Multiple Attribute Decision-making with Interval Numbers Projection and Its Application in the Power grid Planning

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Abstract—**In this paper, the theory of interval numbers was applied into multiple attribute decision-making to overcome the limitation of a single real number for attribute value. The interval positive ideal point of the decision problem was constructed based on the interval numbers of the attributes. The alternatives were projected to the positive ideal point according to the projection theory, and then the optimal closeness can be obtained with which the optimal solution can be chosen. Applying this method to the optimization of power grid planning, the example analysis shows that the method is scientific and practical.**

Keywords-multiple attribute decision-making; interval numbers; projection pursuit; power grid planning.

I. INTRODUCTION

Multiple attribute decision making is becoming an immortal research focus in recent years in the academic circles. Multiple attribute decision-making with multiple goal decision constituting the multi-criteria decision-making system is the principal branch of operations research and management science. Many scholars engaged in the study of the theory of multiple attribute decision making, usually with AHP, fuzzy set theory and TOPSIS method, combined weights theory and methods[1-7], and its range of application is in risk assessment and prediction, etc[8-9].

However, attribute values with an interval number rather than a numerical in a multiple attribute decision can be often more convincing, and the decision results also have more practical application value. In this paper, we assigned interval numbers to the values of attributes, obtained the interval positive ideal point of the decision problem, constructed the interval projection multiple attribute decision-making method, and applied it to the power grid planning optimization. The rest of this article is arranged as following: the second section presents the theoretical basis of the study; the third section is the description of the modeling process; the fourth section has carried on the empirical analysis; and the fifth section is the conclusion.

II. METHODOLOGY

Definition 1: Set weighted normalized decision matrix $\tilde{Y} = (\tilde{y}_{ij})_{n \times m}$, call $\tilde{y}^+ = (\tilde{y}_1^+, \tilde{y}_2^+, \cdots, \tilde{y}_m^+)$ interval positive ideal point, where $\tilde{y}_{ij} = [y_{ij}^L, y_{ij}^U]$, $\tilde{y}_{ij} = \omega_j \tilde{r}_{ij}, i \in N, j \in M$, $\tilde{y}_j^+ = [y_j^{+L}, y_j^{+U}] = [\max_i(y_{ij}^L), \max_i(y_{ij}^U)]$, $j \in M$.

Definition 2: set $\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_m)$, $\beta = (\beta_1, \beta_2, \cdots, \beta_m)$ as two vectors, define

$$PRJ_\beta(\alpha) = \frac{\sum_{j=1}^m \alpha_j \beta_j}{\sqrt{\sum_{j=1}^m \beta_j^2}} \qquad (1)$$

as the projection of α on β . In general, the bigger the value of $PRJ_\beta(\alpha)$, the closer these two vectors are. Similarly,

$$PRJ_{\tilde{y}^+}(\tilde{y}_i) = \frac{\sum_{j=1}^m [\tilde{y}_{ij}^L y_j^{+L} + \tilde{y}_{ij}^U y_j^{+U}]}{\sqrt{\sum_{j=1}^m [(y_j^{+L})^2 + (y_j^{+U})^2]}} \qquad (2)$$

where $\tilde{y}_i = (\tilde{y}_{i1}, \tilde{y}_{i2}, \cdots, \tilde{y}_{im}), i \in N$. Obviously, the bigger the value of $PRJ_{\tilde{y}^+}(\tilde{y}_i)$, the closer the alternative x_i and the interval positive ideal point are, that is to say, the better the alternative x_i is.

III. MODELLING PROCESS

The steps of interval projection multiple attribute decision-making method are as follows.

Step1, measuring x_i according to attribute u_j , we can get the attribute values $\tilde{a}_{ij}$ ($\tilde{a}_{ij} = [a_{ij}^L, a_{ij}^U]$), which constitute the decision matrix $\tilde{A} = (\tilde{a}_{ij})_{n \times m}$. There are two kinds of the most common attribute types: the efficiency type and the cost type. Let $I_j (j = 1,2)$ denote the index sets of the efficiency type and the cost type, respectively. In order to eliminate the influence of different physical dimension to the decision result, we can use the following

formulas to transform the decision matrix $\tilde{A}$ into normalized matrix $\tilde{R} = (\tilde{r}_{ij})_{n \times m}$, $\tilde{r}_{ij} = [r_{ij}^L, r_{ij}^U]$,

$$\tilde{r}_{ij} = \tilde{a}_{ij} / \|\tilde{a}_j\|, i \in N, j \in I_1 \tag{3}$$

$$\tilde{r}_{ij} = (1/\tilde{a}_{ij})/(\|1/\tilde{a}_j\|), i \in N, j \in I_2 \tag{4}$$

where $\|\tilde{a}_j\| = \sqrt{\sum_{i=1}^{n} \tilde{a}_{ij}^2}, \|1/\tilde{a}_j\| = \sqrt{\sum_{i=1}^{n} (1/\tilde{a}_{ij})^2}$.

According to interval algorithms, the formulas (3) and (4) can be written as the following forms:

$$\begin{cases} r_{ij}^L = a_{ij}^L / \sqrt{\sum_{i=1}^{n} (a_{ij}^U)^2} \\ r_{ij}^U = a_{ij}^U / \sqrt{\sum_{i=1}^{n} (a_{ij}^L)^2} \end{cases}, i \in N, j \in I_1 \tag{5}$$

$$\begin{cases} r_{ij}^L = (1/a_{ij}^U) / \sqrt{\sum_{i=1}^{n} (1/a_{ij}^L)^2} \\ r_{ij}^U = (1/a_{ij}^L) / \sqrt{\sum_{i=1}^{n} (1/a_{ij}^U)^2} \end{cases}, i \in N, j \in I_2 \tag{6}$$

Step2, use WAA operator to compile the attributes of the alternatives $x_i (i \in N)$, and obtain the comprehensive attribute values $\tilde{y}_i(\omega)(i \in N)$:

$$\tilde{y}_i(\omega) = \sum_{j=1}^{m} \omega_j \tilde{r}_{ij} \tag{7}$$

Then, we can get the weighted normalized decision matrix $\tilde{Y} = (\tilde{y}_{ij})_{n \times m}$.

Step3, determine the interval type positive ideal point $\tilde{y}^+$.

Step4, find the projection of x_i on the interval ideal point: $PRJ_{\tilde{y}^+}(\tilde{y}_i), i \in N$.

Step5, according to the values of $PRJ_{\tilde{y}^+}(\tilde{y}_i), i \in N$, sort the alternatives and select the biggest one as the optimal decision.

IV. COMPUTATIONAL EXAMPLE

Power grid planning is a typical multiple attribute decision-making problem, in which many factors should be considered. A regional power grid construction has three alternatives $x_i (i = 1,2,3)$. The attributes need to be considered are project costs (u_1), life cycle (u_2), average duration (u_3), availability of land (u_4) and reliability (u_5). The decision matrix $\tilde{A}$ is shown as Table 1.

According to the earlier studies of research group, the attribute weight vector $\omega = (0.2189, 0.2182, 0.1725, 0.2143, 0.1761)$ is obtained by AHP method.

Step1, according to formulas (5) and (6), transform decision matrix $\tilde{A}$ into normalized decision matrix $\tilde{R}$, and the result is shown as Table2.

Step2, Use the attribute weight vector ω and normalized decision matrix $\tilde{R}$ to construct weighted normalized decision matrix $\tilde{Y}$, and the result is shown as Table3.

TABLE I. DECISION MATRIX

	u_1	u_2	u_3	u_4	u_5
x_1	[58.9,59.0]	[200,250]	[1.9,2.1]	[0.990,0.991]	[0.907,0.909]
x_2	[58.5,58.7]	[340,350]	[3.4,3.5]	[0.990,0.992]	[0.910,0.912]
x_3	[58.0,58.5]	[290,310]	[2.0,2.2]	[0.992,0.993]	[0.914,0.917]

TABLE II. NORMALIZED DECISION MATRIX

	u_1	u_2	u_3	u_4	u_5
x_1	[0.5721，0.5757]	[0.3772,0.5106]	[0.6080,0.1265]	[0.5762,0.5775]	[0.5738,0.5765]
x_2	[0.5750,0.5796]	[0.6413,0.7149]	[0.3648,0.4098]	[0.5762,0.5781]	[0.5757,0.5784]
x_3	[0.5770,0.5846]	[0.5470,0.6332]	[0.5803,0.6967]	[0.5774,0.5787]	[0.5782,0.5816]

TABLE III. WEIGHTED NORMALIZED DECISION MATRIX

	u_1	u_2	u_3	u_4	u_5
x_1	[0.1252，0.1260]	[0.0823,0.1114]	[0.1049,0.1265]	[0.1235,0.1238]	[0.1010,0.1015]
x_2	[0.1259,0.1269]	[0.1399,0.1560]	[0.0629,0.0707]	[0.1235,0.1239]	[0.1014,0.1019]
x_3	[0.1263,0.1280]	[0.1194,0.1382]	[0.1001,0.1202]	[0.1237,0.1240]	[0.1018,0.1024]

Step3, Calculate the interval positive ideal point according to the definition1:

$$\widetilde{y}^+ = ([0.1263,0.1280],[0.1399,0.1560],[0.1049,0.1265],[0.1237,0.1240],[0.1018,0.1024])$$

Step4, according to formula (2), calculate the projection of alternative x_i on the interval positive ideal point:

$PRJ_{\widetilde{y}^+}(\widetilde{y}_1) = 0.3537$, $PRJ_{\widetilde{y}^+}(\widetilde{y}_2) = 0.2717$, $PRJ_{\widetilde{y}^+}(\widetilde{y}_3) = 0.3758$.

Step5, according to the values of $PRJ_{\widetilde{y}^+}(\widetilde{y}_i)(i = 1,2,3)$, the alternatives are sorted as $x_3 \succ x_1 \succ x_2$. So, we can draw a conclusion that x_i is the optimal planning.

V. CONCLUSIONS

Multiple attribute decision-making method has wide basis, and its application has broad prospects. The scientific nature of the attribute value is the basis of the reasonability for decision-making results. In this research, the attribute values were regarded as interval numbers, which is more in line with the nature of things. Interval positive ideal point makes the decision problems have a scientific reference standard. Interval projection multiple attribute decision-making method was applied to power grid planning, and the results showed that the method is feasible and has certain applicability.

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