Nonlinear Element for Concrete Beam Reinforced with Prestressed Hybrid C/G Sheet

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Abstract. The consolidation technology of prestressed hybrid C/G sheet takes full advantages of the initiative of the prestressed reinforcement and high strength of the novel composite material, which belongs to relatively advanced reinforcement strategies in reinforcement area. This paper derives nonlinear composite element and shell element to simulate concrete beam and the prestressed hybrid C/G sheet. Besides, the behaviors of nonlinear concrete beams are described by the Owen yielding criterion and the Hinton crushing criterion. Meanwhile, the three-dimensional prestressing effects of the composite C/G materials have been realized. This element can be used for the nonlinear analysis of concrete beam reinforced with prestressed hybrid C/G sheet.

Introduction

The reinforced technology by using FRP sheet has a certain application in practical engineering, which primarily adopts the passive consolidation of non prestressed FRP sheet to raise the bearing capacity of components[1-2]. The weight of a bridge is often a significant proportion of the total load and the passive paste FRP is mainly used for resisting live loads, but it is a pity that the consolidation effect on flexural member is not always satisfactory. Therefore, strengthened with prestressed FRP sheet paste is better than that of non-prestressed FRP sheet paste method. In order to improve the effectiveness of reinforcement, the FRP sheet can be prestressed first by using the appropriate tension machine and then pasted on the structure, which belongs to the new method called active consolidation[3-4]. This kind of reinforcement technology can more effectively use of FRP materials, can solve the above problems in all aspects[5-6]. Compared with the study of non-prestressed FRP sheet, the study of prestressed FRP sheet appears in recent years[7]. Like traditional prestressed construction, the initial prestress can be used to balance structure of self-respect and part of the load, delay the development of the cracks, reduce the width of the crack and deflection of structure, alleviate stress of the steel and improve the ultimate bearing capacity of concrete beams.

Mechanical Model of Concrete Beam Reinforced with Hybrid Prestressed C/G Sheet

In the past achievements of a single FRP or hybrid FRP reinforced concrete beam, the most are focused on a certain amount of the experimental study of the specimens, because of the high nonlinear performance of hybrid fiber reinforced concrete beam element and the complexity of three dimensional nonlinear programming, the theoretical calculation and analysis is researched by using the commercial finite element software. Commercial finite element software is relatively mature in the elastic analysis, but it has some problems necessary to be solved before used in the nonlinear analysis on the material constitutive numerical stability and convergence. In this paper, the concrete beam with prestressed CFRP/GFRP fiber sheet is researched by nonlinear combination element and shell element simulation.

As shown in Fig.1, concrete beam is simulated by entity degradation element. The midsection has nine controlling nodes in Fig.2, which is used to describe the degradation element of information about the corresponding element decreased significantly. Concrete, stirrups and structural reinforcement in the concrete beams can take concrete layer element to simulate. For vertical reinforced (main), using layer element simulating is too approximate, this paper uses composition element to simulate concrete beams. The displacement of 9 nodes degradation element is as follows:

$$\boldsymbol{\delta}_i = [u_i \quad v_i \quad w_i \quad b_{1i} \quad b_{2i}]^T \tag{1}$$

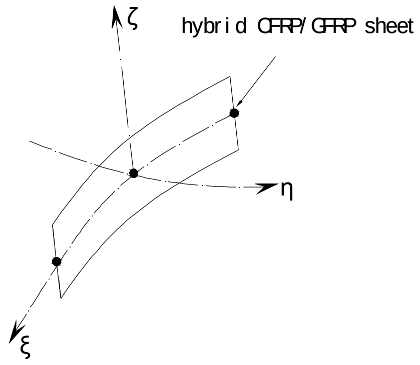

Fig.1 Hybrid CFRP/GFRP sheet of the derived element

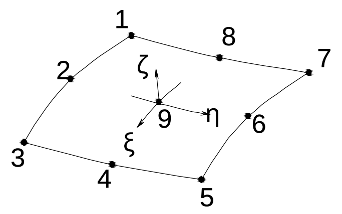

Fig.2 Local coordinate of the mid surface

In this formula: $[u_i\ v_i\ w_i]$ is the line displacement of the node i in the overall system, $[b_{1i}\ b_{2i}]$ is the angle displacement of the i node in the node coordinates. Displacement field by interpolation can be expressed as the shape function:

$$u = \sum_{i=1}^{n} N_i u_i + \sum_{i=1}^{n} N_i \frac{h_i}{2} z\,(n_{1i}^{x} b_{1i} - n_{2i}^{x} b_{2i}) \tag{2}$$

$$v = \sum_{i=1}^{n} N_i v_i + \sum_{i=1}^{n} N_i \frac{h_i}{2} z\,(n_{1i}^{y} b_{1i} - n_{2i}^{y} b_{2i}) \tag{3}$$

$$w = \sum_{i=1}^{n} N_i w_i + \sum_{i=1}^{n} N_i \frac{h_i}{2} z\,(n_{1i}^{z} b_{1i} - n_{2i}^{z} b_{2i}) \tag{4}$$

Where N_i is the shape function of node i, h_i is the element thickness of node i, n_{1i}^{x} is a cosine between the node coordinate $\mathbf{v}_1$ and the overall system x of node i (the same to others). As shown in Fig.1, the main reinforcement in the combination element in concrete beams starts with point A and ends with point B. By using the displacement interpolation Eqs.(2) to (4), the node displacement in main reinforcement elements can be expressed as:

$$\begin{bmatrix} u_A \\ v_A \\ w_A \\ u_B \\ v_B \\ w_B \end{bmatrix} = \sum_{i=1}^{n} \begin{bmatrix} N_i^A & & & N_i^A \frac{h_i}{2} z_A n_{1i}^{x} & -N_i^A \frac{h_i}{2} z_A n_{2i}^{x} \\ & N_i^A & & N_i^A \frac{h_i}{2} z_A n_{1i}^{y} & -N_i^A \frac{h_i}{2} z_A n_{2i}^{y} \\ & & N_i^A & N_i^A \frac{h_i}{2} z_A n_{1i}^{z} & -N_i^A \frac{h_i}{2} z_A n_{2i}^{z} \\ N_i^B & & & N_i^B \frac{h_i}{2} z_B n_{1i}^{x} & -N_i^B \frac{h_i}{2} z_B n_{2i}^{x} \\ & N_i^B & & N_i^B \frac{h_i}{2} z_B n_{1i}^{y} & -N_i^B \frac{h_i}{2} z_B n_{2i}^{y} \\ & & N_i^B & N_i^B \frac{h_i}{2} z_B n_{1i}^{z} & -N_i^B \frac{h_i}{2} z_B n_{2i}^{z} \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ w_i \\ b_{1i} \\ b_{2i} \end{bmatrix} = \sum_{i=1}^{n} \mathbf{R}_i \boldsymbol{\delta}_i \tag{5}$$

Where N_i^A is the value of node A about the main reinforcement element in the shape function, N_i^A is the value of node B; z_A is the value of element's starting point A in the local z coordinates, z_B is the value of element's end point B in the local z coordinates, $\boldsymbol{\delta}_i$ is the array of shell element node displacement , $\mathbf{R}_i$ is the transformation matrix. Take $\boldsymbol{\delta}_s = [u_A\ v_A\ w_A\ u_B\ v_B\ w_B]^{\mathrm{T}}$, $\mathbf{R} = [\mathbf{R}_1\ \mathbf{R}_2\ \mathbf{L}\ \mathbf{R}_n]$, $\boldsymbol{\delta}_c = [\boldsymbol{\delta}_1^{\mathrm{T}}\ \boldsymbol{\delta}_2^{\mathrm{T}}\ \mathbf{L}\ \ \boldsymbol{\delta}_n^{\mathrm{T}}]^{\mathrm{T}}$. Then Eq.(5) can be written as :

$$\boldsymbol{\delta}_s = \mathbf{R}\boldsymbol{\delta}_c \tag{6}$$

Application of virtual work principle, the winner muscle cell's contribution can be pushed to the composition element stiffness matrix:

$$\mathbf{K}_{\mathrm{S}} = \mathbf{R}^T \overline{\mathbf{K}}_{\mathrm{S}} \mathbf{R} \tag{7}$$

Where $\mathbf{K}_{\mathrm{S}}$ is the element's contribution to the composition element stiffness matrix; $\overline{\mathbf{K}}_{\mathrm{S}}$ is the steel stiffness matrix of the overall coordinate system. The contribution of the concrete layer, the stirrup layer and the reinforcement layer to the stiffness matrix can use Gaussian formula to calculate. Hybrid reinforcement of structure with CFRP/GFRP fiber sheet thickness is small, single layer thickness is between 0.15 mm to 0.25 mm. The CFRP/GFRP fiber sheet can use the shell element is shown in Fig.3 like this paper, and initial stress equivalent load method is adopted to realize hybrid the space prestressing effect of CFRP/GFRP sheet. Because of adopting advanced steel plate bolt anchorage technique and paste for prestressed CFRP/GFRP sheet, the failure mode of test reinforcement beam is broken in the form of fiber blah or rib fracture, the failure mode of fiber sheet strip damage does not happen in this paper. Therefore, this article does not consider the interface bond-slip effect, the hybrid fiber sheet element with concrete beam element use the node processing.

Criterion for the concrete beam reinforced with prestressed C /G fiber sheet

Concrete Yielding model

The Owen triaxial yielding criterion is utilized as:

$$f(I_1, J_2) = \left(aI_1 + 3bJ_2\right)^{1/2} = s_0 \tag{8}$$

Where I_1 is the first invariant stress tensor, J_2 is the second invariant stress tensor, s_0 is equivalent stress, f_c is the concrete compressive strength, a and b are the material parameters which are respectively determined according to the uniaxial compression and biaxial compressive test calibration.

$$a = \frac{1-t^2}{2t-t^2} s_0,\ b = \frac{2t-1}{2t-t^2} \tag{9}$$

Where $t = f_{cc} / f_c$, f_{cc} is bidirectional compressive strength.

Concrete Hardening model

Hardening model determine the subsequent yield surface movement in the process of plastic deformation. It determines the loading surface and the relationship between the cumulative plastic strains. By using the effective stress and effective plastic strain, it makes the description of the mechanical behavior of uniaxial test about concrete to uniaxial test. The parameters H' of the elastic-plastic matrix can be represented by a slope of effective stress s on the effective plastic strain e_p, the expression shows as follows:

$$H' = \frac{ds}{de_p} \tag{10}$$

The expression of the relation between uniaxial effective stress s and effective plastic strain e_p based on the Madrid parabolic shows as follows:

$$s = E_0(e_e + e_p) - \frac{1}{2}\frac{E_0}{e_0}(e_e + e_p)^2 \tag{11}$$

Where E_0 is the initial elastic modulus, e_0 is the total strain at the uniaxial compressive strength f_c and its value is $2f_c/E_0$. e_e is the elastic strain, and its value is s/E_0 . The expression can be determined by uniaxial effective stress and effective plastic strain equation:

$$s = -E_0 e_p + \sqrt{2E_0^2 e_0 e_p} \qquad (0.3 f_c < s \le f_c) \tag{12}$$

Thus the parameter H' can be solved by Eqs. (9) and (11).

Concrete Crushing model

Concrete crush is controlled by strain, the expression of Hinton crush model shows as follows:

$$F(I_1', J_2') = \left(aI_1' + 3bJ_2'\right)^{1/2} = e_u \tag{13}$$

Where I_1' is the first invariant strain tensor, J_2' is the second invariant of deviator strain, e_u is the ultimate compressive strain of concrete. a and b is the parameters like Eq.(9).

Conclusion

This paper derives nonlinear composite element and shell element to simulate concrete beam and the prestressed hybrid C/G sheet. Besides, the behaviors of nonlinear concrete beams are described by the Owen yielding criterion and the Hinton crushing criterion. Meanwhile, the three-dimensional prestressing effects of the composite C/G materials have been realized. This element can be used for the nonlinear analysis of concrete beam reinforced with prestressed hybrid C/G sheet.

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