective volume	6.90×10^8 N·m^{-2}
w	gradient for servo valve	3.14×10^{-2}m
C_d	valve port flow coefficient	6.10×10^{-1}
G	Stiffness of connecting shaft	3.5×10^3 N·m·rad^{-1}
B_c	viscous damping coefficient	8.00 N·m·(rad·s^{-1})$^{-1}$
G_{sv}	servo valve gain	3.25×10^{-2} m/A

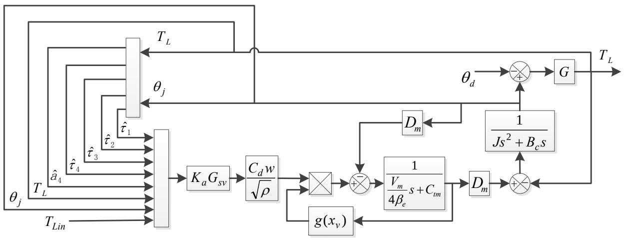
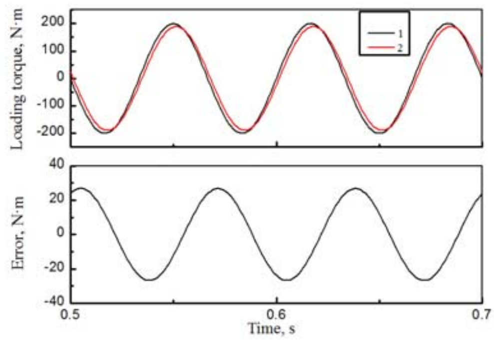
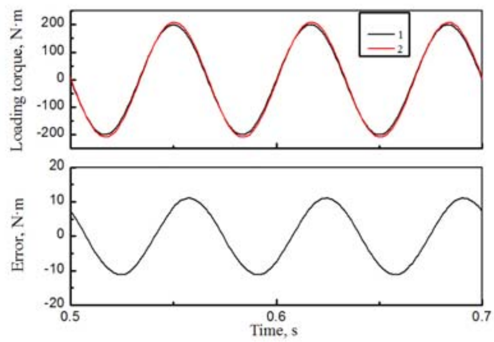


FIGURE I. BLOCK DIAGRAM OF NONLINEAR CONTROL SYSTEM



(a) Torque output with PID control



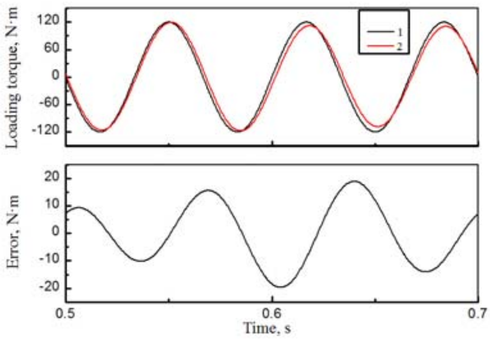
(b) Torque output with ABSC control

FIGURE II. SIMULATION CURVES OF LINEAR LOADING

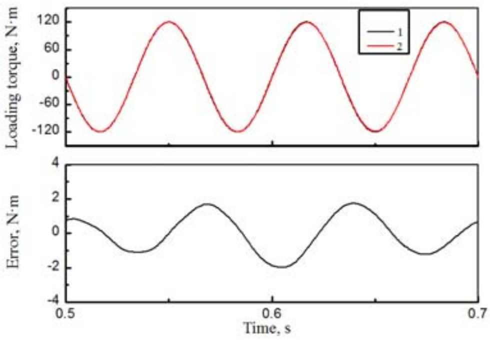
To test the effectiveness of the designed controller, according to various loading conditions, the simulation analysis of the ABSC controller and PID contoller is conducted as follows. The process is sinusoidal loading. In Figure 2, Figure 3, and Figure 4, curve 1 is the command signal, curve 2 is the output of the controller.

Figure 2 is the output torque and error curve under the linear loading, of which the loading system is 200N.m, the bearing system amplitude is about ±5º, the frequency is 15Hz. As shown, the effect of PID controller is poor when frequency is 15Hz, the error is 26N.m, fail to track the performance requirements of double-ten indicators; the amplitude tracking is improved obviously when using the ABSC controller, the error is only 8N.m. The inhibition of the ABSC controller in terms of the extra torque interference are superior to those of the PID controller.

The nonlinear load simulation is shown in Figure 3. For a load torque of 120 N·m with a frequency of 15 Hz, the bearing system has an amplitude of 2 degrees with a frequency of 10 Hz. As shown in Figure 3, when using the PID controller, the output torque curve shows bias, with the curve showing the "beating" state, and there is a certain lag phase. In contrast, the ABSC controller does not show the phenomenon above, so the ABSC controller has a good control effect on the nonlinear load.



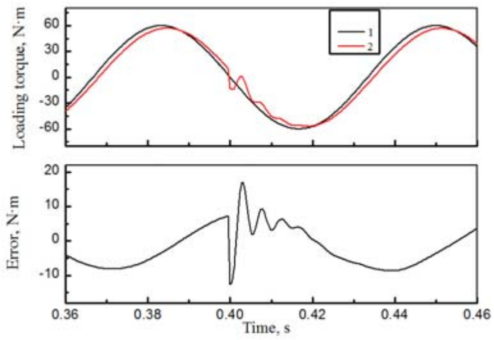
(a) Torque output with PID control



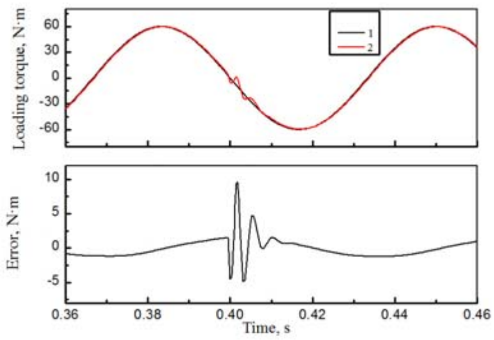
(b) Torque output with ABSC control

FIGURE III. SIMULATION CURVES OF NONLINEAR LOADING

To verify the performance of the disturbance rejection controller design for the outside signal, for T_L=60 N·m, the bearing system amplitude is 1º with a sine wave frequency of 15 Hz, and the output torque is appended to the end of the 20 N·m amplitude, with a period of 0.8 s, a width of 0.4 s, and wave signal interference. The simulation results are shown in Figure 4, the system is disturbed by the outside square wave signal at 0.4 s. The ABSC controller can inhibit the interference better than the PID controller, so the ABSC controller has strong inhibition ability for external disturbances.



(a) Torque output with PID control



(b) Torque output with ABSC control

FIGURE IV. SIMULATION CURVES WITH SQUARE WAVE DISTURB ON LOADING SYSTEM

V. CONCLUSION

Aimed at the problem of the passive electro-hydraulic servo loading system with the external disturbance and parameter uncertainties, the adaptive backstepping sliding mode control strategy is proposed. The strategy not only overcoming the influences of external disturbance and uncertainty parameters but also solving the problem of the control quantity and the adaptive law being nested. The simulation results show that the ABSC controller has a better control effect and can effectively improve the loading performance of the system under various conditions of loading and external disturbance.

ACKNOWLEDGMENT

This study is supported by the National Natural Science Foundation of China (51175148).

REFERENCES

[1] WANG C W, JIAO Z X, LUO C J, An improved velocity synchronization control on electro-hydraulic load simulator[J]. Acta Aeronautica ET Astronautica Sinica2012, 09:1717-1725.

[2] HAO J J. Application of dual value in electro-hydraulicsimulator[J]. China Mechanical Engineering, 2002,13(10): 814-816.

[3] OUYANG X P, LI F, ZHU Y, et al. Nonlinear optimal feedforward compensation controller for heavy load aviation load simulator[J]. Acta Aeronautica ET Astronautica Sinica, 2016, 02:669-679.

[4] GUO D, FU Y L, LU N, et al. Application of ADRC technology in electro-hydraulic force servo system [J]. Journal of Beijing University of Aeronautics and Astronautics, 2013, 01:115-119.

[5] FANG Y M,SHI S L,LI J X, et al. Adaptive backstepping control for electro-hydraulic servo system with input saturation based on command filter[J]. Electric Machines and Control, 2013, 09:105-110.

[6] CAO J, ZHANG B, ZHAO K D. Backstepping control of passive force control system with parameter uncertainties[J]. Journal of Harbin Institute of Technology, 2010, 07:1071-1075.