Robust Optimization Analysis on Green Manufacturing Process Evaluation Based on Ordinal Interval Preference Information

Yaqin Lin, Chunxiang Guo
Business School of Sichuan University, Chengdu, 610065, China

***Abstract*—With regard to the problem of ranking alternatives in green manufacturing process evaluation based on ordinal preference information with uncertain weight vector of attributes, a robust optimization analysis method is proposed. Firstly, the basic concepts and characters of ordinal interval preference information are introduced. Secondly, according to the probability matrix of ordinal preferences, a robust optimization model based on uncertain scenario with mini-max regret criterion is suggested. Then, a group preference gathered from individual preference is obtained by solving the model to acquire an overall robust solution. Finally, a numerical example is given to illustrate the feasibility of the method.**

Keywords-green manufacturing process; ordinal interval; robust optimization; mini-max regret criterion.

I. INTRODUCTION

For achieving the goal of green manufacturing, the decision making process of selecting the best green manufacturing alternatives is necessary. In the actual operation environment, due to the lack of empirical data, there are much preference information judged by experts may in the form of uncertain parameters. At present, researchers have considered various types of uncertain information, such as preference order, linguistic variables, fuzzy variables, interval numbers, etc.[1-3]. Ordinal interval preference, which represents all possible ordinals of an alternative among the feasible solutions, has considerable advantages to deal with the imprecise, difficult and unstructured decision-making problems [1].

González-Pachón, Rodríguez-Galliano and Romero [4] propose the concepts of ordinal interval information and elaborated its forms and properties. Then, an integer goal programming model is constructed to depict it. For addressing the deficiencies of the integer goal programming, González-Pachón[5]et al design an approach with a decomposition and aggregation phase to obtain the ranking result of alternatives. Wang et al [6]. suggest a preference aggregation approach to ranking alternatives by combining preference rankings of alternatives provided by experts. However, there are still some disadvantages exist: on the one hand, the uncertain risks of preference information have not been well considered[7], on the other hand, the uncertainty of weight vectors have not been taken into account yet. For this situation, robust optimization which finds robust correspondence for the uncertain data is an effective tool to solve uncertainty decision.[8-9]

In this paper, we focus on the green manufacturing process evaluation with uncertain ordinal interval preference information; a robust optimization model is established by the mini-max regret criterion. In order to solve the model, pairwise comparison of feasible solutions is used to transform the robust optimization model into a linear programming model.

II. NOTATIONS AND DEFINITIONS

Notations

a_j Alternative j e_i Attribute i k Ranking position

m Number of ranking positions or alternatives n Number of attributes

$\tilde{w}_i$ Weight interval of the attributes i, where $\tilde{w}_i \in [w_i^L, w_i^U]$

M Set of ranking positions, where $M = \{1, 2, \ldots, m\}$

A Set of alternatives, where $A = \{a_1, a_2, \ldots, a_m\}$

$\tilde{W}$ Weight vector of attributes, where $\tilde{W} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)$

x_j^k Binary variable, whether alternative j could be ranked in the position k

Definition 1. Let $\tilde{r}_{ij}$ be the ordinal interval preference towards alternative j of attribute i judged by experts, where $\tilde{r}_{ij} = [r_{ij}^L, r_{ij}^U] = \{r_{ij}^L, r_{ij}^L+1, r_{ij}^L+2, \ldots r_{ij}^U\}$, $r_{ij}^L \le r_{ij}^U$, $r_{ij}^L, r_{ij}^U \in M$ .Particularly, if $r_{ij}^L = r_{ij}^U$, then $\tilde{r}_{ij}$ is deteriorated to a ranking ordinal.

Definition 2. Let $\tilde{p}_{ij} = \{p_{ij}^1 \ldots, p_{ij}^k, \ldots, p_{ij}^m\}$ be the overall probability vector of $\tilde{r}_{ij}$, where p_{ij}^k is given by

$$p_{ij}^{k} = \begin{cases} 0 \\ 1/(r_{ij}^{U} - r_{ij}^{L} + 1) \\ 0 \end{cases} \quad (1)$$

p_{ij}^{k} represents the probability that alternative j is ranked in the position k based on the estimate of attribute i. It is obvious that $\sum_{k=1}^{m} p_{ij}^{k} = 1$ and $0 \le p_{ij}^{k} \le 1$. If m =5, $\tilde{r}_{ij}$ =[3,4], then the probability vector of $\tilde{r}_{ij}$ is $\tilde{p}_{ij}$ =(0,0,1/2,1/2,0).

According to the definition 2, the probability matrix of the attributes could be constructed as

$$\tilde{P} = (\tilde{p}_{ij})_{n\times m} = \begin{vmatrix} \tilde{p}_{11} & \tilde{p}_{12} & \cdots & \tilde{p}_{1m} \\ \tilde{p}_{21} & \tilde{p}_{22} & \cdots & \tilde{p}_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ \tilde{p}_{n1} & \tilde{p}_{n2} & \cdots & \tilde{p}_{nm} \end{vmatrix} \quad (2)$$

Definition 3. Let $S = \{((\tilde{P}, \tilde{W}) \mid \tilde{P} = (\tilde{p}_{ij})_{n\times m}, \tilde{W} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)$, $\tilde{w}_i \in [w_i^L, w_i^U], i = 1, 2, \ldots, n, j = 1, 2, \ldots, m\}$ be the scenario set of the green manufacturing process evaluation problems, the element $s = (\tilde{P}, \tilde{W}) \in S$ is defined as a possible scenario. Obviously, if there are m alternatives, there are $m!$ total order arrangements.

Definition 4. If there exists $X \in \Omega$ then X must satisfy: for any element x_j^k in X, $j, k = 1, 2, \ldots, m$ if $x_j^k = 1$ then $\sum_{i=1}^{n} p_{ij}^{k} > 0$

III. THE ROBUST OPTIMIZATION MODEL

In order to lower the conservation of solution, we choose the mini-max regret criterion to construct the robust optimization model.

Let the weighted expectation of feasible solution X under the scenario $s \in S$ be expressed as $f(X,s) = \sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{i-1}^{n} w_i^s p_{ij}^k x_j^k$. The maximum value of $f(X,s)$ under the scenario s can be expressed as $f^*(s) = \max_{X\in\Omega} f(X,s)$. The calculation of $f^*(s)$ is equivalent to solving a classic optimization problem. According to the relative concept, the regret value of feasible X is recorded as $re(X,s) = f^*(s) - f(X,s)$.

The worst scenario refers to the situation which maximizes the value of $re(X,s)$.

The purpose of this paper is to find a feasible solution X which minimizes the maximum regret value, so the relative robust optimization model for the group decision-making problem under the ordinal interval preference information (F1) can be formulated as follows:

$$\min_{X\in\Omega} \max_{s\in S} re(X,s) \quad (3)$$

s.t.

$$\sum_{j=1}^{m} x_j^k = 1, k = 1, 2, \ldots, m \quad (4)$$

$$\sum_{k=1}^{m} x_j^k = 1, k = 1, 2, \ldots, m \quad (5)$$

$$x_j^k = \{0,1\}, j, k = 1, 2, \ldots, m \quad (6)$$

$$w_i^L \le w_i^s \le w_i^U, i = 1, 2, \ldots, n \quad (7)$$

$$\sum_{i=1}^{n} w_i^s = 1, \forall s \in S \quad (8)$$

The object function of F1 is nonlinear. For convenience, it can be converted to an equivalent linear form (F2):

$$\min \ z \quad (9)$$

s.t.

$$re(X,s) \le z \quad (10)$$

Constrain (4)-(8).

Note that the variable z is continuous.

As the weight of attributes can take any value within the range, so $|S| \to +\infty$. It is visible that solving this programming is an NP-hard problem. In this paper, we apply the pairwise comparison method proposed by Wang [10]to determine the maximum regret value for each feasible solution X under the scenario s.

Let $X, Y \in \Omega$. Then the weighted expectations for them under the scenario s are $f(X,s) = \sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{i-1}^{n} w_i^s p_{ij}^k x_j^k$ and $f(Y,s) = \sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{i-1}^{n} w_i^s p_{ij}^k y_j^k$.

Let $re^Y(X,s) = f(Y,s) - f(X,s) = \sum_{j=1}^{m}\sum_{k=1}^{m}\sum_{i-1}^{n} w_i^s p_{ij}^k (y_j^k - x_j^k)$ be the relative regret value between the feasible solution X and feasible solution Y **.**Specially, if $X = Y$ then $re^Y(X,s) = 0$.

Theorem 1. For any feasible solution $Y \in \Omega$, there exists $re(X,s) = \max_{Y\in\Omega} re^Y(X,s)$

Since $\tilde{P}$ is known, the scenario s is only determined by the weight vector of attributes $\tilde{W}$. We define the corresponding weight vector of attributes of the scenario s is $\tilde{W}^s = (\tilde{w}_1^s, \tilde{w}_1^s, \ldots, \tilde{w}_n^s)$.

Theorem 2 The scenario s^* is determined by the following model (F3) which makes $re^Y(X, s^*) = \max_{s \in S} re^Y(X, s)$

$$\max \sum_{j=1}^{m} \sum_{k=1}^{m} \sum_{i-1}^{n} w_i^s p_{ij}^k (y_j^k - x_j^k) \quad (11)$$

$$s.t.$$

$$w_i^L \le w_i^s \le w_i^U, i = 1, 2, \ldots, n \quad (12)$$

$$\sum_{i=1}^{n} w_i^s = 1 \quad (13)$$

$$x_j^k \in X, y_j^k \in Y \quad (14)$$

Theorem 3 For any feasible solution $X \in \Omega$, $Z(X) = \max_{Y \in \Omega} re^Y(X, s^*)$,

Theorem 4 The feasible solution X^* is the solution of model F2 if it satisfies $Z(X^*) = \min_{X \in \Omega} Z(X)$.

In sum, the steps to solve the linear robust optimization model F2 are presented as follows:

Step 1: Constructing all solutions by arranging and combing the m alternatives .Generate the feasible solution set Ω according to definition 4.

Step 2: Using the pairwise comparison method and creating the model F3 for every feasible solution $X \in \Omega$ and solve it to obtain all $re^Y(X, s^*)$ of X

Step 3: Compare all $re^Y(X, s^*)$ of X to find the maximum regret value $Z(X)$.

Step 4: Compare all $Z(X)$ where $X \in \Omega$ to find the solution of model F2 X^* based on theorem 4.

IV. ILLUSTRATIVE EXAMPLE

There are five alternatives ($a_1, \ldots, a_5$) on the production of rolling bearing. Experts evaluate these processes based on the following attributes: Time (T), Quality (Q), Cost (C), Resource consumption (R) and Environment impact (E). The results are expressed as ordinal interval preference information shown in Table I. The weight interval vectors of attributes provided by the decision maker are

$$\tilde{w}_1 \in [0.15, 0.25], \tilde{w}_2 \in [0.25, 0.4], \tilde{w}_3 \in [0.25, 0.4],$$

$$\tilde{w}_4 \in [0.15, 0.25], \tilde{w}_5 \in [0.15, 0.25]$$

TABLE I. THE PREFERENCE INFORMATION ON THE ALTERNATIVE

	a_1	a_2	a_3	a_4	a_5
T	[3,4]	[3,4]	[4,5]	[1,3]	[2,3]
Q	[3,5]	[2,4]	[3,5]	[1,3]	[1,2]
C	[4,5]	[2,2]	[4,5]	[3,3]	[1,1]
R	[3,5]	[2,3]	[3,4]	[1,3]	[1,3]
E	[3,4]	[1,3]	[3,5]	[2,3]	[2,3]

The probability matrix $\tilde{P} = (\tilde{p}_{ij})_{5 \times 5}$ is constructed by the definition 2. For example, $\tilde{p}_{11} = (0,0,1/2,1/2,0)$, $\tilde{p}_{12} = (0,0,1/2,1/2,0), \tilde{p}_{21} = (0,0,1/3,1/3,1/3)$ .Based on probability matrix, the feasible solution set is revealed in Table II.

TABLE II. THE FEASIBLE SOLUTION SET Ω

Feasible solution	Total order arrangement	Feasible solution	Total order arrangement
X_1	$(a_2, a_4, a_5, a_3, a_1)$	X_2	$(a_2, a_4, a_5, a_1, a_3)$
X_3	$(a_2, a_5, a_4, a_1, a_3)$	X_4	$(a_2, a_5, a_4, a_3, a_1)$
X_5	$(a_4, a_2, a_5, a_1, a_3)$	X_6	$(a_4, a_2, a_5, a_3, a_1)$
X_7	$(a_4, a_5, a_1, a_2, a_3)$	X_8	$(a_4, a_5, a_2, a_1, a_3)$
X_9	$(a_4, a_5, a_2, a_3, a_1)$	X_{10}	$(a_4, a_5, a_3, a_2, a_1)$
X_{11}	$(a_5, a_2, a_4, a_1, a_3)$	X_{12}	$(a_5, a_2, a_4, a_3, a_1)$
X_{13}	$(a_5, a_4, a_1, a_2, a_3)$	X_{14}	$(a_5, a_4, a_2, a_1, a_3)$
X_{15}	$(a_5, a_4, a_2, a_3, a_1)$	X_{16}	$(a_5, a_4, a_3, a_2, a_1)$

For every feasible solution $X_L, L = 1, 2, \ldots, 16$ in Table 2, constructing the model F3 as Theorem 2 shows, then solve the model by LINGO to get the value of $re^Y(X_L, s^*)$ for all feasible solution X_L .The maximum regret value of X_L is $Z(X_L) = \max_{Y \in \Omega} re^Y(X_L, s^*)$ shown in Table III

TABLE III. THE MAXIMUM REGRET VALUE OF X_L

X_L	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8
$Z(X_L)$	1.108	1.033	0.608	0.683	0.65	0.725	1.025	0.75
X_L	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}
$Z(X_L)$	0.825	1.175	0	0.1	0.8	0.525	0.6	0.95

From the Table 3, it can be seen that the feasible solution X_{11} has the minimum value of $Z(X_L)$ .So the robust solution of the example is $(a_5, a_2, a_4, a_1, a_3)$

V. CONCLUSIONS

In this paper, we study the problems of ranking alternatives in green manufacturing process evaluation based on the ordinal preference information with uncertain weight vector of experts, and a robust optimization model is conducted. Firstly, the concepts and properties of ordinal interval preference information are reviewed. Then, the probability matrix of the uncertain preference information is constructed to build the relative robust optimization model (F2). When solving the model, pairwise comparison between feasible solutions is used to convert the NP-hard model to a tractable one. Finally, the complete sequence of alternatives is obtained by solving the transformed model.

ACKNOWLEDGEMENTS

The research work was supported by National Natural Science Foundation of China under Grant No.71340009 and No.741471123.

REFERENCES

[1] Fan Zhi-Ping, Yue Qi, Feng Bo, Liu Yang. An approach to group decision-making with uncertain preference ordinals. Compute and Industrial Engineering, 58(1), pp.51-57. 2010.

[2] Xu Ze-Shui. Group decision making model and approach based on interval preference orderings. Compute and Industrial Engineering, 64(3), pp.797-803. 2013.

[3] Chen, Yen-Liang, Cheng Li-Chen. Mining maximum consensus sequences from group ranking data. European Journal of Operational Research, 198(1), pp. 241-51, 2009.

[4] González-Pachón J, Romero C. Aggregation of partial ordinal rankings: An interval goal programming approach. Computers and Operations Research. 28(2), pp. 827-34, 2001.

[5] González-Pachón J, Rodrı Guez-Galiano M. Transitive approximation to pairwise comparison matrices by using interval goal programming. Journal of the Operational Research Society. 54(5), pp.532-538, 2003.

[6] Wang Ying-Ming, Yang Jian-Bo, Xu Dong-Ling. A preference aggregation method through the estimation of utility intervals. Computers and Operations Research. 32(1), pp.2027-2049, 2005.

[7] Wei Gui-Wu, Wang Jia-Min, Chen Jian. Potential optimality and robust optimality in multi-attribute decision analysis with incomplete information: A comparative study. Decision Support Systems, 55(3), pp.679-84, 2013.

[8] Ben-Tal A, Nemirovski A. Robust convex optimization. Mathematics of Operations Research, 23(4), pp.769-805, 1998.

[9] Ben-Tal A, Nemirovski A. Robust solutions of uncertain linear programs. Operations Research Letters, 25(1), pp.1-13, 1999

[10] Wang Jia-Min. Robust optimization analysis for multiple attribute decision making problems with imprecise information. Annals of Operations Research. 197(1), pp.109-22, 2012.