Empirical Test for Pricing of Multi-Stock SPs with Multi-Observation

-----------Huaxia Bank Huiying No.1 as an Example

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Abstract—This paper focuses on the pricing of the multi-stock linked SPs with digital option. "Huiying No. 1 of Huaxia Bank" was characterized by multi-asset linked & multi-observation. To reduce the error, it was priced by means of the modified Cholesky decomposition method and Monte Carlo method. The result demonstrates that the modified Cholesky decomposition increases the accuracy, the pricing process for multi-asset linked SPs with multi-observation is reasonable and the theoretical premium rate of the SPs is 2.74%.

Keywords- **SPs, modified Cholesky decomposition, digital option ,Monte Carlo method, pricing**

I. INTRODUCTION

Structured products flourished in 1980s, but grounded to a halt by the economic crisis. In recent years, as investors are optimistic about the economic situation, structured products market booms.

Under inflation and low interest rates on deposits, the actual rate of return for investors is negative. While the stock market is high-risk, investors invest in structured products in order to obtain a high return while bearing a relatively low risk.

The research of pricing of structured products would help investors choose the right structured products, help issuers price the structured products and be conducive to national regulation of structured products.

II. DEFINITIONS

A. Definition of Structured Products

Structured products(abbreviated as SPs), also named as structured notes, structured bonds, linkage bonds, are a combination of fixed income securities and other basic financial derivatives , it is obvious of SPs to be with bond's characteristics. Satyajit Das (2001) defined the structured products as a combination of products, which are composed of fixed income securities and derivative contracts.

B. Definition of Digital Option

Digital option, also named as binary option, fixed income option, all-or-nothing option, is a type of option out of which the investor can receive only two possible outcomes. There are only two possible outcomes at maturity, based on a standard asset within a specified period of time (for example an hour, a day, a week in the future). Whether the price is below or above the price of the implementation of results, can decide the gain. If the trend of the underlying asset meets predetermined launch conditions, traders of digital option would get a fixed amount of income, otherwise they would lose a fixed amount of the investment. Digital option is one of the most popular variety of the simple options

III. THE DOMESTIC AND FOREIGN LITERATURE REVIEW

A. The Foreign Literature Review

In foreign countries, Chen and Sears (1990) decomposed spin (standard & Poor's 500 index Subordinated note) into bonds and European call option, and put forward the pricing model: the theoretical value of spin = zero coupon bond value + (the value of S&P500 index option x multiplier).Chen and Kensinger (1990) proposed a payoff formula of MICDs (Market-Index Certificates of Deposits): $1 + min\left\{K, \ max\left[\lambda, \ \theta\left(\frac{S_1}{S_0} - 1\right)\right]\right\}$, and the value equation of MICDs was derived from decomposition technique and the maximum and minimum optimization model, the pricing and hedging studies were conducted, empirical studies showed that some SPs were overpriced. Finnerty (1993) decomposed sign into zero coupon bonds and European calls, and drew the following model: the theoretical value of sign = zero coupon bond's value + call option's value + tax deferred income + transaction costs and the value of innovation. Pavel a. Stoimenov, Sascha Wilkens (2004) studied equity-linked SPs in Germany, the result demonstrated that equity-linked SPs in the German primary market were overpriced, transaction price and the theoretical price were the same at the mature and perfect German secondary market. Diethelm and Wallmeier (2009) priced convertible bonds with multiple assets, the result was that 468 multi-asset reverse convertible bonds with

knock-out feature in Switzerland averaged the premium rate of at least 3.4%.

B. The Domestic Literature Review

In China, Renmin and Chen Jinlong (2008) analysed the structure of equity-linked guaranteed structured financial products of foreign exchange, and priced it by means of the Black- Scholes option pricing method.Chen Jinlong, Renmin (2010) studied the single asset equity-linked guaranteed structured financial products with hypothesis of "without limit of return rate" and "limit of return rate" and "proportional transaction costs ", and obtained the formula with the impact of exchange rate and without the impact of exchange rate, and priced "CITIC CSI 300 index- linked No. 1 financial products of RMB". The result demonstrated that the yield design was reasonable. Chen Jinlong, Renmin (2011) priced the multiple assets equity-linked guaranteed structured financial products –"Shenzhen bank Yingfeng 0706" by means of the Monte Carlo method, pointed out the revenue function important, and released the relationship between assets by means of Cholesky decomposition method. The result demonstrated that the price of SPs was high, while the actual rate was low. He Liang (2012) priced "BOC Jinqu 09001A" by means of Cholesky decomposition method and Monte Carlo method, the result was that the premium rate was 1.91%.Tian Hui priced "Huaxia bank Hui ying No. 1" by means of Cholesky decomposition method and the Monte Carlo method. Liu Zongliang (2013) proposed the modified Cholesky method and tested the pricing of at most 2 assets-linked SPs with multi-observations and the trigger conditions.

C. The Literature Review Summary

Because there is a complex relationship among different financial products, if the correlation between assets was not excluded, the accuracy of the SPs value calculation would be weakened. The problem of the multiple assets correlation would be solved by means of Cholesky decomposition method, but the volatility would be reduced by decomposition method to decompose Wiener stochastic process on multiple assets, the error of the pricing would appear. Although Tian Hui priced "Huaxia bank Huiying No. 1" by means of Cholesky decomposition method and Monte Carlo method, but the value of the barrier option is zero, which is not reasonable. This paper would use modified Cholesky decomposition method to transfer the decomposed matrix to be standardized. Liu Zongliang proposed a modified Cholesky method, but only tested the pricing of at most 2 assets linked SPs with multiple observations and the trigger conditions. The paper would price 5 stocks-linked SPs with multi-observation by means of modified Cholesky decomposition method and Monte Carlo simulation method using MATLAB 2013.

IV.   METHODS

A. Cholesky Decompositon Method

Cholesky (Jacobian) decomposition is a method to release correlation. The basic procedure of this method is as follows: make variable change into orthogonal and orthogonal can

release the correlation among the stochastic processes, so that achieve the stochastic process independence can be achieved. S_1, S_2, $\cdots$, S_n, are n random variables, their covariance matrix is as follow:

$$\Sigma = \begin{bmatrix} 1 & \cdots & \sigma_{1,j} & \cdots & \sigma_{1,n} \\ \vdots & & & & \vdots \\ \sigma_{i,1} & & \ddots & & \sigma_{i,n} \\ \vdots & & & & \vdots \\ \sigma_{n,1} & \cdots & \sigma_{n,j} & \cdots & 1 \end{bmatrix}$$

Among them, $\sigma_{i,j} = \sigma_{j,i}$, $\sigma_{i,i} = 1$

In the form of matrix is expressed as

$$AA^T = \Sigma \quad （1）$$

Among them, $A = \begin{bmatrix} a_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}$

The method is known as Cholesky decomposition.

B. The Principle of Modified Cholesky Decomposition Method

Although Cholesky decomposition could solve the correlation problem of multiple assets, the volatility would be reduced by decomposition method to decompose Wiener stochastic processes, the error of the pricing would appear. The paper would use modified Cholesky decomposition method to transfer the decomposed matrix into standardized. Set $B = \Sigma^{-0.5}A$, thus $D(Bz) = D(\Sigma^{-0.5}Az) = \Sigma^{-0.5}AD(z)A^T(\Sigma^{-0.5})^T$. Because $\Sigma^{-0.5}$ is symmetric,$(\Sigma^{-0.5})^T = \Sigma^{-0.5}$, z is a standard random vector.

$$D(Bz) = \Sigma^{-0.5}AD(z)A^T\Sigma^{-0.5} = \Sigma^{-0.5}\Sigma\Sigma^{-0.5} = I$$

Therefore,$W = Bz = \Sigma^{-0.5}Az$, and $D(W) = I$, B is the coefficient matrix of the variance decomposition, $\Sigma^{-0.5}$ is transformation matrix.

C. How to Use Modified Cholesky Decomposition Method

Set (dz_1, dz_2, $\cdots$, dz_n) as independent random processes, and convert dz_{s_1}, dz_{s_2}, $\cdots$, dz_{s_n} into the following N random processes:

$$\begin{cases} dz_{s_1} = b_{11}dz_1 + b_{12}dz_2 + \cdots + b_{1,n}dz_n \\ dz_{s_2} = b_{21}dz_1 + b_{22}dz_2 + \cdots + b_{2,n}dz_n \\ dz_{s_3} = b_{31}dz_1 + b_{32}dz_2 + \cdots + b_{3,n}dz_n \\ \qquad\qquad \vdots \\ dz_{s_n} = b_{n,1}dz_1 + b_{n,2}dz_2 + \cdots + b_{n,n}dz_n \\ dz_i \cdot dz_j = 0 \quad , \quad i,j = 1,2,\cdots,n, \ i \neq j \end{cases} \quad (2)$$

Among them,$B = \Sigma^{-0.5}A = \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nn} \end{bmatrix}$

Set $dz_1 = \varepsilon_1\sqrt{dt}$, $dz_2 = \varepsilon_2\sqrt{dt}$, $\cdots$, $dz_n = \varepsilon_n\sqrt{dt}$, ε_1, ε_2, $\cdots$, ε_n independent of each other, can be obtained,

$$\begin{cases} s_1(t+1) = \\ s_1(t)exp\left[\left(r(t)-\frac{\sigma_{s_1}^2}{2}\right)dt+\sigma_{s_1}\left(b_{11}\cdot\varepsilon_1+b_{12}\varepsilon_2+\cdots+b_{1,n}\varepsilon_n\right)\sqrt{dt}\right] \\ s_2(t+1) = \\ s_2(t)exp\left[\left(r(t)-\frac{\sigma_{s_2}^2}{2}\right)dt+\sigma_{s_2}\left(b_{21}\varepsilon_1+b_{22}\varepsilon_2+\cdots+b_{2,n}\varepsilon_n\right)\sqrt{dt}\right] \quad (3) \\ \vdots \\ s_n(t+1) = \\ s_n(t)exp\left[\left(r(t)-\frac{\sigma_{s_n}^2}{2}\right)dt+\sigma_{s_n}\left(b_{n,1}\varepsilon_1+b_{n,2}\varepsilon_2+\cdots+b_{n,n}\varepsilon_n\right)\sqrt{dt}\right] \end{cases}$$

Monte Carlo simulation could be carried out after σ_{s_i} and b_{ij} are estimated.

D. Monte Carlo Method

Monte Carlo method was derived of probability and statistics theory, was put forward as a numerical calculation method for the first time in the 1940s, also was known as a stochastic simulation method or a statistical simulation method. Monte Carlo method solved computational problems through the repeatedly producing pseudo random numbers with some statistical characteristics using computer. The law of large number ensures the convergence of Monte Carlo method, and central limit theorem provides an analysis approach for Monte Carlo method. Because of the progress of modern electronic computer, the development of Monte Carlo method was accelerated. Monte Carlo method is widely used in finance, operational research, physics, biology, etc.

The basic idea of Monte Carlo method constructs random variables or random processes whose numerical characteristics is similar to that of solution for the problem. After random sampling of the random variables or processes, calculate the parameter values corresponding to each sample as the random solution. Finally, analyse the random solutions according to the sampling processes, and find out the probability of solution and the accuracy of the estimation of the solution. The prediction accuracy will increase with increasing sample number gradually.

The Monte Carlo pricing process is as follows:

1) Input parameters of assets and options: S_0，S_T，T，μ，σ，r，the number of periods n,the number of simulations m,and calculate $\Delta t = T/n$.

2) When $i = 1,2,\cdots,m$, simulation and calculation are as follows: $S_{k+1} = S_k exp(\mu\Delta t + \sigma z\sqrt{\Delta t})$, $k = 0，1，\cdots，n-1$,simulate $S_T = S_n$ starting from S_0 , $C_T = max\{S_T - S_X，0\}$ or $P_T = max\{S_X - S_T，0\}$.

3) Calculate $E[C_T]$ and $E[P_T]^{[14]}$.

V. EMPIRICAL TEST

A. Basic Terms of "Huaxia Bank Hui Ying No.1" Stock-linked Guaranteed SPs Introduction

TABLE I   BASIC TERMS OF "HUIYING NO.1"

"Huiying No.1"	RMB, US dollar denominated notes, with guaranteed profit limit
Issuer	Huaxia Bank
The date of issue	From June 15th, 2007 to July 2nd, 2007 (inclusive)
The investment period	July 5th, 2007 -- July 7th,2008
Investment currency	RMB, US dollar
Par value	As the starting point for the amount of 50000 RMB, every 1000 RMB increase; As the starting point for the amount of $50000 , every $1000 increase
Issued price	100%
Underlying Stocks	BOC 3988.HK, BoCom 3328.HK, CCB 0939. HK, CMB 3968.HK, ICBC 1398. HK
（S_0）Stock reference price	the closing price on the day of the establishment (be postponed to the next trading day in the case of the stock suspension)
Barrier price	110%× S_0
Observation of the No.1 period	The observation period: July 5th, 2007 - October 5th, 2007; the final observation on October 5th, 2007
Observation of the No. 2 period	The observation period: October 6th, 2007 - January 6th,2008; the final observation on January 6th, 2008
Observation of the No.3 period	The observation period: January 8th, 2008 - April 7th, 2008 ; the final observation on April 7th, 2008
Observation of the No.4 period	The observation period: April 8th, 2008 - July 2nd ,2008; the final observation on July 2nd, 2008
Guaranteed rate	100%

The principal and interest payment	customers can not be paid in advance.
Yield to maturity	There are a total of four observation days, **parallel observation of 5 stocks**. As long as the price of each stock on each observation day is more than 110% of initial price, trigger condition to achieve the highest profit whose return rate is 6.75% , while the return rate of the dollar product is 13.5%. If the trigger condition is not reached, the yield is 0%. So investors get return rate either 0% or 6.75% of RMB products.

B. Case Analysis

Because "Huiying No.1" financial products linked to the stocks and there was a barrier option in it, we could price it by means of the decomposition method and Monte Carlo numerical simulation method to estimate the theoretical value of the equity-linked SPs.

Assuming that investors invested in financial products on July 5th, 2007, with par value 50000 RMB, which is the minimum investment as the sales price of commercial banks. The following was to calculate the theoretical value of the product through the analysis of the financial products.

According to the basic terms of "Huaxia Bank Hui ying No.1", the guaranteed rate was 100%, the minimum yield was 0, so Huaxia bank could invest in bond of which value was 50000 RMB on the expiry date.

The floating profit of the product was derived from option. Because the assets-linked were bank shares, the correlation between the assets could not be ignored. We released the correlation by means of modified Cholesky decomposition.

Assuming $\Omega_i(i = 1, 2, 3, 4, 5.)$ as the recorded set which contained the closing prices of stocks every day in the investment period. Respectively corresponding to Bank of China (3988.HK), Bank of Communications(3328.HK), China Construction Bank (0039.HK), China Merchants Bank(3968.HK)and the Industrial and Commercial Bank of China (1398. HK) , the initial price of the five stocks were $S_{i0}, (i = 1, 2, 3, 4, 5)$. This trigger event could be defined as $\forall i, \exists S \in \Omega_i$, making $S_i \geq m \times S_{i0}$, payoff function of the financial products was as follow:

$$payoff = \begin{cases} D \times (1 + \tau) & ,if \quad min(S_{i,j}/S_{i,0}) \geq m \\ D \times (1 + \gamma) & ,if \quad min(S_{i,j}/S_{i,0}) < m \end{cases} \quad (4)$$

Among them,D：principal，$\tau = 5\%$，$\gamma = 0$，m = 1.1.

Thus, there were two results after investors invest the financial products: Set the trigger event B, if the event B happened, the rate of return was τ, and the income rate was greater than the bank deposit rate of return. If the event B did not happen, investors could only ensure safety of initial principal to obtain a positive profit and loss of opportunity cost was a deposit income from commercial banks of one year. Obviously, comparing the characteristics of a financial product with rainbow option, this type of financial product was embedded in a rainbow option. The analysis could be launched from the characteristics of rainbow option.

In the case , because the financial products could not be redeemed and its price depended on the path of the underlying stocks' trends. SPs' features were multi-asset and multiple observations, the paper priced the structured product by means of Monte Carlo method to simulate the stock prices, observe the trigger conditions and meet the waiver of the simulation, otherwise continue until maturity. The best way was to sum expected return and average the sum, so as to achieve the discounted value of the corresponding option. Lastly, the theoretical price of this financial product equaled to the sum of the fixed income security price and the option price.

C. Pricing of Fixed Income Securities

Structured financial products, which is divided into fixed income and floating income. There are three types of fixed guaranteed, floating guaranteed and non guaranteed. This product was guaranteed floating type, then split it into fixed income and floating income. Fixed income could be used as earnings of without risk. Because China's deposit interest rate was controlled and not fully realized the marketization of interest rate, the one-year deposit interest rate could be regarded as risk-free interest rate. The research base was based on the minimum standard of 50000 RMB .

Set the par value of SPs D, the bond interest rate r, bond issuers need to pay a total of N point cash flow to owners. Issuers need to pay $A = D \times r$ interest payments at t = 1, 2,..., n-1. Issuers need to pay interest $A = D \times r$ and principal D. Assuming the discounted rate k within this period, the bond pricing formula was as follow:

$$P = \sum_{t=1}^{n-1} \frac{D \times r}{(1+k)^t} + \frac{D \times (1+r)}{(1+k)^n} \quad （5）$$

In the paper, for the sake of simplicity, set fixed deposit interest rate to bond bank within the specified period of time as the discounted rate. So the bond pricing could be realized accurately.

The fixed income portion was the sum of the principal and interest. We had to calculate the discounted price to price the bond. Risk-free interest rate was derived from the one-year deposit interest rate 3.06% in July, 2007 from the people's Bank of China website. The principal was 50000 RMB, so the discounted present value is 50000 / (1+3.06%) =48515.43 RMB. A RMB's discounted present value is 1/(1+3.06%) = 0.9703 RMB.

D. Parameter Determination

The investment period of the product was one year. This paper selected closing price data of five stocks, Bank of China, Bank of communications, China Construction Bank, China Merchants Bank, Industrial and Commercial bank of China, from July 5th 2006 to July 5th 2007 to estimate the historical volatility and return rate. But the IPO date of Industrial and Commercial bank of China is October 27th, 2006, so we

couldn't get data of 1 year to estimate. In order to ensure the consistency of the data sample, we selected data from November 7th, 2006 to July 4th ,2007, in total closing prices of 160 trading days. Downloaded historical data of 5 stocks within sample time interval from Yahoo Finance and drawn out five stocks charts with MATLAB software 2013, as shown below:
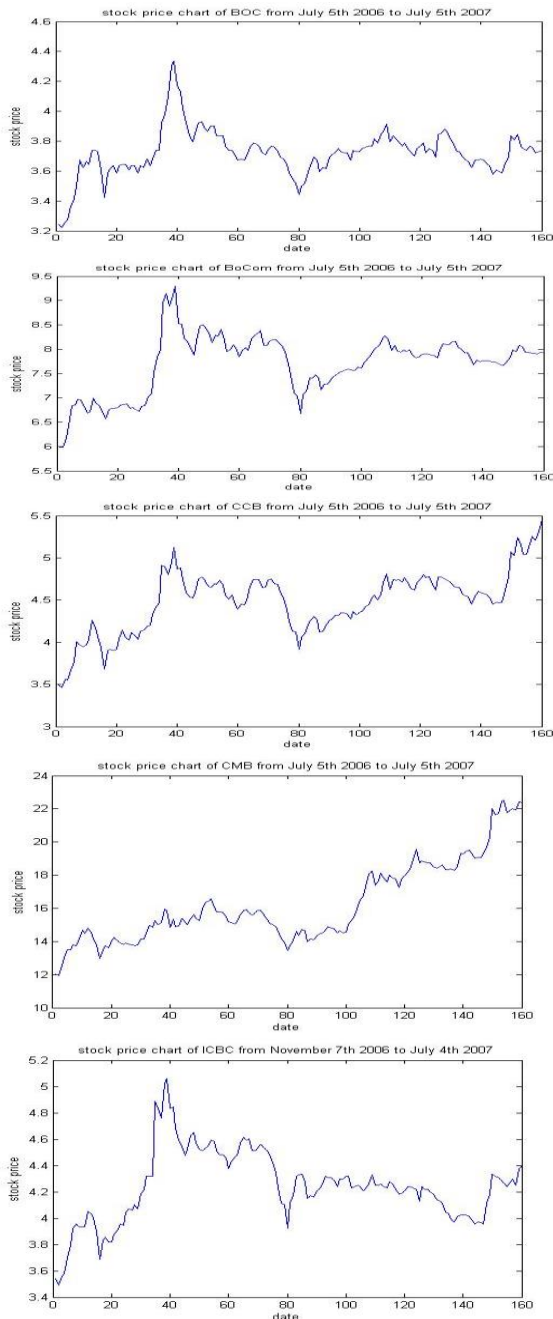

Figure 1. Price charts of 5stocks

The trigger condition was applied in historical data, he trigger price of BOC was HKD4.38, the trigger price of Bof Com was HKD9.24, the trigger price of CCB was HKD6.35, the trigger price of CMB was HKD26.51,the trigger price of ICBC was HKD4.98. According to the rough observation of 5 stocks charts and sample data that five stocks price had

reached the trigger price. Because the stock market was bullish in 2007, investors were optimistic, so investors were willing to invest.

According to the sample data, uses graph analysis to distinguish if 5 stocks' prices were in accordance with normal distribution. The results were as shown below:
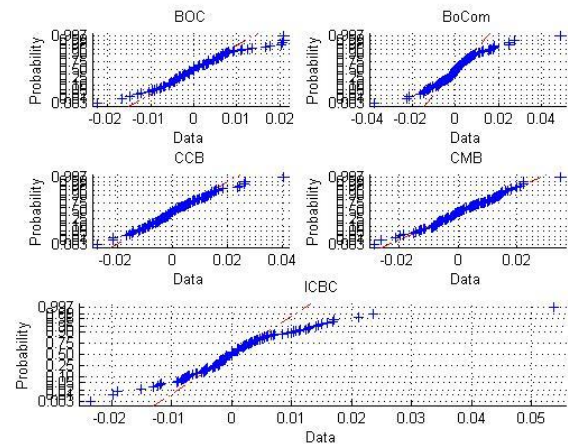

Figure 2. QQ graph of the return rates of 5 stocks

According to the Quantile-Quantile results, the return rates of 5 stocks almost obeyed normal distribution. Hypothesized the return rate of 5 stocks to obey normal distribution in this paper.

The floating portion of the structured financial products could be treated as the pricing of call option. According to the historical data from November 7th, 2006 to July 4th, 2007, the return rates of 5 stocks were measured. Obtain the estimator of stock volatility of five stocks' return rates with MATLAB 2013, the correlation coefficient matrix was as follow:.

$$\rho = \begin{bmatrix} \rho_{1,1} & \rho_{1,2} & \rho_{1,3} & \rho_{1,4} & \rho_{1,5} \\ \rho_{2,1} & \rho_{2,2} & \rho_{2,3} & \rho_{2,4} & \rho_{2,5} \\ \rho_{3,1} & \rho_{3,2} & \rho_{3,3} & \rho_{3,4} & \rho_{3,5} \\ \rho_{4,1} & \rho_{4,2} & \rho_{4,3} & \rho_{4,4} & \rho_{4,5} \\ \rho_{5,1} & \rho_{5,2} & \rho_{5,3} & \rho_{5,4} & \rho_{5,5} \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0.652132 & 0.785365 & 0.608046 & 0.776650 \\ 0.652132 & 1 & 0.768745 & 0.544411 & 0.793887 \\ 0.785365 & 0.768745 & 1 & 0.640743 & 0.847228 \\ 0.608046 & 0.544411 & 0.640743 & 1 & 0.616502 \\ 0.776650 & 0.793887 & 0.847227 & 0.616502 & 1 \end{bmatrix} \quad (6)$$

Among them, $\rho_{i,j} = \rho_{j,i}$, $(i, j = 1, 2, 3, 4, 5)$,the subscript 1 for BOC; subscript 2 for BoCom; subscript 3 for CCB; subscript 4 for CMB; subscript 5 for ICBC.

Then calculate the historical volatility of stock return rate. Firstly, the 5 stocks' closing prices from July 7th,2006 to July 4th 2007 were processed by logarithm with the formula (7); secondly, calculated the five stocks' volatility according to the formula (8)

$$X_i = ln\left(\frac{P_{i+1}}{P_i}\right) = ln(P_{i+1}) - ln(P_i) \quad (7)$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(X_i - \bar{X})^2}{N-1}}, \quad \bar{X} = \frac{\sum_{i=1}^{N}(X_i - \bar{X})^2}{N} \quad (8)$$

Here, N was the number of observations of 160, σ was the logarithmic return average standard deviation. There are

about 230 trading days a year in Hongkong stock market, so the annualized fluctuation rate was $\sigma\sqrt{230}$. Finally, it was concluded that stocks' year fluctuation rate: $\sigma_1 = 0.01011255$, $\sigma_2 = 0.01966528$, $\sigma_3 = 0.02089417$, $\sigma_4 = 0.02142528$, $\sigma_5 = 0.01529613$.

On July 5th, 2007, the one-year deposit interest rate as the risk-free rate was 3.06%. The prices at the beginning of the period on July 5th, 2007 of five stocks are HKD3.98 for the BOC, HKD8.4 for BoCom, HKD5.77 for CCB, HKD24.1 for CMB, HKD4.53 for ICBC,respectively.

The covariance matrix between the return rates of 5 stocks was as follow:

Table2.Covariance matrix between the 5 stocks' return rate

	BOC	BoCom	CCB	CMB	ICBC
BOC	4.396759 45940878 e-05	3.998415 06454507 e-05	4.963482 83894364 e-05	3.891365 24920465 e-05	4.199702 16002405 e-05
BoCom	3.998415 06454507 e-05	8.550123 70773705 e-0	6.775124 95552691 e-05	3.998415 06454507 e-05	5.986481 89505587 e-05
CCB	4.963482 83894364 e-05	6.775124 95552691 e-05	9.084419 68692598 e-05	5.894290 97746937 e-05	6.585299 58235708 e-05
CMB	3.891365 24920465 e-05	4.858617 15599050 e-05	5.894290 97746937 e-05	9.315338 37842979 e-05	4.852448 47345182 e-05
ICBC	4.199702 16002405 e-05	5.986481 89505587 e-05	6.585299 58235708 e-05	4.852448 47345182 e-05	6.650490 14699833 e-05

To release the correlation between stocks by means of MATLAB(Chol) function , the A matrix was achieved.

$$A = \begin{bmatrix} 0.006631 & 0 & 0 & 0 & 0 \\ 0.006030 & 0.007010 & 0 & 0 & 0 \\ 0.007485 & 0.003226 & 0.004940 & 0 & 0 \\ 0.005868 & 0.001883 & 0.001810 & 0.007204 & 0 \\ 0.006333 & 0.003092 & 0.001714 & 0.0003376 & 0.003712 \end{bmatrix}$$

By means of modified Cholesky decomposition method, $B = \Sigma^{-0.5}A$, the decomposition of matrix was standardized as follow:

$$B = \begin{bmatrix} 0.777554 & -0.426796 & -0.382644 & -0.178771 & -0.186763 \\ 0.255906 & 0.883408 & -0.320739 & -0.094729 & -0.205559 \\ 0.385058 & 0.122528 & 0.858620 & -0.206014 & -0.238844 \\ 0.268347 & 0.040527 & 0.085165 & 0.956460 & -0.0654198 \\ 0.331123 & 0.144150 & 0.078920 & -0.0425508 & 0.928191 \end{bmatrix}$$

E. Pricing the Option

In the previous discussion, we learned that a digital option with a trigger condition was embedded into SPs, there were four observation dates. when the prices of five underlying stocks are above 110% of the price at the beginning of the period on the observation day, the stock return rate was 6.75 percent. When numerical simulated the option, in addition to the establishment of path simulations of 10000 times, also need to observe if the simulation prices reached 110% of the initial stock price. If five stocks' simulation prices were above 110% of the initial stock price, this option would be carried out in the final time. Through Monte Carlo simulation, it was found that the simulation prices of five stocks had triggered 0, 2, 10, 19 times on the 1st, 2nd, 3rd, 4th observation day. We achieve the option price 0.0032 with the MATLAB 2013. The

value of option within the SPs is $50000 \times 0.0032 = 160RMB$.

In the MATLAB environment, the procedures to reach the theoretical value of SPs by means of modified Cholesky decomposition method were as follows:

1) The correlation coefficient matrix was converted to a covariance matrix and decomposed it by means of Cholesky method. Used function Chol in the MATLAB to compute Cholesky decomposition, got the matrix A. Calculated $B = \Sigma^{-0.5}A$, B was obtained.
2) Simulated the price path value by means of Monte Carlo simulation method on the stocks according to simulation formula such as in Section 4.3, using random function to generate normal random variables.
3) Calculated the product revenue according to the random simulation path.
4) Repeated the second step and the third step to produce a large amount of samples. Calculated each sample path of product's prices according to the risk neutral pricing technique. Finally count average to reach the theoretical value of the product.

Coding nested loops programs for Monte Carlo simulation of five stocks prices, implemented by 10000 times, five stocks simulation were as shown in the diagram below:
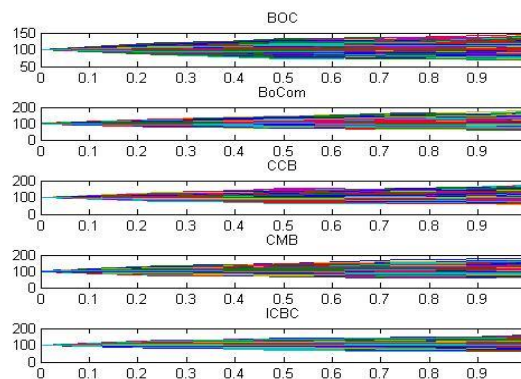

Figure 3. Monte Carlo simulation of 5 stocks' prices

F. The Price of Financial Products

After reaching the price of fixed income and floating income, summed them and reached the theoretical value of financial products. The theoretical value of per RMB is 0.97335 which is the sum of 0.9703 and 0.0032. That is to say, the theoretical value of "Huaxia Hui Ying No.1" is $Val^T = 48668$ by means of modified Cholesky method and Monte Carlo method .Relative premium rate by Wilkens and Stoimenov [6] formula:

$$rp = \frac{Val^M - Val^T}{Val^T} \qquad (9)$$

Among them: Val^M is the market price, rp is the premium rate . When $Val^M = 50000$, $rp = 2.74\%$,and the theoretical premium rate of the SPs is 2.74%, Huaxia Bank would obtain the theoretical premium of 1332 RMB after issuing "Huiying No.1" SPs of 50000 RMB.

VI. CONCLUSIONS

This paper not only priced structured financial products more accurately by means of modified Cholesky decomposition method and Monte Carlo method, but also described the whole pricing process for multi-asset linked SPs with multi-observation. The result demonstrates that the value of digital option was obviously different from the result of zero in [10]. This paper concludes that the modified cholesky method increases the accuracy and the process of pricing is reasonable. Actually 5 stocks' prices reached the trigger condition in the first observation day. This product achieved the highest expected rate of return at maturity because of the performance in line with the provisions. The product could obtain the highest income, mainly because the Hong Kong stock market within observation period was in the growth.

The probability to realize the highest rate was low. But investors had got the highest income. The main reason was that the estimator of volatility was much different from the actual volatility. In addition, this case reminded issuer very important to hedge. If the issuer used the back-to-back hedging method, he would not lose much.

Further study on the pricing of SPs focus on the volatility estimation and how to improve Monte Carlo simulation calculation method, such as using the GARCH model to estimate the volatility, using the dual variable method and control variable method of Monte Carlo simulation to obtain more accurate numerical results. In this paper, study on the pricing of products with more stringent assumptions which was the asset price following geometric Brownian motion assumption. But From 5.4 of this paper, asset price changed according to other laws, which required further research.

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