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Standard Grey Potential and Its Application in Grey Target Decision

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***Abstract*—Grey target decision-making is one of important problems of decision-making theory. It is critical to express uncertain information effectively and depose them in reasonable and simple way. Firstly this paper makes a brief overview of the existing method for Grey target decision. Then the conception of Grey potential degree system is introduced and the conception of standard Grey Potential Degree is build up in the paper. And a new Grey potential-based method on Grey target clustering is proposed. At the same time the standard Grey potential and its application in multiple criteria is studied. It is presented by three examples, that the Grey potential-based method could be used to solve the problem of Grey target decision conveniently and effectively.**

Keywords-grey target; decision-making; grey number; grey potential degree

I. INTRODUCTION

In traditional decision-making theory, the parameters of model are usually of determined values. However in real life, because of objective conditions and human cognition limitation, it is very hard to obtain complete information; hence uncertainty theory becomes an important branch of decision-making problem.

Grey systems theory is another method to do with uncertain decision-making problems. Grey number is used to describe uncertain information in the theory. Deng is the pioneer of Grey systems theory who first used described uncertain problems with Grey numbers [1].

The thought of Gray target [2] is, for a set of patterns in the sequence, to find the data which is nearest to target value and construct a standard mode, each mode and standard mode constitutes gray target, the standard mode is the bull's-eye. Grey Target is a method for processing multiple program evaluation and optimization of multiple objective problem. The earliest introduction of Interval Number into Grey Target Decision Model is Yaoguo Dang, Sifeng Liu and Bin Liu's research. Important research Grey Target include information theory established Shannon, using entropy to measure the amount of information; Chanyao Gu and Wanhua Qiu[3] made definition of entropy and introduced into decision-making; Entropy is used by Mon et al. To decide weight[4]; Wang et al study using objective programming, quadratic programming, and dynamic programming model to determine the index weight [5]. Sifeng Liu [6], Yaoguo Dang [7], Yongming Chen [8] all studied on Grey Target decision-making. Grey Target has also been applied and Evaluation. To increase the amount of information to improve decision-making accuracy, Guangzhi Bu built up fuzzy comprehensive evaluation of three parameters interval numbers [9]. In addition Qizhou Hu [10], Xinfang Wang [11], Rong Lan [12], Luo Dang [13] and Jie Song [14] studied and explored three-parameter interval gray numbers.

To compare Grey number is very important to obtain scientific and reasonable results for a Grey target decision-making problem. But there is very little study on comparing or ranking Grey numbers. Xie and Liu, in paper [15], put forward novel methods on comparing Grey number, of which the probability density function is used. In Zeng's research on Grey target decision-making model, the information distribution of Grey number is asked to know [16].

However, in the actual application of Grey number, there are some problems. (1) It is difficult to know the probability density function or the whiten function of Grey number. (2) If Grey number is compared and deposed through whiten value which is most commonly used, much information of Grey number will be lost, at the same time the value of Grey number in uncertainty is lost partly. (3) In normal condition, it is hard to supply additional information for decision-making.

So that, this paper focuses on handling directly Grey number without whiten value and without distribution function in Grey target decision-making. Furthermore, approach of Grey potential degree could treat the data with both comparing and standardization work. Grey potential could be used as a form of metrics in Grey target decision-making.

In this paper, we will propose the conception of Grey potential degree and discuss its application in Grey target decision. The remaining paper is organized as follows. In Section II, the conception system of Grey number and Grey potential degree is put forward. And the structure of Grey potential degree is presented and especially Standard Grey Target and Standard Grey potential degree. In Section III, the application of standard Grey potential degree, the model of Basic Grey Target Clustering Model is studied. Finally, the typical numerical example are presented to demonstrate the effectiveness of the method in Section IV. Section V comes to the conclusions.

II. CONCEPTIONS OF GREY POTENTIAL DEGREE

A. Grey Potential Degrees

Definition 2.1 (Grey number) because of the defect of information due to various reasons, the exact value of the characteristics cannot be determined but the scope of exact value can be decided, so the description of the characteristics can use Grey number $\otimes_i$, denoted as $\otimes_i \in [a_{ij}, b_{ij}]$, $a_{ij} \neq b_{ij}$.

Definition 2.2 (Grey number whitening and true value) for Grey number $\otimes_i$, when more and more information is provided, the scope of Grey number $\otimes_i$ becomes smaller and smaller, this process is called Grey number whitening. When complete information is supplied, the Grey number whitening is finished and true value d_i^* of characteristics description $\otimes_i$ is determined.

When no further information is provided and Grey number has to be deposed without any whitening. So Zhigeng Fang put forward the concept of Grey potential degree to build a comparative framework for the interval Grey number in his doctoral thesis [17]. The improved conceptions of Grey potential degree are put forward in the past studies [18,19] and are further developed here.

Definition 2.3 (Grey Superiority Potential Degree of Grey Numbers $i-to-j$, $GSPD_{i\to j}$ in short) for any two Grey numbers $\otimes_i \in [a_i, b_i]$, $a_i \neq b_i$ and $\otimes_j \in [a_j, b_j]$, $b_i - b_j$ is called Grey superior dominance. $\frac{b_i - b_j}{b_i - a_i}$ is called Grey Superiority Potential Degree of Grey Number $\otimes_i$ to Grey number $\otimes_j$, denoted as $GSPD_{i\to j}$.

Definition 2.4 (Grey Inferior Potential Degree of Grey Numbers $i-to-j$, $GIPD_{i\to j}$ in short) for any two Grey numbers $\otimes_i \in [a_i, b_i]$, $a_i \neq b_i$ and $\otimes_j \in [a_j, b_j]$, $a_i - a_j$ is called Grey inferior dominance. $\frac{a_i - a_j}{b_i - a_i}$ is called Grey Inferior Potential Degree of Grey Number $\otimes_i$ to Grey number $\otimes_j$, denoted as $GIPD_{i\to j}$.

The realistic condition of Grey numbers are more complex, only $GSPD_{i\to j}$ or $GIPD_{i\to j}$ is not enough for full description of the relationship between the Grey numbers $\otimes_i$ and $\otimes_j$. Therefore $GSPD_{i\to j}$ and $GIPD_{i\to j}$ need to be combined.

At the same time, Grey Superiority Potential Degree of Grey Numbers focus on superiority potential and Grey Inferior Potential Degree focus on inferior potential. Superiority potential means good and positive potential and inferior potential means bad and negative possibility. In real situation, the decision-maker has different focus in different situation. Therefore, Grey Total Potential Degree of Grey Numbers is defined as following way.

Definition 2.5 (Grey Total Potential Degree of Grey Numbers $\otimes_i$ to $\otimes_j$, $GTPD_{i\to j}$ in short) For any two Grey numbers $\otimes_i \in [a_i, b_i]$, $a_i \neq b_i$, and $\otimes_j \in [a_j, b_j]$, the sum of Grey Superiority Potential Degree and Grey Inferior Potential Degree of Grey number $\otimes_i$ to Grey number $\otimes_j$, is called Grey Total Potential Degree of Grey number $\otimes_i$ to Grey number $\otimes_j$, denoted as $GTPD_{i\to j}$, that is, $GTPD_{i\to j} = \alpha \cdot GSPD_{i\to j} + \beta \cdot GIPD_{i\to j}$, $\alpha + \beta = 1, \alpha, \beta \in [0,1]$. Here, α is Superiority coefficient, and β is called Inferior coefficient. The value of these two coefficients displays the proportional relationship between Superiority Potential Degree which is representative of advantage potential, and Grey Inferior Potential Degree which is on behalf of risk, when the issue is considered in integrated way

Here $GTPD_{i\to j}$ is the sum of $GSPD_{i\to j}$ and $GIPD_{i\to j}$, and it can be used to compare two Grey numbers. In order to convenient to compare the Grey numbers by Grey Total Potential Degree, as follows give judging rules.

Definition 2.6 the Judgment Rules of Grey Total Potential Degree of Grey Numbers $\otimes_i$ to $\otimes_j$ are shown as follows.

For any two Grey numbers $\otimes_i \in [a_i, b_i]$, $a_i \neq b_i$, and $\otimes_j \in [a_j, b_j]$,

1) When $GTPD_{i\to j} = 0$, Grey numbers $\otimes_i$ and $\otimes_j$ have the Equal Grey Total Potential Degree, expressed as $\otimes_i \overset{p}{=} \otimes_j$.

2) When $GTPD_{i\to j} \succ 0$, Grey Number $\otimes_i$ has Positive Grey Total Potential Degree to Grey number $\otimes_j$, denoted as $\otimes_i \overset{p}{\succ} \otimes_j$. And $\otimes_i$ is Superior Potential Grey number, $\otimes_j$ is Inferior Potential Grey number.

3) When $GTPD_{i\to j} \prec 0$, Grey Number $\otimes_i$ has Negative Grey Total Potential Degree to Grey number $\otimes_j$, denoted as $\otimes_i \overset{p}{\prec} \otimes_j$. And $\otimes_i$ is Inferior Potential Grey number, $\otimes_j$ is Superior Potential Grey number.

There are some characters about Grey potential degree.

Theorem 2.1 for any three interval Grey numbers, $\otimes_i \in [a_i, b_i]$ ($a_i \neq b_i$), $\otimes_j \in [a_j, b_j]$ ($a_j \neq b_j$), and $\otimes_k \in [a_k, b_k]$, if $\otimes_i \overset{p}{\prec} \otimes_j$ and $\otimes_j \overset{p}{\prec} \otimes_k$ exit, there will be $\otimes_i \overset{p}{\prec} \otimes_k$.

Theorem 2.2 for any four interval Grey numbers, $\otimes_i \in [a_i, b_i]$ ($a_i \neq b_i$), $\otimes_j \in [a_j, b_j]$,

$\otimes_s \in [a_s, b_s]$ $\otimes_s \in [a_s, b_s]$ ($a_s \neq b_s$) and $\otimes_t \in [a_t, b_t]$, if $\otimes_i \overset{p}{\prec} \otimes_j$ and $\otimes_s \overset{p}{\prec} \otimes_t$ exit, there will be $\otimes_i + \otimes_s \overset{p}{\prec} \otimes_j + \otimes_t$.

The values of Grey Potential Degrees tell us the relationship between Grey numbers. Therefore the distribution of the values of Grey Potential Degrees should be discussed further in order to compare Grey numbers.

B. Standard Grey Target and Standard Grey potential degree

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In order to compare alternatives, consistent standard should be built up. In Grey target decision problem, there are two levels of standard. The alternatives should be selected by values of indexes. So different indexes should be compared and integrated. For each index, which maybe have different source or evaluated by many experts, it needs to combine different information by consistent standard.

It is considered that a standard Grey target is set for each index. And then compared to standard Grey target, the Grey potential degree could be used to compare and integrate.

Assume that there is evaluation Grey matrix $M(\otimes) = (\otimes_{ij})_{m\times n}$, $\otimes_{ij} \in [a_{ij}, b_{ij}]$, $a_{ij} \neq b_{ij}$, $i = 1, \cdots, m$, $j = 1, \cdots, n$. $\otimes_{ij}$ is the Grey evaluation of Object-i on Index-j, or $\otimes_{ij}$ is the Grey evaluation of Object-i by Expert-j, $\otimes_{ij}$ is the Grey evaluation of Object-i by or Source-j which may be judged by different ways. How to set standard Grey target?

Because different index has different measurement scale, each standard Grey target, denoted as $\otimes_s$, should be set for each index-j according to the character of index. However the most important thing is that if every standard Grey target $\otimes_s$ for every index follows the same principle and their Grey potential degrees are calculated by the same approach, it is possible to integrate the judgment of different criteria directly. Hence principle of value-covered set of data of index-j, $(j = 1, \cdots, n)$ is adopted.

Definition 2.7(Standard Grey Target of Assessment) Assume that $\otimes_{ij} \in [a_{ij}, b_{ij}]$, $(j = 1, \cdots, n)$ is a series assessments of number-n on fixed subject-i, which could be object-i, index-i, or source-i, and $\otimes_{ij}$ $(j = 1, \cdots, n)$ have the same measurement scale. Set standard Grey number, that is $\otimes_{Si} = \otimes_{total} \in [\, a_{st}, b_{st}\,]$, $a_{st} \neq b_{st}$, $a_{st} = \min_j a_{ij}, b_{st} = \max_j b_{ij}$ $(j = 1, \cdots, n)$. $\otimes_{si}$ is defined as Standard Grey Target of Assessment of $\otimes_{ij}$ $(j = 1, \cdots, n)$.

Definition 2.8(Standard Grey Potential Degree) To each judgment $\otimes_{ij} \in [a_{ij}, b_{ij}], (j = 1, \cdots, n)$, set Standard Grey Target $\otimes_{si}$ for fixed subject-i, Standard Grey Superiority Potential Degree of $\otimes_{i\_j}$ is, $GSPD^S_{si\to ij} = \dfrac{b_{st} - b_{ij}}{b_{st} - a_{st}}$; Standard Grey Inferior Potential Degree of $\otimes_{ij}$ is $GIPD^S_{si\to ij} = \dfrac{a_{st} - a_{ij}}{b_{st} - as_t}$, Standard Grey Total Potential Degree of $\otimes_{i\_j}$ is $GTPD^S_{Si\to ij} = \alpha \cdot GSPD^S_{si\to ij} + \beta \cdot GIPD^S_{si\to ij}$, $\alpha + \beta = 1, \alpha, \beta \in [0,1]$.

It is easy to proof that the natures of Standard Grey Potential Degree as followed, there are $0 \le GSPD^S_{si\to ij} \le 1$, $-1 \le GIPD^s_{si\to ij} \le 0$ and $-1 \le GTPD^s_{Si\to ij} \le 1$.

By the ways of Standard Grey Potential Degree, Grey target problem with Grey matrix could be deposed.

III. GREY TARGET DECISION-MAKING MODELS

In this paper the Basic Grey Target Clustering Model is discussed.

When a Grey target is provided, many subjects can be divided into different clusters which are in different relationship with Grey target. This kind of decision-making problem is called Grey target clustering.

Here the value of target and subjects are presented as Grey numbers. It is assumed that there is a series of evaluation $\otimes_i \in [a_i, b_i]$ $(i = 1, \cdots, n)$ and target $\otimes_T$.

Above all, the realistic condition of target and each subject is complex because the relationship between two Grey numbers is so complex that $GSPD_{i\to j}$ and $GIPD_{i\to j}$ should be discussed separately and further combined to analyze.

Based on the conception and characters of Grey potential degree discussed above, combined with the Grey superiority and inferiority, the Grey potential relationship between Grey numbers $\otimes_i$ and $\otimes_j$ can be classified in to six kinds according to the Grey superiority potential degree $GSPD_{i\to j}$ and Grey inferiority potential degree $GIPD_{i\to j}$, as shown in Table I.

Grey potential degree of $\otimes_i$ to $\otimes_j$ is mainly composed of Grey superiority potential and Grey inferiority potential which present the relationship between Grey numbers $\otimes_i$ and $\otimes_j$. The case of "Relative Superiority and None Inferiority" is marked as the type of "RSNI" in abbreviated. And the case of "Relative Inferiority and None Superiority" is

defined as the type of "RINS" in abbreviated. The conditions of "Relative Superiority and Relative Inferiority" and "None Superiority and None Inferiority" are similar and named as the type "RSRI" and "NSNI". Therefore by the Grey potential degree, there are six regions of Grey potential relationship, shown in Figure I.

In Figure I, the oval area is $\otimes_j$ and each kind of relationship has it typical Grey number. According to the relationship map of Grey numbers, for a series of evaluation $\otimes_i \in [a_i, b_i]$ $(i = 1, \cdots, n)$ and target $\otimes_T$, the basic Grey target clustering is processed as follows.

TABLE I. THE GREY POTENTIAL RELATIONSHIP BETWEEN GREY NUMBERS $\otimes_i$ TO $\otimes_j$

Grey Potential degree of $\otimes_i$ to $\otimes_j$	$GSPD_{i\to j}$	$GIPD_{i\to j}$	**Mark of Relationship**
Absolutely Superiority	$GSPD_{i\to j} \succ 1$		AS
Relative Superiority and None Inferiority	$0 \le GSPD_{i\to j} \le 1$	$GIPD_{i\to j} \succ 0$	RSNI
Relative Superiority and Relative Inferiority	$0 \le GSPD_{i\to j} \le 1$	$-1 \le GIPD_{i\to j} \le 0$	RSRI RE1
None Superiority and None Inferiority	$GSPD_{i\to j} \prec 0$	$GIPD_{i\to j} \succ 0$	NSNI RE2
Relative Inferiority and None Superiority	$GSPD_{i\to j} \prec 0$	$-1 \le GIPD_{i\to j} \le 0$	NSRI

Step one. Computer the Grey superiority potential degree $GSPD_{i\to T}$ and Grey inferior potential degree $GIPD_{i\to T}$ separately.

Step two. According to Grey potential relationship between Grey numbers in Table I and make the relationship map of Grey target clustering.

This is the basic Grey target clustering method. In many situations, Grey target clustering is asked based on multiple criteria. It will be discussed in other papers.

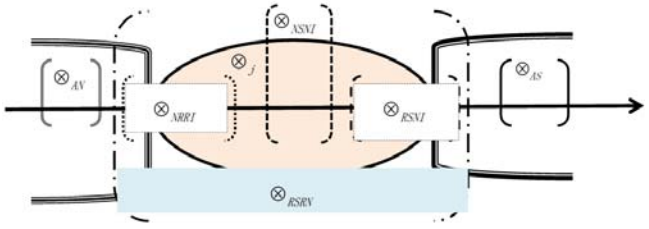

FIGURE I. THE RELATIONSHIP MAP BETWEEN GREY NUMBERS BY GREY POTENTIAL DEGREES

IV. EXAMPLES

With the Grey Potential Degree and its application model in Section II and III, in this section one example of Grey Target Clustering will be presented to explain the effect of Grey potential-based method on Grey target decision.

When a Grey target [80, 85] is provided, there are 20 evaluations of subjects shown in Table II, which need to be divided into different clusters according to different relationship with Grey target.

There is a Grey target clustering problem. Here the value of target and subjects are presented as Grey numbers. There is a series of evaluation $\otimes_i \in [a_i, b_i]$ $(i = 1, \cdots, 20)$ and target $\otimes_T \in$ [80, 85].

TABLE II. TWENTY EVALUATIONS DATA OF SUBJECTS

NO.	**Score**	**NO.**	**Score**
1	[88,92]	11	[86,91]
2	[80,85]	12	[78,85]
3	[82,84]	13	[75,80]
4	[83,88]	14	[63,68]
5	[81,84]	15	[90,93]
6	[82,88]	16	[80,84]
7	[75,82]	17	[75,85]
8	[78,86]	18	[75,87]
9	[58,63]	19	[76,81]
10	[85,90]	20	[80,88]

According to the model of Grey target clustering in Section III, the realistic condition of target and each subject depends on the relationship between Grey superiority potential degree $GSPD_{i\to T}$ and Grey inferior potential degree $GIPD_{i\to T}$.

Based on the conception and characters of Grey potential degree discussed above, combined with the Grey superiority and inferiority, the Grey potential relationship between Grey numbers $\otimes_i$ and $\otimes_T$ can be classified in to six kinds, as shown in Table III and IV.

TABLE III. $GSPD_{i\to T}$, $GIPD_{i\to T}$ AND GREY POTENTIAL RELATIONSHIP

NO.	**GSPD**	**GIPD**	**Grey Potential Relationship**
1	1.750	2.000	AS
2	0.000	0.000	RSRI
3	-0.500	1.000	NSNI
4	0.600	0.600	RSNI
5	-0.333	0.333	NSNI
6	0.500	0.333	RSNI
7	-0.429	-0.714	NSRI
8	0.125	-0.250	RSRI
9	-4.400	-4.400	AI
10	1.000	1.000	RSNI
11	1.2	1.2	AS

TABLE III, CONT. $GSPD_{i\to T}$, $GIPD_{i\to T}$ AND GREY POTENTIAL RELATIONSHIP

12	0.000	-0.286	RSRI
13	-1.000	-1.000	NSRI
14	-3.400	-3.400	AI
15	2.667	3.333	AI
16	-0.250	0.000	NSRI
17	0.000	-0.500	NSRI
18	0.167	-0.417	RSRI
19	-0.800	-0.800	NSRI
20	0.375	0.000	RSRI

TABLE IV. SIX CLUSTERS OF TWENTY SUBJECTS

Grey Potential Relationship	**NO.**	**Grey Potential Relationship**	**NO.**
AI	9, 14, 15	RSNI	4, 6, 10
AS	1, 11	NSRI	7, 13, 16, 17, 19
NSNI	3, 5	RSRI	2, 8, 12, 19, 20

V. CONCLUSIONS

The description and treatment of uncertain information is very important to Grey target decision-making problem. This paper focuses on handling directly Grey number without whiten value and without distribution function in Grey target decision-making. The paper firstly built up the definition system of Grey potential degrees, the Grey Potential Degrees, including Grey Superiority Potential Degree, Grey Inferior Potential Degree, Grey Total Potential Degrees, and their characters are analyzed. In order to compare and sort different kinds of Grey numbers, standard Grey potential degree is put forward. Then the application of standard Grey potential degree, the model of Basic Grey Target Clustering Model is built up. Finally, three typical numerical example is presented to demonstrate the effectiveness of the method in Section IV.

Furthermore, approach of Grey potential degree could treat the data with both comparing and standardization work. Grey potential could be used as a form of metrics in Grey target decision-making in many situations of Grey target, including real number and interval number, whether single source or multiple sources.

As is presented in the results, the method proposed this paper can describe uncertain information better. The Grey potential-based method can makes full use of Grey number information without losing information and without additional information. And it can substitute stochastic and interval methods to solve uncertain problems in the situation that enough information of uncertainty is unavailable. It will be widely used in uncertain decision-making problems.

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