Stability Analysis of an N-Unit Series Repairable System with a Repairman Doing Other Work

A. Haji
College of Mathematics and System Sciences
Xinjiang University
Urumqi 830046, China

B. Yunus
College of Mechanical Engineering
Xinjiang University
Urumqi 830047, China

Huxur
College of Mathematics and System Sciences
Xinjiang University
Urumqi 830046, China

Abstract—**We investigate an N-unit series repairable system with a repairman doing other work. By analysing the spectral distribution of the system operator and taking into account the irreducibility of the semigroup generated by the system operator we show that the dynamic solution converges strongly to the steady state solution. Thus we obtain asymptotic stability of the dynamic solution.**

Keywords-series repairable system; C_0-semigroup; dynamic solution; asymptotic stability

I. INTRODUCTION

In [1], Liu and Tang studied an N-unit series repairable system with a repairman doing other work, and obtained the expression of Laplace transforms of some primary reliability indices of the system by using the supplementary variable method, the generalized Markov progress method and the Laplace-transform technique. In [2], we proved the well-posedness and the existence of a unique positive dynamic solution of the system, by using C_0-semigroup theory of linear operators from [3] and [4]. In this paper, we study the asymptotic stability of the dynamic solution. We first reformulate the system as an abstract Cauchy problem as in [2], and then, we prove that the time-dependent solution converging to its static solution in the sense of the norm through studying the spectrum of the operator and irreducibility of the corresponding semigroup, thus we obtain the asymptotic stability of the time-dependent solution of this system.

The system can be described by the following equations (see [1]):(R)

$$\begin{cases} \frac{dp_0(t)}{dt} = (c+\Lambda)p_0(t) + \int_0^\infty \mu(x)p_2(t,x)dx + \sum_{i=1}^{n}\int_0^\infty \mu_i(y)p_{1i}(t,y)dy, \\ \frac{\partial p_{1i}(t,y)}{\partial t} + \frac{\partial p_{1i}(t,y)}{\partial y} = -\mu_i(x)p_{1i}(t,y), i=1,2,\cdots,n, \\ \frac{\partial p_2(t,x)}{\partial t} + \frac{\partial p_2(t,x)}{\partial x} = -(c+\mu(x)+\Lambda)p_2(t,x), \\ \frac{\partial p_{3i}(t,x)}{\partial t} + \frac{\partial p_{3i}(t,x)}{\partial x} = -\mu(x)p_{3i}(t,x) + \lambda_i p_4(t,x), i=1,2,\cdots,n, \\ \frac{\partial p_4(t,x)}{\partial t} + \frac{\partial p_4(t,x)}{\partial x} = -(\Lambda+\mu(x))p_4(t,x) + cp_2(t,x), \end{cases}$$

Where $\Lambda = \sum_{i=1}^{n} \lambda_i$, it's the boundary condition

$$(BC)\begin{cases} p_{1i}(t,0) = \lambda_i p_0(t) + \int_0^\infty \mu(x)p_{3i}(t,x)dx, i=1,2,\cdots,n, \\ p_2(t,0) = cp_0(t) + \int_0^\infty \mu(x)p_4(t,x)dx, \\ p_{3i}(t,0) = 0, i=1,2,\cdots,n, \\ p_4(t,0) = 0, \end{cases}$$

And its initial condition

$$(IC)\begin{cases} p_0(0) = 1, \\ p_{1i}(0,y) = 0, i=1,2,\cdots,n, \\ p_2(0,x) = 0, \\ p_{3i}(0,x) = 0, i=1,2,\cdots,n, \\ p_4(0,x) = 0, \end{cases}$$

where $(t,x)\in[0,+\infty)\times[0,+\infty)$, $(t,y)\in[0,+\infty)\times[0,+\infty)$, $p_0(t)$ gives the probability that at time t all the units are in working state and the repairman is idle; Analogously, $p_{1i}(t,y)dy$ represents the probability that at time t the repairman is repairing the failed unit i, and the hours that the failed unit has been repaired lies in $(y, y+dy]$; $p_2(t,x)dx$ represents the probability that at time t all the units are in working state and repairman is servicing for customer, $p_{3i}(t,x)dx$ represents the probability that at time t the failed unit i is waiting to be repaired and the repairman is servicing for customer, and the hours that the customer has spent on the service lies in $(x, x+dx]$, $p_4(t,x)dx$ represents the probability that at time t all the units are in working state and repairman is servicing for one customer, the other customers is waiting for service.

Throughout the paper we require the following assumption for the functions $\mu(x)$ and $\mu_i(x)$.

© 2015. The authors - Published by Atlantis Press

General assumption. The functions μ and $\mu_i : R_+ \to R_+$ is measurable and bounded such that

$$\mu := \lim_{x\to\infty} \mu(x) > 0 \;,\; \mu_\infty^{(i)} := \lim_{x\to\infty} \mu_i(x) > 0, i = 1,2,\dots,n,$$

$$\mu_\infty := \min\left(\mu, \mu_\infty^{(1)}, \mu_\infty^{(2)}, \dots, \mu_\infty^{(n)}\right).$$

II. THE PROBLEM AS AN ABSTRACT CAUCHY PROBLEM

To apply semigroup theory we transform in this section the system (R), (BC), (IC) into an abstract Cauchy problem [3,Def.II.6.1] on the Banach space $(X, \|.\|)$, where

$$X = C \times \left(L_y^1[0,\infty)\right)^n \times \left(L_x^1[0,\infty)\right)^{n+2}$$

And

$$\|p\| = |p_0| + \sum_{i=1}^n \|p_{1i}\|_{L_y^1[0,\infty)} + \|p_2\|_{L_x^1[0,\infty)} + \sum_{i=1}^n \|p_{3i}\|_{L_x^1[0,\infty)} + \|p_4\|_{L_x^1[0,\infty)}$$

$$P = (P_0, P_{11}(y), P_{12}(y), \dots, P_{1n}(y), P_2(x), P_{31}(x), p_{32}(x), \dots, P_{3n}(x), P_4(x))^t \in X$$

The Space $(X, \|.\|)$ will also be called state space.

In a first step we introduce a ``maximal operator" $(A_m, D(A_m))$ describing only (R). It is given by

$$A_m = \begin{pmatrix} -(c+\Lambda) & \psi_1 & \dots & \psi_n & \psi & 0 & 0 & \dots & 0 & 0 \\ 0 & D_{11} & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & D_{1n} & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & D_2 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & \lambda_1 & D_{31} & 0 & \dots & 0 & \lambda_1 \\ 0 & 0 & \dots & 0 & \lambda_2 & 0 & D_{32} & \dots & 0 & \lambda_2 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & \lambda_n & 0 & 0 & \dots & D_{3n} & \lambda_n \\ 0 & 0 & \dots & 0 & c & 0 & 0 & \dots & 0 & D_4 \end{pmatrix}, \quad (1)$$

$$D(A_m) = C \times \left(W_y^{1,1}[0,\infty)\right)^n \times \left(W_x^{1,1}[0,\infty)\right)^{n+2}. \quad (2)$$

Here and in the following $\psi_i, i = 1,2,\dots,n$ and ψ denote the linear functionals

$$\psi_i : L_y^1[0,\infty) \to C, \quad f \mapsto \psi_i(f) = \int_0^\infty \mu_i(y) f(y) dy,$$

$$\psi : L^1[0,\infty) \to C, \quad f \mapsto \psi(f) = \int_0^\infty \mu(x) f(x) dx. \quad (3)$$

Moreover, the operators $D_{1i}, i = 1,2,\dots,n, D_2, D_{3i}, i = 1,2,\dots,n$ and D_4 on $W^{1,1}[0,\infty)$ are defined respectively as

$$D_{1i} = -\frac{d}{dy} - \mu_i(y), i = 1,2,\dots,n, \quad D_2 = -\frac{d}{dx} - (c + \mu(x) + \Lambda),$$

$$D_{3i} = -\frac{d}{dx} - \mu(x), i = 1,2,\dots,n, \quad D_4 = -\frac{d}{dx} - (\Lambda + \mu(x)). \quad (4)$$

To model the boundary conditions (BC) we use an abstract approach as in [5]. To this purpose we introduce the boundary space $\partial X = C^{2n+2}$ and then define the following boundary operators" L and Φ by

$$L : D(A_m) \to \partial X \quad \begin{pmatrix} p_0 \\ p_{11}(y) \\ p_{12}(y) \\ \vdots \\ p_{1n}(y) \\ p_2(x) \\ p_{31}(x) \\ p_{32}(x) \\ \vdots \\ p_{3n}(x) \\ p_4(x) \end{pmatrix} \mapsto L \begin{pmatrix} p_0 \\ p_{11}(y) \\ p_{12}(y) \\ \vdots \\ p_{1n}(y) \\ p_2(x) \\ p_{31}(x) \\ p_{32}(x) \\ \vdots \\ p_{3n}(x) \\ p_4(x) \end{pmatrix} = \begin{pmatrix} p_{11}(0) \\ p_{12}(0) \\ \vdots \\ p_{1n}(0) \\ p_2(0) \\ p_{31}(0) \\ p_{32}(0) \\ \vdots \\ p_{3n}(0) \\ p_4(0) \end{pmatrix} \quad (5)$$

And

$$\Phi : X \to \partial X$$

$$\Phi \begin{pmatrix} p_0 \\ p_{11}(y) \\ p_{12}(y) \\ \vdots \\ p_{1n}(y) \\ p_2(x) \\ p_{31}(x) \\ p_{32}(x) \\ \vdots \\ p_{3n}(x) \\ p_4(x) \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 & \dots & 0 & 0 & \psi & 0 & \dots & 0 & 0 \\ \lambda_2 & 0 & \dots & 0 & 0 & 0 & \psi & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \lambda_n & 0 & \dots & 0 & 0 & 0 & 0 & \dots & \psi & 0 \\ c & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & \psi_0 \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 \end{pmatrix} \begin{pmatrix} p_0 \\ p_{11}(y) \\ p_{12}(y) \\ \vdots \\ p_{1n}(y) \\ p_2(x) \\ p_{31}(x) \\ p_{32}(x) \\ \vdots \\ p_{3n}(x) \\ p_4(x) \end{pmatrix}. \quad (6)$$

Now the system operator $(A, D(A))$ on X given by

$$Ap = A_m p, \quad D(A) = \{p \in D(A_m) \mid Lp = \Phi p\} \quad (7)$$

describes the system completely. The above equations $(R),(BC)$ and (IC) are equivalent to the abstract Cauchy problem in the Banach space X as follows.

$$\begin{cases} \dfrac{dp(t)}{dt} = Ap(t), t \in [0,\infty) \\ p(0) = (1,0,\dots,0)^t \in X \end{cases} \quad (ACP)$$

We start from the operator $(A_0, D(A_0))$ defined by

$$D(A_0) := \{p \in D(A_m) \mid Lp = 0\}, \quad A_0 p := A_m p \quad (8)$$

Lemma 2.1: For $\gamma \in \rho(A_0)$, we have

$$p \in \ker(\gamma - A_m) \quad (9)$$

$$\Leftrightarrow$$

$$p = (p_0, p_{11}(y), p_{12}(y), \dots, p_{1n}(y), p_2(x), p_{31}(x), p_{32}(x), \dots, p_{3n}(x), p_4(x))^t \in D(A_m)$$

$$p = \frac{1}{\gamma + c + \Lambda}\left[\sum_{i=1}^n \int_0^\infty c_{1i}\mu_i(y) e^{-\gamma y - \int_0^y \mu_i(\xi)d\xi} dy + c_2 \int_0^\infty \mu(x) e^{-(\gamma+c\Lambda)x - \int_0^x \mu(\xi)d\xi} dx\right] \quad (10)$$

$$p_{1i} = c_{1i} e^{-\gamma y - \int_0^y \mu_i(\xi)d\xi}, i = 1,2,\dots,n$$

$$p_2(x) = c_2 e^{-(\gamma+c+\Lambda)x-\int_0^x \mu(\xi)d\xi} \tag{11}$$

$$p_{3i}(x) = c_{3i} e^{-\gamma x-\int_0^x \mu(\xi)d\xi} + (c_2 + c_4)\frac{\lambda_i}{\Lambda}\left(1-e^{-\Lambda x}\right)e^{-\gamma x-\int_0^x \mu(\xi)d\xi}, i = 1,2,\dots,n \tag{12}$$

$$p_4(x) = c_2\left(1-e^{-cx}\right)e^{-(\gamma+\Lambda)x-\int_0^x \mu(\xi)d\xi} + c_4 e^{-(\gamma+\Lambda)x-\int_0^x \mu(\xi)d\xi} \tag{13}$$

Using [6, Lemma 1.2], the domain $D(A_m)$ of the maximal operator A_m decomposes as

$$D(A_m) = D(A_0) \oplus \ker(\gamma - A_m). \tag{14}$$

Moreover, since L is surjective, $L|_{\ker(\gamma-A_m)}: \ker(\gamma - A_m) \to \partial X$ is invertible for each $\gamma \in \rho(A_0)$, see [6, Lemma 1.2]. We denote its inverse by

$$D_\gamma := \left(L|_{\ker(\gamma-A_m)}\right)^{-1} : \partial X \to \ker(\gamma - A_m) \tag{15}$$

and call it ``Dirichlet operator".

We can give the explicit form of D_γ as follows.

Lemma 2.2: For each $\lambda \in \rho(A_0)$, the operator D_λ has the form

$$D_\gamma = \begin{pmatrix} d_{1,1} & d_{1,2} & \cdots & d_{1,n} & d_{1,n+1} & 0 & 0 & \cdots & 0 & 0 \\ d_{2,1} & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & d_{3,2} & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & d_{n+1,n} & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & d_{n+2,n+1} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & d_{n+3,n+1} & d_{n+3,n+2} & 0 & \cdots & 0 & d_{n+3,2n+2} \\ 0 & 0 & \cdots & 0 & d_{n+4,n+1} & 0 & d_{n+4,n+3} & \cdots & 0 & d_{n+4,2n+2} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & & & & & & & & & \\ 0 & 0 & \cdots & 0 & d_{2n+2,n+1} & 0 & 0 & \cdots & d_{2n+2,2n+1} & d_{2n+2,2n+2} \\ 0 & 0 & \cdots & 0 & d_{2n+3,n+1} & 0 & 0 & \cdots & 0 & d_{2n+3,2n+2} \end{pmatrix} \tag{16}$$

Where

$$d_{1,i} = \frac{1}{\gamma+c+\Lambda}\int_0^\infty \mu_i(y)e^{-\gamma y-\int_0^y \mu_i(\xi)d\xi}dy, \quad i = 1,2\cdots,n, \tag{17}$$

$$d_{1,n+1} = \frac{1}{\gamma+c+\Lambda}\int_0^\infty \mu(x)e^{-(\gamma+c+\Lambda)x-\int_0^x \mu(\xi)d\xi}dx, \tag{18}$$

$$d_{i+1,i} = e^{-\gamma y-\int_0^y \mu_i(\xi)d\xi}, \quad i = 1,2\cdots,n,$$

$$d_{n+2,n+1} = e^{-(\gamma+c+\Lambda)x-\int_0^x \mu(\xi)d\xi}, \tag{19}$$

$$d_{n+3,n+1} = \frac{\lambda_1}{\Lambda}(1-e^{-\Lambda x}), \quad d_{n+3,n+2} = e^{-\gamma x-\int_0^x \mu(\xi)d\xi}, \tag{20}$$

$$d_{n+3,2n+2} = \frac{\lambda_1}{\Lambda}(1-e^{-\Lambda x})e^{-\gamma x-\int_0^x \mu(\xi)d\xi},$$

$$d_{n+4,n+1} = \frac{\lambda_1}{\Lambda}(1-e^{-\Lambda x})e^{-\gamma x-\int_0^x \mu(\xi)d\xi}, \tag{21}$$

$$d_{n+4,n+3} = e^{-\gamma x-\int_0^x \mu(\xi)d\xi}, \tag{22}$$

$$d_{n+4,2n+2} = \frac{\lambda_2}{\Lambda}\left(1-e^{-\Lambda x}\right)e^{-\lambda x-\int_0^x \mu(\xi)d\xi},$$

$$d_{2n+2,n+1} = \frac{\lambda_n}{\Lambda}\left(1-e^{-\Lambda x}\right), \tag{23}$$

$$d_{2n+2,n+2} = e^{-\gamma x-\int_0^x \mu(\xi)d\xi}, \tag{24}$$

$$d_{2n+2,2n+2} = \frac{\lambda_n}{\Lambda}\left(1-e^{-\Lambda x}\right)e^{-\gamma x-\int_0^x \mu(\xi)d\xi},$$

$$d_{2n+3,2n+1} = e^{-(\gamma+\Lambda)x-\int_0^x \mu(\xi)d\xi}\left(1-e^{-cx}\right), \tag{25}$$

$$d_{2n+3,2n+2} = e^{-(\gamma+\Lambda)x-\int_0^x \mu(\xi)d\xi}. \tag{26}$$

For $\gamma \in \rho(A_0)$, the operator ΦD_λ can be represented by the $(2n+2)\times(2n+2)$-matrix

$$\Phi D_\gamma = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} & a_{1,n+1} & 0 & 0 & \cdots & 0 & a_{1,2n+2} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} & a_{2,n+1} & 0 & a_{2,n+3} & \cdots & 0 & a_{2,2n+2} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n} & a_{n,n+1} & 0 & 0 & \cdots & a_{n,2n+1} & a_{n,2n+2} \\ a_{n+1,1} & a_{n+1,2} & \cdots & a_{n+1,n} & a_{n+1,n+1} & 0 & 0 & \cdots & 0 & a_{n+1,2n+2} \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix} \tag{27}$$

Where

$$a_{1,i} = \frac{\lambda_1}{\gamma+c+\Lambda}\int_0^\infty \mu_i(y)e^{-\gamma y-\int_0^y \mu i(\xi)d\xi}dy, \quad i = 1, 2,\cdots, n, \tag{28}$$

$$a_{1,n+1} = \frac{\lambda_1}{\gamma+c+\Lambda}\int_0^\infty \mu(x)e^{-(\gamma+c+\Lambda)x-\int_0^x \mu(\xi)d\xi}dx + \frac{\lambda_1}{\Lambda}\int_0^\infty \mu(x)\left(1-e^{-\Lambda x}\right)e^{-\gamma x-\int_0^x \mu(\xi)d\xi}dx, \tag{29}$$

$$a_{1,2n+2} = \frac{\lambda_1}{\Lambda}\int_0^\infty \mu(x)\left(1-e^{-\Lambda x}\right)e^{-\gamma x-\int_0^x \mu(\xi)d\xi}dx, \tag{30}$$

$$a_{2,i} = \frac{\lambda_2}{\gamma+c+\Lambda}\int_0^\infty \mu_i(y)e^{-\gamma y-\int_0^y \mu_i(\xi)d\xi}dy, \quad i = 1, 2, \cdots, n, \tag{31}$$

$$a_{2,n+1} = \frac{\lambda_2}{\gamma+c+\Lambda}\int_0^\infty \mu(x)e^{-(\gamma+c+\Lambda)x-\int_0^x \mu(\xi)d\xi}dx + \frac{\lambda_2}{\Lambda}\int_0^\infty \mu(x)\left(1-e^{-\Lambda x}\right)e^{-\gamma x-\int_0^x \mu(\xi)d\xi}dx, \tag{32}$$

$$a_{2,n+3} = \int_0^\infty \mu(x)e^{-\gamma x-\int_0^x \mu(\xi)d\xi}dx, \tag{33}$$

$$a_{2,2n+2} = \frac{\lambda_2}{\Lambda}\int_0^\infty \mu(x)\left(1-e^{-\Lambda x}\right)e^{-\gamma x-\int_0^x \mu(\xi)d\xi}dx, \tag{34}$$

$$a_{n,i} = \frac{\lambda_n}{\gamma+c+\Lambda}\int_0^\infty \mu_i(y)e^{-\gamma y-\int_o^y \mu_i(\xi)d\xi}dy, \quad i = 1, 2,\cdots,n, \tag{35}$$

$$a_{n,n+1} = \frac{\lambda_n}{\gamma+c+\Lambda}\int_0^\infty \mu(x)e^{-(\gamma+c+\Lambda)x-\int_0^x \mu(\xi)d\xi}dx + \frac{\lambda_n}{\Lambda}\int_0^\infty \mu(x)\left(1-e^{-\Lambda x}\right)e^{-\gamma x-\int_0^x \mu(\xi)d\xi}dx, \tag{36}$$

$$a_{n,2n+1} = \int_0^\infty \mu(x)e^{-\gamma x-\int_0^x \mu(\xi)d\xi}dx, \tag{37}$$

$$a_{n+1,i} = \frac{c}{\gamma+c+\Lambda}\int_0^\infty \mu_i(y)e^{-\gamma y-\int_0^y \mu_i(\xi)d\xi}dy, \quad i = 1, 2,\cdots,n, \tag{38}$$

$$a_{n+1,n+1} = \frac{c}{\gamma+c+\Lambda}\int_0^\infty \mu(x)e^{-(\gamma+c+\Lambda)x-\int_0^x \mu(\xi)d\xi}dx + \int_0^\infty \mu(x)\left(1-e^{-cx}\right)e^{-(\gamma+\Lambda)x-\int_0^x \mu(\xi)d\xi}dx, \tag{39}$$

$$a_{n+1,2n+2} = \int_0^\infty \mu(x) e^{-(\gamma+\Lambda)x - \int_0^x \mu(\xi)d\xi} dx. \quad (40)$$

The Following result, which can be found in [7], plays important role for us to obtain our main result in Section 3.

Lemma 2.3(The characteristic equation): Let $\gamma \in \rho(A_0)$, then

(i) $\gamma \in \sigma_p(A) \Leftrightarrow 1 \in \sigma_p(\Phi D_\gamma)$ (41)

(ii)If $\gamma \in \rho(A_0)$ and there exists $\gamma_0 \in C$ such that $1 \notin \sigma(\Phi D_{\gamma 0})$, then

$$\gamma \in \sigma(A) \Leftrightarrow 1 \in \sigma(\Phi D_\gamma) \quad (42)$$

III. STABILITY OF THE SOLUTION

In this section, we will investigate the asymptotic stability of the dynamic olution of the system. We show first the following lemmas:

Lemma 3.1: For the operator $(A, D(A))$ we have $0 \in \sigma(A)$.

Applying Lemma 2.3(ii) we can show that 0 is the only spectral value of A on the imaginary axis.

Lemma 3.2: Under the General Assumption 1.1, the spectrum $\sigma(A)$ of A satisfies $\sigma(A) \cap iR = \{0\}$.

Lemma 3.3: If $\gamma \in \rho(A_0) \cap \rho(A)$, then

$$R(\gamma, A) = R(\gamma, A_0) + D_\gamma (Id - D_\gamma)^{-1} \Phi R(\gamma, A_0). \quad (43)$$

Lemma 3.4: The semigroup $(T(t))_{t\geq 0}$ generated by $(A, D(A))$ is irreducible.

With this at hand one can then show the convergence of the semigroup to a one dimensional equilibrium point, see [7, Thm. 3.11].

Theorem 3.5: The space X can be decomposed into the direct sum

$$X = X_1 \oplus X_2, \quad (44)$$

Where $X_1 = fix(T(t))_{t\geq 0} = \ker A$ is one-dimensional and spanned by a strictly positive eigenvector $\hat{p} \in \ker A$ of A. In addition, the restriction $(T(t)|_{X_2})_{t\geq 0}$ is strongly stable.

Corollary 3.6: There exists $p^{/} \in X$, $p^{/} >> 0$, such that for all $p \in X$

$$\lim_{t\to\infty} T(t)p = \langle p^{/}, p \rangle \hat{p}, \quad (45)$$

where $\ker A = \langle \hat{p} \rangle$, $\hat{p} >> 0$.

Corollary 3.7: The dynamic solution of the system (R), (BC) and (IC) converges strongly to the steady-state solution as time tends to infinity, that is,

$$\lim_{t\to\infty} p(t, .) = \alpha \hat{\mathrm{p}}, \quad (46)$$

Where $\alpha > 0$ α >0 and $\hat{p}$ as in Corollary 3.6.

ACKNOWLEDGEMENT

This work was supported by the National Natural Science Foundation of China (No.11361057), the National Natural Science Foundation of Xinjiang Uighur Autonomous Region (No. 2014211A002) and the Students' Innovative Training Program of Xinjiang University (No. XJU-SRT-13157).

REFERENCES

[1] Liu, R.B, Tang, Y.H. and Luo, C.Y, An N-unit series repairable system with a repairman doing other work, J. Natu. Science .Heilongjiang Uni, 2005, 22(4), 493--496.

[2] Abdukerim Haji, Bilikiz Yunus and Abdulla Hoxur, Existence and uniqueness of solution of an N-unit series repairable system with a repairman doing other work, Journal of Modern Mathematics Frontier (accepted).

[3] K.-J. Engel, R. Nagel, One-Parameter Semigroups for Linear Evolution Equations, Graduate Texts in Mathematics, vol. 194, Springer-Verlag, 2000.

[4] R. Nagel, One-parameter Semigroups of Positive Operators, Springer-Verlag, 1986.

[5] V. Casarino and K.-J. Engel and R. Nagel and G. Nickel, A semigroup approach to boundary feedback systems, Integr. Equ. Oper. Theory, 2003, 47, 289--306.

[6] G. Greiner, Perturbing the boundary conditions of a generator, Houston J. Math. 1987, 13, 213--229.

[7] A. Haji and A. Radl, A semigroup approach to queueing systems, Semigroup Forum, 2007, 75, 610--624.