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Trellis Coded Multi-*h* CPM for Serially Concatenated Schemes

Xian LIU[1, a], Hua FANG[1, b], Kegang PAN[1, c], Xiaofei PAN[1, d], Bin GAO[1, e], Nan SHA[1, f]

[1]College of Communications Engineering, PLA University of Science and Technology, Nanjing 210007, China

[a]email: 13584013508@163.com, [b]email:13951753065@163.com, [c]email:13701581949@163.com
[d]email: 13915956811@163.com, [e]email:15895977608@163.com, [f]email:13814070036@163.com

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Abstract. Trellis coded multi-*h* CPM schemes have been shown in the literature to have attractive power-bandwidth performance at the expense of increased receiver complexity. In this paper, trellis coded multi-*h* CPM is adopted in the serially concatenated schemes as the inner encoder. With such schemes, interleaving gain could be improved comparatively while convergence threshold failed to improve. As a result, the performance in error-floor region is much better than traditional SCCPM systems based on the union bound analysis. Both analysis and simulations show that proper use of multiple indices combined with trellis code is much more suitable for systems with very low bit error rate probability.

Introduction

With the adoption of MIL-STD-188-181B on 20 March 1999 [1], multi-*h* Continuous Phase Modulation (CPM) became a mandatory requirement of US. Military UHF SATCOM terminals. A dual-*h* CPM scheme was selected as a Tier-II waveform in IRIG-106 Aeronautical Telemetry (ARTM) standard in 2004 because of its superior spectral efficiency and error performance as compared to the legacy PCM/FM waveforms [2]. Among these multi-*h* CPM schemes, a properly chosen cyclic set of modulation indices results in delayed merging of neighboring phase trellis paths and, therefore, provides a larger minimum Euclidean distance than conventional single-*h* CPM schemes.

In the recent years, several parallel and serially concatenated coding schemes have been proposed for use with CPM with iterative demodulation and decoding [3] [4] [5]. The main idea in most of these schemes is to exploit the inherent recursive nature of CPM to treat the modulator as a recursive encoder, which is essential for the interleaving gain. Ming Xiao and Tor M. Aulin investigated Serially Concatenated Continuous Phase Modulation (SCCPM) with Convolutional Codes (CC) over rings, and compared with previous SCCPM with binary CC, CC over rings in SCCPM showed an improvement concerning the convergence threshold or error floors [3]. However, single index was used in such system and the convergence threshold and error floors are paradoxical in optimizing the system design. Multiple indices sometime could benefit both on convergence threshold and error floors. Krishna R. Narayanan designed serially concatenated convolutional coded and Low-Density Parity-Check (LDPC) coded MSK schemes [4], and based on density evolution, make the design matched to the iterative decoding algorithm, and get valuable results. However, such schemes just paid attention to MSK signal, and other CPM signals were not concerned. High order modulation is more urgent for some bandwidth-limited systems. Hasung Kim and Gordong L. Stüber also discussed the serially concatenated CPM systems with multiple indices [5], however, its analysis failed to tell us how to choose the multiple indices to achieve superior performance.

In this paper, trellis coded multi-*h* quaternary CPM for serially concatenated system has been investigated with the maximum-likelihood upper bounds on error probability [6] and convergence analysis [7]. Both the chernoff union bound and the convergence threshold are analyzed and optimized based on the maximum-likelihood principle and the density evolution respectively. Firstly, the error event of multi-*h* CPM was analyzed which is employed as the inner code of the serially concatenated system. Because the multi-*h* CPM is actually rate-1 accumulator, the interleaving gain could not be improved effectively. Then trellis coded multi-*h* CPM was employed as inner code to optimize the interleaver gain. Based on analysis and simulations, valuable results are also concluded.

Signal Model

The multi-*h* SCCPM system model is described in Fig. 1. At the transmitter side, the serially concatenated system is composed by the outer convolutional encoder, interleaver and multi-*h* CPM modulator, and such modulator just acts as the inner encoder of the serially concatenated system. At the receiver side, the extrinsic information $P(C,I)$ and $P(U,I)$ are exchanged between two soft-in soft-out modules, which are used as APP processors to demodulate CPM signals and decode the convolutional coded signals respectively.

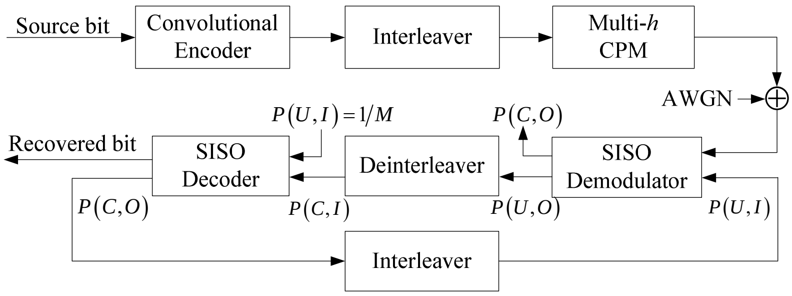

Fig. 1 SCCPM system model with multiple indices

According to the analysis of the error events for single-*h* CPM, the theorem on determining error events was presented as follows [3].

$$\sum_i \tilde{l}_i \bmod P = 0 \tag{1}$$

Where $\tilde{l}_i$ is the input difference sequence, P is the denominator of the modulation index. Based on such theorem, the inference which determines error events of multi-*h* CPM signals could be derived as follows.

$$\left(\sum_i \tilde{l}_i \cdot K_{(i)_H}\right) \bmod P = 0 \tag{2}$$

Whereas these multiple indices could be expressed as $h=[K_1, K_2,\ldots, K_H]/P$, and H is the number of multiple indices. The range of the input difference sequence $\tilde{l}_i$ is $\{-M+1,\ldots,0,\ldots,M-1\}$ for M-ary modulated symbols. The effective length of error events for multi-*h* CPM signals is the number of non-zero symbols of the input difference sequence.

According to expression (2), it could be concluded that the largest effective length of multi-*h* CPM error events is 2 no matter how many indices are adopted. For such conclusion, the input difference sequence $[-x, 0\ldots0, x]$ (H-1 zeros) inevitably results in an error event with $h=[K_1, K_2,\ldots, K_H]/P$, because

$$-x\cdot K_1 + 0\cdot K_2 + \cdots + 0\cdot K_H + x\cdot K_1 \equiv 0 \tag{3}$$

For partial-response CPM signals, the largest effective length is still 2, that is because no matter what value of pulse memory L is, the input difference sequence $[-x, 0\ldots0, x, 0\ldots0]$ (H-1 zeros between $-x$ and x, L-1 zeros following x) must result in an error event.

Union Bound Analysis

For SCCPM system with single-*h*, the union bound could be derived as [3]

$$P_{b,e} \le \sum_{w}\sum_{\tilde{l}^o}\sum_{\tilde{l}}\sum_{n^{\text{in}}}\sum_{n^{\text{out}}} \frac{w\cdot\binom{N}{n^{\text{out}}}\cdot\binom{N}{n^{\text{in}}}}{N\cdot R_c^o\cdot\log_2 M\cdot T} A^{o}_{w,\tilde{l}^o,n^{\text{out}}} \times A^{\text{in}}_{\tilde{l},d^2_{\text{in}},n^{\text{in}}} Q\left(\sqrt{d^2_{\text{in}}\cdot\log_2 M\cdot R_c^o\cdot SNR}\right) \tag{4}$$

Assuming $\max\left\{n^{\text{in}}, n^{\text{out}}, \left|\tilde{l}^o\right|\right\} \ll N$ and ignoring the terms not related with N, then expression (4) becomes

$$P_{b,e} \le \sum_{w}\sum_{\tilde{l}^o}\sum_{\tilde{l}}\sum_{n^{\text{in}}}\sum_{n^{\text{out}}} \frac{w\cdot N^{n^{\text{in}}+n^{\text{out}}-\left|\tilde{l}^o\right|-1}}{R_c^o\cdot\log_2 M} A^{o}_{w,\tilde{l}^o,n^{\text{out}}} \times A^{\text{in}}_{\tilde{l},d^2_{\text{in}},n^{\text{in}}} Q\left(\sqrt{d^2_{\text{in}}\cdot\log_2 M\cdot R_c^o\cdot SNR}\right) \tag{5}$$

Both expression (4) and (5) are very complex, however, there are two factors which mainly determines the performance: one is the exponent of the interleaver length N, which is called interleaving gain; the other is the Euclidean distance, since the $Q(\cdot)$ function decreases exponentially with distance d^2_{in}.

For serially concatenated system with single-h, the interleaving gain is determined by the maximal value of the exponent

$$\alpha_{\max} = \max\left\{n^{\text{in}} + n^{\text{out}} - \left|\tilde{l}^o\right| - 1\right\} \tag{6}$$

Where $\left|\tilde{l}^o\right|$ is the number of nonzero symbols of the input difference sequence, n^{in} and n^{out} are the number of error events of the inner code (CPM) and outer code respectively.

The analysis of interleaving gain in expression (6) is also applied for multi-h SCCPM system as the inner encoder was replaced by multi-h CPM. Based on this, much effort is then made to improve the interleaving gain or increase the Euclidean distance otherwise with multi-h schemes.

In order to improve the interleaving gain, multiple indices with different values are tried to increase the effective length of the inner encoder. Unfortunately, the increase of indices and proper value of them failed to benefit the interleaving gain. Based on analysis in Section II.B, the effective length of multi-h CPM is limited as 2, because the input difference sequence $\{-x, 0\cdots 0, x\}$ $\left(x\in\{0,1,\cdots,M-1\}\right)$ inevitably result in an error event no matter how many indices are used. As a result, the maximal value of n^{in} is $\left|\tilde{l}^o\right|/2$, so the maximal exponent of the interleaver length in multi-h SCCPM system is

$$\alpha_{\max} = \left\lfloor \left|\tilde{l}^o_{\min}\right|/2 \right\rfloor + n^{\text{out}} - \left|\tilde{l}^o_{\min}\right| - 1 \tag{7}$$

Where $\left|\tilde{l}^o_{\min}\right|$ is the minimal number of nonzero symbols of an effective difference sequence. The value of $\alpha_{\max}$ is the same with single-h SCCPM, in other words, multiple indices could not improve the interleaving gain.

Although the interleaving gain could not be improved, the Euclidean distance of error events could be increased in multi-h SCCPM system. As an example, for quaternary single-h CPM with h=1/4, the normalized Euclidean distance is as follows (with difference sequence {-x, x}).

$$d^2_{\text{in}} = \begin{cases} 1.33, & (x=1) \\ 3.94, & (x=2) \\ 5.30, & (x=3) \end{cases} \tag{8}$$

For quaternary dual-h CPM with h=[6,7]/16, the normalized Euclidean distance is as follows (with difference sequence {-x,0, x}).

$$d^2_{\text{in}} = \begin{cases} 5.08, & (x=1) \\ 7.25, & (x=2) \\ 9.86, & (x=3) \end{cases} \tag{9}$$

The minimal Euclidean distance is 5.08 for dual-h SCCPM, which is about four times of minimal Euclidean distance for single-h SCCPM. With the increase of multiple indices, the minimal

Euclidean distance could be further enlarged as shown in Table 1.

Table 1 Minimum Euclidean distance of multiple indices

H	2	3	4	5
$h=[K_1,\ldots K_H]/P$	[6,7]/16	[6,7,11]/16	[12,13,17,23]/32	[23,25,31,37,49]/64
$d^2_{\min}$	5.08	6.67	8.36	10.21

As shown in the table above, the minimal Euclidean distance is 10.21 when five indices were adopted. As a result, the performance in expression (5) could also be improved.

Convergence Analysis Based on Density Evolution

The SCCPM system is different with other serially concatenated codes in [6] that the inner CPM is a rate-1 accumulator encoder. As a result, convergence analysis of such scheme needs to take the probability density functions for input and output extrinsic information of outer convolutional code and inner CPM encoder into account. Based on density evolution in [8], convergence characteristics of dual-*h* and multi-*h* SCCPM system was simulated in Fig. 2 and Fig.3 respectively.

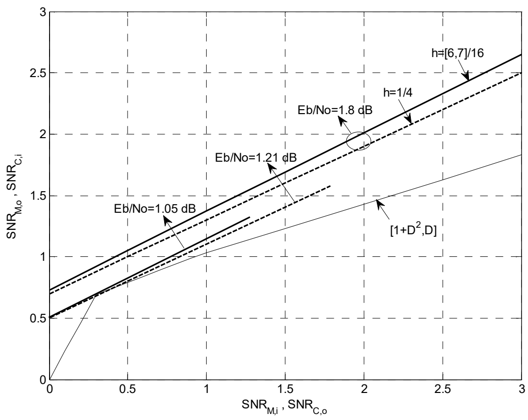

Fig.2 Convergence analysis of dual-*h* schemes

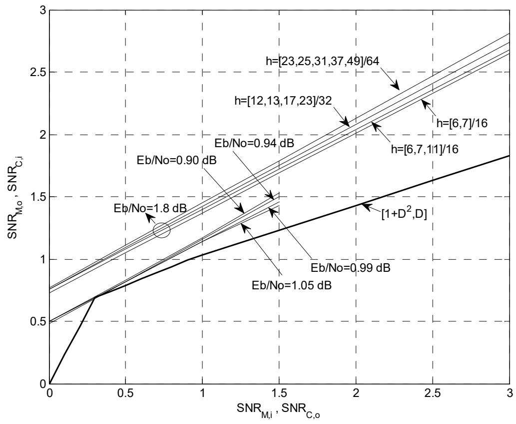

Fig.3 Convergence analysis of multi-*h* schemes

As shown in Fig. 2, input and output SNRs for single-*h* and dual-*h* CPM are depicted in dashed and solid line respectively. Obviously, the initial value of dual-*h* is a little higher than the single-*h*. When combined with the rate-1/2 outer encoder with generation matrix [1+D^2,D], the convergence threshold for dual-*h* is 0.16 dB lower than the single-*h* concatenated system.

As shown in Fig. 3, the curve of input and output SNR is becoming higher gradually with the increase of multiple indices. The convergence threshold for five-*h* is 0.9 dB when combined with the same outer encoder as in Fig. 2, which is 0.15 dB lower than the convergence threshold of dual-*h* concatenated system.

Performance of several concatenated systems with single-*h* or multi-*h* CPM is simulated as shown in Fig. 4. The outer encoder of all these concatenated systems is a rate-1/2 binary convolutional code with generation matrix [1+D^2,D].

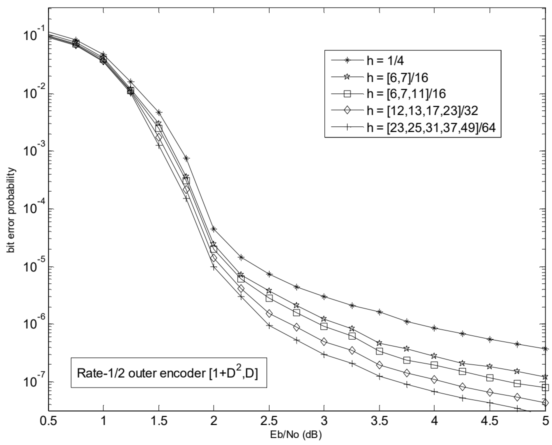

Fig.4 Performance of multi-*h* SCCPM

As shown in Fig. 4, the performance of single-*h* CPM is the worst one among all these schemes. For dual-*h* CPM, its performance is a little better than single-*h* with about 0.25 dB performance gain. As the number of multiple indices increases to three, four or five, the performance of the concatenated system could be further improved gradually, no matter in the error-floor region or in the water-fall region, which is coincidence with the union bound analysis in section III and convergence analysis in section IV.

The expense of such performance gain with multiple indices is the growing complexity of the concatenated system. As we know, the synchronization for CPM is a big challenge for the receiver always. The carrier and timing recovery would become even more complicated with multiple indices. Besides, superbaud timing is an additional task with multiple indices which is not necessary for single index.

Trellis Coded CPM As the Inner Code

From analysis and simulation results in section IV, performance gain could be achieved when multiple indices were adopted in the SCCPM systems. However, the benefit of union bound and convergence threshold is few because the inner encoder is a rate-1 accumulator whose encoding characteristic is very limited. In this section, trellis coded multi-*h* CPM is adopted as the inner code, the interleaver gain could be improved when multiple indices is designed to be a specific pattern. As shown in Fig. 5, because the trellis coded CPM was used as the inner code of the serially concatenated system, the effective length of the error event of the inner code could be increased apparently.

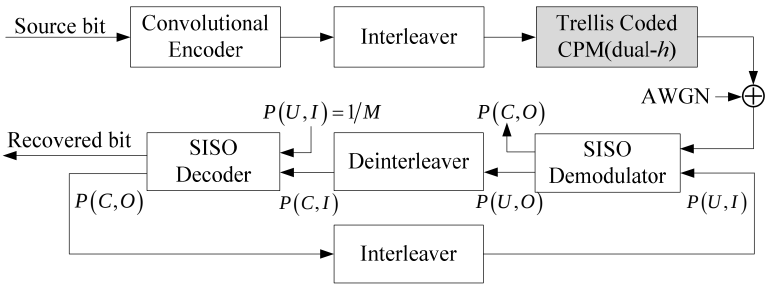

Fig.5 SCCPM system with Trellis coded CPM as the inner code

To achieve impersonal results, three serially concatenated schemes were proposed with comparative complexity. The scheme (a) uses the dual-*h* quaternary CPM ($[h_0\ h_1]$=[8/32,9/32]) as the inner code and rate 1/3 code as the outer one, whose generation matrix is [1+D^2, 1+D+D^2, 1+D+D^2]. The trellis diagram of the outer code is shown in Fig. 6.

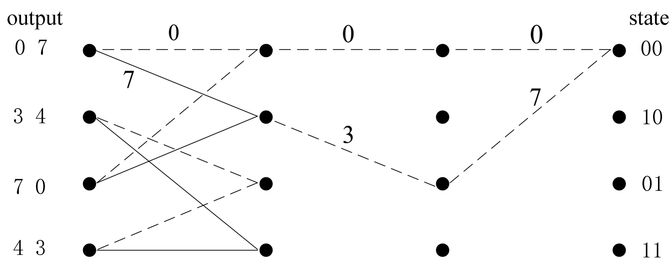

Fig. 6 Trellis diagram of the rate 1/3 outer code in (a).

The scheme (b) uses the rate 1/2 trellis coded dual-*h* quaternary CPM ($[h_0\ h_1]$=[4/16,5/16]) as the inner code with generation matrix [1, 1/1+D] and rate 2/3 outer code with generation matrix [1+D+D^2+D^3, D+D^2, 1+D+D^2]. The trellis diagram of the inner and outer code is shown in Fig. 7 and Fig. 8 respectively.

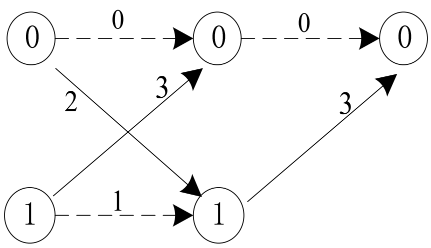

Fig.7 Trellis diagram of the inner code in (b)

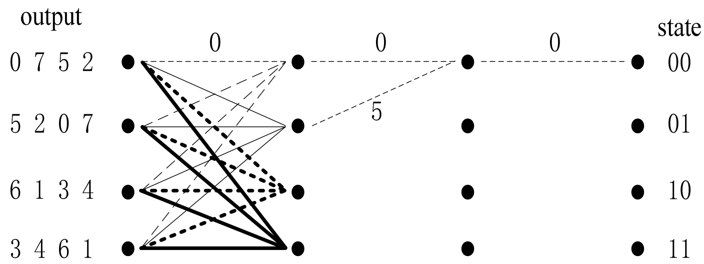

Fig. 8 Trellis diagram of the outer code in (b).

The scheme (c) uses the same inner and outer code with the second scheme. The only difference is that the dual-h quaternary CPM $[h_0\ h_1]$=[4/16,5/16] employed such pattern $[h_0\ h_0\ h_0\ h_1\ h_1\ h_1]$ to improve the effective length of the error event of the inner code [9].

Obviously, the overall code rate of these three concatenated systems is 1/3, and the number of states of inner and outer code is 4 and 32 respectively, which is all the same with these three schemes. The interleaver gain of these three schemes is analyzed as follows.

The minimal error event of the outer code in scheme (a) is 000-737, so the overall minimal error in scheme (a) is 000737-737000. Such minimal event will result in four error event of inner code. If the minimal error event was transformed as binary denotation, the number of nonzero symbols with difference sequence of outer code would be $\left|\tilde{\boldsymbol{l}}^{\boldsymbol{o}}\right|=9$. The number of error events of outer code would be $n^{\text{out}}=2$, and the number of error events of inner code would be $n^{\text{in}}=4$. The interleaver gain of scheme (a) is computed as follows.

$$\alpha_{\max}^{(a)}=\max\left\{n^{\text{in}}+n^{\text{out}}-\left|\tilde{\boldsymbol{l}}^{\boldsymbol{o}}\right|-1\right\}=2+4-9-1=-4 \tag{10}$$

For scheme (b), the minimal error event of the inner code is 0023-2300, and the free distance of outer code would be d_f=3. So the number of nonzero symbols with difference sequence of outer code would be $\left|\tilde{\boldsymbol{l}}^{\boldsymbol{o}}\right|=4$. The number of error events of outer code would be $n^{\text{out}}=1$, and the number of error events of inner code would be $n^{\text{in}}=1$. The interleaver gain of scheme (b) is computed as follows.

$$\alpha_{\max}^{(b)}=\max_{w,l}\left\{n^{\text{in}}+n^{\text{out}}-\left|\tilde{\boldsymbol{l}}^{\boldsymbol{o}}\right|-1\right\}=1+1-4-1=-3 \tag{11}$$

For scheme (c), the minimal error event of the inner code is 000213213000-213000000213, so the number of nonzero symbols with difference sequence of outer code would be $\left|\tilde{\boldsymbol{l}}^{\boldsymbol{o}}\right|=8$. The number of error events of outer code would be $n^{\text{out}}=2$, and the number of error events of inner code would be $n^{\text{in}}=1$. The interleaver gain of scheme (c) is computed as follows.

$$\alpha_{\max}^{(c)}=\max_{w,l}\left\{n^{\text{in}}+n^{\text{out}}-\left|\tilde{\boldsymbol{l}}^{\boldsymbol{o}}\right|-1\right\}=2+1-8-1=-6 \tag{12}$$

Performance of such three concatenated schemes with dual-h quaternary CPM is simulated as shown in Fig. 9.

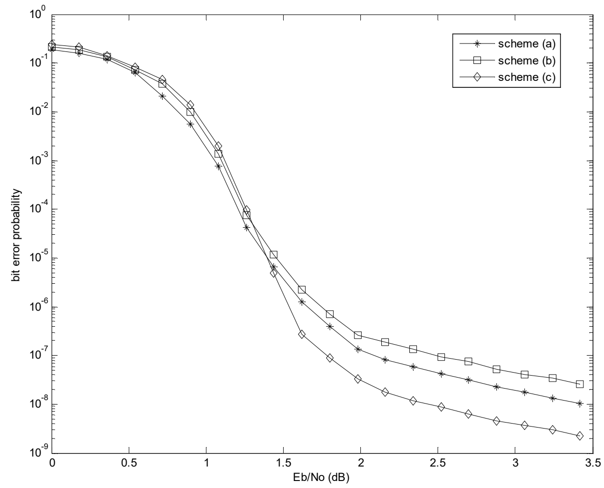

Fig.9 Performance comparison of three concatenated schemes

As shown in Fig. 9, the performance of scheme (a) and (b) is very close, and the error floor of scheme (c) is the lowest one among all the three schemes. However, the water-fall performance of scheme (c) is a little poor. If more than two indices CPM are used, the error floor performance would be even better because the interleaver gain could be improved even greatly.

Conclusion

In this paper, the union bound and convergence threshold of SCCPM system were analyzed when adopting multiple indices. The effective length of error events with multi-h CPM is derived in order to optimize the performance. It is disappointing that increasing the number of multiple indices could not benefit the interleaving gain. As a result, trellis coded multi-h CPM is employed as the inner code to improve the interleaver gain. When adopting a specific pattern of multiple indices, the performance of error floor region would be even better.

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