Oscillation of Neutral Nonlinear Impulsive Parabolic Equations with Continuous Distributed Deviating Arguments

G.J. Liu
School of Mathematics and Physics, China University of Geosciences,
P. R. China

T. Liu*
School of Mathematics and Physics, China University of Geosciences,
P. R. China
Corresponding author

A.P. Liu
School of Mathematics and Physics, China University of Geosciences,
P. R. China

M. Wang
School of Mathematics and Physics, China University of Geosciences,
P. R. China

***Abstract*—This paper investigated oscillatory properties of solutions for nonlinear parabolic partial differential equations with impulsive effects under two different boundary conditions, by using integral averaging method, variable substitution and functional differential inequalities, established a series of sufficient conditions. It solved a new problem to some extent. We provided two examples to illustrate the results.**

Keywords-oscillation; parabolic equation; impulsive; neutral type; continuous distributed deviating arguments

I. INTRODUCTION

In this article, we discuss oscillatory properties of solutions for the nonlinear impulsive parabolic equations of neutral type with continuous distributed deviating arguments

$$\begin{aligned}&\frac{\partial}{\partial t}\left[u(t,x)+\int_\alpha^\beta g(t,\xi)u(\rho(t,\xi),x)d\eta(\xi)\right]\\&=a(t)h(u)\Delta u+\sum_{i=1}^{l}b_i(t)h_i(u(\tau_i(t),x))\Delta u(\tau_i(t),x)\\&-\int_\gamma^\delta q(t,x,\zeta)f(u(\sigma(t,\zeta),x))d\omega(\zeta),\quad t\neq t_k,(t,x)\in\mathbb{R}^+\times\Omega=G,\end{aligned}\tag{1}$$

$$u(t_k^+,x)-u(t_k^-,x)=b_k u(t_k,x),\quad k=1,2,\cdots\tag{2}$$

with the boundary conditions

$$u=0,(t,x)\in\mathbb{R}^+\times\partial\Omega,\tag{3}$$

$$\frac{\partial u}{\partial n}+\varphi(t,x)u=0,(t,x)\in\mathbb{R}^+\times\partial\Omega\tag{4}$$

and the initial condition

$$u(t,x)=\Phi(t,x),(t,x)\in[-\lambda,0]\times\Omega.$$

Here $\Omega\subset\mathbb{R}^N$ is a bounded domain with boundary $\partial\Omega$ smooth enough and n is the unit exterior normal vector of $\partial\Omega$, $\max_{t\in\mathbb{R}^+}\{t-\rho(t,\xi),t-\tau_i(t),t-\sigma(t,\zeta)\}\le\lambda$ a positive constant, $\Phi(t,x)\in C^2([-\lambda,0]\times\Omega,\mathbb{R})$.

We will use the following conditions:

(H1) $a(t)$, $b_i(t)\in PC(\mathbb{R}^+,\mathbb{R}^+)$, $q(t,x,\zeta)\in C(\mathbb{R}^+\times\bar{\Omega}\times[\gamma,\delta],\mathbb{R}^+)$, $g(t,\xi)\in PC(\mathbb{R}^+\times[\alpha,\beta],\mathbb{R}^+)$; $\rho(t,\xi)\in C(\mathbb{R}^+\times[\alpha,\beta],\mathbb{R})$, $\tau_i(t)\in C(\mathbb{R}^+,\mathbb{R})$, $\sigma(t,\zeta)\in C(\mathbb{R}^+\times[\gamma,\delta],\mathbb{R})$ such that $t-\rho(t,\xi)>0$, $t>t-\iota>\sigma(t,\zeta)$, $\lim_{t\to\infty}\min_{\xi\in[\alpha,\beta]}\rho(t,\xi)=\infty$, $\lim_{t\to\infty}\tau_i(t)=\infty$, $\lim_{t\to\infty}\min_{\zeta\in[\gamma,\delta]}\sigma(t,\zeta)=\infty$, $-\int_\alpha^\beta g(t,\xi)d\eta(\xi)\le h_0<1$, where h_0 and ι are constants, PC denote the class of functions which are piecewise continuous in t with discontinuities of first kind only at $t=t_k$, and left continuous at $t=t_k$, $k=1,2,\cdots$, $\rho(t_k,\xi)\neq t_j$, $j<k$, $j=1,2,\cdots$, $g(t_k^+,\xi)=(1+b_k)g(t_k^-,\xi)$.

(H2) $-\eta(\xi)$, $\omega(\zeta)$ are nondecreasing functions on $[\alpha,\beta]$ and $[\gamma,\delta]$, respectively; $h(u)$, $h_i(u)\in C^1(\mathbb{R},\mathbb{R})$, $f(u)\in C(\mathbb{R},\mathbb{R})$; $f(u)/u\ge C$ a positive constant, $u\neq 0$; $uh'(u)\ge 0$, $uh_i'(u)\ge 0$, $h(0)=h_i(0)=0$, $\varphi(t,x)\in C(\mathbb{R}^+\times\partial\Omega,\mathbb{R})$, $h(u)\varphi(t,x)\ge 0$, $h_i(u)\varphi(\tau_i(t),x)\ge 0$, $i=1,2,\cdots,l$, $0<t_1<t_2<\cdots$, $\lim_{k\to\infty}t_k=\infty$.

(H3) $u(t,x)$ is piecewise continuous in t with discontinuities of first kind only at $t=t_k$, and left continuous at $t=t_k$, $u(t_k,x)=u(t_k^-,x)$, $k=1,2,\cdots$.

Let us construct the sequence $\{\bar{t}_k\} = \{t_k\} \cup \{t_{j\xi}\} \cup \{t_{j\tau}\} \cup \{t_{j\zeta}\}$, where $\{t_{j\xi}\} = \{t \mid \rho(t,\xi) = t_j\}$, $\{t_{j\tau}\} = \{t \mid \tau_i(t) = t_j\}$, $\{t_{j\zeta}\} = \{t \mid \sigma(t,\zeta) = t_j\}$ and $\bar{t}_k < \bar{t}_{k+1}$, $i = 1,2,\cdots l$; $k, j = 1,2,\cdots$.

We introduce the notations:

$$\Gamma_k = \{(t,x) : t \in (\bar{t}_k, \bar{t}_{k+1}), x \in \Omega\},\ \Gamma = \bigcup_{k=0}^{\infty} \Gamma_k,\ v(t) = \int_\Omega u(t,x)dx$$

$$\bar{\Gamma}_k = \{(t,x) : t \in (\bar{t}_k, \bar{t}_{k+1}), x \in \bar{\Omega}\},\qquad \bar{\Gamma} = \bigcup_{k=0}^{\infty} \bar{\Gamma}_k,$$

$Q(t,\zeta) = \min_{x\in\Omega} q(t,x,\zeta)$.

The solution $u \in C^2(\Gamma) \cap C^1(\bar{\Gamma})$ of problems (1), (3) ((4)) is called non-oscillatory in the domain G if it is either eventually positive or eventually negative. Otherwise, it is called oscillatory.

As is well-known, oscillatory properties of partial differential equations are significantly important both in theory and application. The developing theory of partial differential equations have been applied in many fields, such as biology, chemistry, engineering, theoretical physics, generic repression, climate model, and so on. In the last few years, the fundamental theories of partial differential equations with deviating arguments have undergone intensive development. We can see lots of studies [1-4] which have been done under the assumption that the state variables and system parameters change continuously. However, one may easily visualize situations in nature where abrupt change such as harvesting or disasters may occur [5]. These phenomena are short-time perturbations whose duration is negligible in comparison with the duration of the whole evolution process. Consequently, it is natural to assume, in modelling these problems that these perturbations act instantaneously, that is, in the form of impulses.

In 1991, the first paper [6] on this class of equations was published. However, we only find a few of papers on oscillation theory of impulsive partial differential equations. Recently, Bainov, Minchev, Luo, Fu, Liu, Xiao [7-13] investigated the oscillation of solutions of impulsive partial differential equations with or without deviating arguments and Du, Zhang, Shoukaku [3,14,15] discussed the oscillation of solutions of partial differential equations with continuous distributed deviating arguments. However, there is a scarcity in the study of oscillation theory of nonlinear impulsive parabolic equations of neutral type with continuous distributed deviating arguments.

II. OSCILLATION PROPERTIES OF THE PROBLEM (1), (4)

For the main result of this article, we need following lemma:

Lemma 2.1 [16] Let $\rho = const. > 0$, $a_0(t)$, $p(t) \in ([0,+\infty), R)$ be locally summable functions and $p(t) > 0$; $y(t_k) = y(t_k^-)$, $k = 1,2,\cdots$. If the following condition is satisfied

$$\liminf_{t\to+\infty} \int_{t-\rho}^{t} p(s)\exp(\int_{s-\rho}^{s} a_0(r)dr) \prod_{s-\rho<t_k<s} (1+d_k)^{-1} ds > \frac{1}{e},$$

then the following differential inequality has no eventually positive solution.

$$y'(t) + a_0(t)y(t) + p(t)y(t-\rho) \le 0, t \ge 0, t \ne t_k$$

$$y(t_k^+) - y(t_k^-) = d_k y(t_k), k = 1,2,\cdots.$$

The following theorem is the first main result of this article.

Theorem 2.1 Suppose that the conditions (H1)-(H3) and the following condition (5) hold

$$\liminf_{t\to+\infty} \int_{t-t}^{t} \prod_{s-t<t_k<s} (1+b_k)^{-1} ds \int_\gamma^\delta C(1-h_0)Q_0(s,\zeta)d\omega(\zeta) > \frac{1}{e} \qquad (5)$$

Then each solution of the problem (1)-(3) oscillates in G.

Proof Suppose that the assertion is not true and $u(t,x)$ is a non-oscillatory solution of problem (1), (3) in G. Without loss of generality, we may assume that there exists a $T > 0$, $t_0 > T$ such that $u(t,x) > 0$, $u(\rho(t,\xi),x) > 0$, $u(\tau_i(t),x) > 0$, $i = 1,2,\cdots l$, $u(\sigma(t,\zeta),x) > 0$ for any $(t,x) \in [t_0,+\infty) \times \Omega$.

For $t \ge t_0$, $t \ne \bar{t}_k$, $k = 1,2,\cdots$, integrating (1) with respect to x over Ω yields

$$\frac{d}{dt}\left[\int_\Omega u(t,x)dx + \int_\alpha^\beta g(t,\xi)d\eta(\xi)\int_\Omega u(\rho(t,\xi),x)dx\right] = a(t)\int_\Omega h(u)\Delta u dx + \sum_{i=1}^{l} b_i(t)\int_\Omega h_i(u(\tau_i(t),x))\Delta u(\tau_i(t),x)dx - \int_\gamma^\delta d\omega(\zeta)\int_\Omega q(t,x,\zeta)f(u(\sigma(t,\zeta),x))dx.$$

By Green's formula and the boundary condition, we have

$$\int_\Omega h(u)\Delta u dx = \int_{\partial\Omega} h(u)\frac{\partial u}{\partial n}ds - \int_\Omega h'(u)\left|grad u\right|^2 dx = -\int_\Omega h'(u)\left|grad u\right|^2 dx \le 0,$$

$$\int_\Omega h_i(u(\tau_i(t),x))\Delta u(\tau_i(t),x)dx \le 0.$$

From condition (H2), we can easily obtain

$$\int_\Omega q(t,x,\zeta)f(u(\sigma(t,\zeta),x))dx \ge CQ(t,\zeta)\int_\Omega u(\sigma(t,\zeta),x)dx$$

From the above it follows that

$$\frac{d}{dt}[v(t) + \int_\alpha^\beta g(t,\xi)v(\rho(t,\xi))d\eta(\xi)] + C\int_\gamma^\delta Q(t,\zeta)v(\sigma(t,\zeta))d\omega(\zeta) \le 0. \qquad (6)$$

In inequality (6), set $w(t) = \Pi_{t_0 \le \bar{t}_k < t}(1+b_k)^{-1}v(t)$, we obtain the following results: (1) $w(t)$ is continuous on $[t_0,+\infty)$; (2) Inequality (6) has no eventually positive solution if inequality (7) has no eventually positive solution.

$$\frac{d}{dt}[w(t)+\int_\alpha^\beta G(t,\xi)w(\rho(t,\xi))d\eta(\xi)]+C\int_\gamma^\delta Q_0(t,\zeta)w(\sigma(t,\zeta))d\omega(\zeta)\le 0, t\ge t_0, t\ne \bar{t}_k, \frac{d}{dt}[w(t)+\int_\alpha^\beta G(t,\xi)w(\rho(t,\xi))d\eta(\xi)]+C\int_\gamma^\delta Q_0(t,\zeta)w(\sigma(t,\zeta))d\omega(\zeta)\le 0, t\ge t_0, t\ne \bar{t}_k, \quad (7)$$

where

$$G(t,\xi)=\prod_{\rho(t,\xi)\le \bar{t}_k<t}(1+b_k)^{-1}g(t,\xi),$$

$$Q_0(t,\zeta)=\prod_{\sigma(t,\zeta)\le \bar{t}_k<t}(1+b_k)^{-1}Q(t,\zeta).$$

In fact, $v(t)$ is continuous on each interval $(\bar{t}_k,\bar{t}_{k+1}]$, and in view of $v(\bar{t}_k^+)=(1+b_k)v(\bar{t}_k)$, it follows that for all $t\ge t_0$,

$$w(\bar{t}_k^+)=\prod_{t_0\le \bar{t}_j\le \bar{t}_k}(1+b_j)^{-1}v(\bar{t}_k^+)=\prod_{t_0\le \bar{t}_j<\bar{t}_k}(1+b_j)^{-1}v(\bar{t}_k)=w(\bar{t}_k),$$

$$w(\bar{t}_k^-)=\prod_{t_0\le \bar{t}_j\le \bar{t}_{k-1}}(1+b_j)^{-1}v(\bar{t}_k^-)=\prod_{t_0\le \bar{t}_j<\bar{t}_k}(1+b_j)^{-1}v(\bar{t}_k)=w(\bar{t}_k),$$

which implies that $w(t)$ is continuous on $[t_0,+\infty)$.

$$\frac{d}{dt}\left[w(t)+\int_\alpha^\beta G(t,\xi)w(\rho(t,\xi))d\eta(\xi)\right]+C\int_\gamma^\delta Q_0(t,\zeta)w(\sigma(t,\zeta))d\omega(\zeta)$$
$$=\frac{d}{dt}\left[\prod_{t_0\le \bar{t}_k<t}(1+b_k)^{-1}v(t)+\int_\alpha^\beta \prod_{\rho(t,\xi)\le \bar{t}_k<t}(1+b_k)^{-1}g(t,\xi)\prod_{t_0\le \bar{t}_k<\rho(t,\xi)}(1+b_k)^{-1}v(\rho(t,\xi))d\eta(\xi)\right]$$
$$+C\int_\gamma^\delta \prod_{\sigma(t,\zeta)\le \bar{t}_k<t}(1+b_k)^{-1}Q(t,\zeta)\prod_{t_0\le \bar{t}_k<\sigma(t,\zeta)}(1+b_k)^{-1}v(\sigma(t,\zeta))d\omega(\zeta)$$
$$=\prod_{t_0\le \bar{t}_k<t}(1+b_k)^{-1}\left(\frac{d}{dt}\left[v(t)+\int_\alpha^\beta g(t,\xi)v(\rho(t,\xi))d\eta(\xi)\right]+C\int_\gamma^\delta Q(t,\zeta)v(\sigma(t,\zeta))d\omega(\zeta)\right)\le 0,$$

which implies that $w(t)$ is a positive solution.

Now in inequality (7), set

$$y(t)=w(t)+\int_\alpha^\beta G(t,\xi)w(\rho(t,\xi))d\eta(\xi). \quad (8)$$

Hence we have

$$y'(t)+C\int_\gamma^\delta Q_0(t,\zeta)w(\sigma(t,\zeta))d\omega(\zeta)\le 0, t\ge t_0, t\ne \bar{t}_k, \quad (9)$$

For $t\ge t_0$, $t=\bar{t}_k$, $k=1,2,\cdots$, since $w(t)$ is continuous on $[t_0,+\infty)$ and $G(\bar{t}_k^+,\xi)=G(\bar{t}_k,\xi)$, it is easy to verify that

$$y(\bar{t}_k^+)=y(\bar{t}_k) \quad (10)$$

From inequality (9) and (10), it is easy to see that $y(t)$ is nonincreasing on $[t_0,+\infty)$. Noting that $-\eta(\xi)$ is nondecreasing, then we can obtain

$$w(t)=y(t)-\int_\alpha^\beta G(t,\xi)w(\rho(t,\xi))d\eta(\xi)$$
$$=y(t)-\int_\alpha^\beta G(t,\xi)[y(\rho(t,\xi))-\int_\alpha^\beta G(\rho(t,\xi),\xi)w(\rho(\rho(t,\xi),\xi))d\eta(\xi)]d\eta(\xi)$$
$$\ge y(t)-\int_\alpha^\beta G(t,\xi)y(\rho(t,\xi))d\eta(\xi)$$
$$\ge (1-\int_\alpha^\beta G(t,\xi)d\eta(\xi))y(t)$$
$$\ge (1-h_0)y(t),$$

$$w(\sigma(t,\zeta))\ge (1-h_0)y(\sigma(t,\zeta))\ge (1-h_0)y(t-\iota).$$

Therefore, we get

$$y'(t)+C(1-h_0)y(t-\iota)\int_\gamma^\delta Q_0(t,\zeta)d\omega(\zeta)\le 0, t\ge t_0, t\ne \bar{t}_k, \quad (11)$$

Hence, we obtain that $y(t)\ge 0$ is an eventually positive solution of differential inequality (10), (11). But according to Lemma 2.1 and condition (5), the differential inequality (10), (11) has no eventually positive solution. This is a contradiction. This ends the proof of the theorem.

III. OSCILLATION PROPERTIES OF THE PROBLEM (1), (4)

The following theorem is the second main result of this article.

Theorem 3.1 Suppose that conditions (H1)-(H3) and the following conditions (12) hold

$$\liminf_{t\to\infty}\int_{t-\iota}^t \prod_{s-\iota<t_k<s}(1+b_k)^{-1}ds\int_\gamma^\delta C(1-h_0)Q_0(s,\zeta)d\omega(\zeta)>\frac{1}{e}, \quad (12)$$

Then every solution of the problem (1), (4) oscillates in G.

Proof Suppose that the assertion is not true and $u(t,x)$ is a non-oscillatory solution of problem (1), (4) in G. Without loss of generality, we may assume that there exists a $T>0$, $t_0>T$ such that $u(t,x)>0$, $u(\rho(t,\xi),x)>0$, $u(\tau_i(t),x)>0$, $i=1,2,\cdots l$, $u(\sigma(t,\zeta),x)>0$ for any $(t,x)\in[t_0,+\infty)\times\Omega$.

For $t\ge t_0$, $t\ne \bar{t}_k$, $k=1,2,\cdots$, integrating (1) with respect to x over Ω yields

$$\frac{d}{dt}\left[\int_\Omega u(t,x)dx+\int_\alpha^\beta g(t,\xi)d\eta(\xi)\int_\Omega u(\rho(t,\xi),x)dx\right]=a(t)\int_\Omega h(u)\Delta u dx+$$
$$\sum_{i=1}^l b_i(t)\int_\Omega h_i(u(\tau_i(t),x))\Delta u(\tau_i(t),x)dx-\int_\gamma^\delta d\omega(\zeta)\int_\Omega q(t,x,\zeta)f(u(\sigma(t,\zeta),x))dx.$$

By Green's formula and the boundary condition, we have

$$\int_\Omega h(u)\Delta u dx = \int_{\partial\Omega} h(u)\frac{\partial u}{\partial n}ds - \int_\Omega h'(u)\left|gradu\right|^2 dx$$
$$= -\int_{\partial\Omega} uh(u)\varphi(t,x)ds - \int_\Omega h'(u)\left|gradu\right|^2 dx$$
$$\le -\int_\Omega h'(u)\left|gradu\right|^2 dx \le 0$$

$$\int_\Omega h_i(u(\tau_i(t),x))\Delta u(\tau_i(t),x)dx \le 0.$$

The rest of the proof is similar to the one in Theorem 2.1, so we omit it.

IV. REMARKS AND EXAMPLES

Remarks From the theoretical viewpoint, the results of this paper, uncovered the essential difference between partial differential equations with impulses, functional arguments and partial differential equations without impulses, functional arguments; from a practical standpoint, they are very convenient because these criteria only depend on the coefficients of the equations, impulsive term and the time-delays. The results of this article improve the results in the papers [10, 17, 18]. For example, paper [19] discussed the case with discrete distributed deviating arguments; however, we consider a more complex case with continuous distributed deviating arguments in this article.

The following are examples to illustrate the applicability of the conditions.

Example 4.1 Consider the equation

$$\frac{\partial}{\partial t}\left[u(t,x)+\int_0^{\pi}\frac{e^{-t-\xi}}{4}u(t+\xi-\frac{3\pi}{2},x)d(-\xi)\right]=u^2\Delta u+e^t u^2(t-\frac{\pi}{2},x)\Delta u(t-\frac{\pi}{2},x)$$
$$-\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}(x^2+1)e^{t-\zeta}u(t-\zeta,x)e^{u^2(t-\zeta,x)}d\zeta,\quad t>1, t\neq 2^{k+1}, (t,x)\in\mathbb{R}^+\times\Omega=G,$$
$$u((2^{k+1})^+,x)-u((2^{k+1})^-,x)=\frac{1}{2^{k+1}}u(2^{k+1},x),\quad k=1,2,\cdots,$$

with the boundary condition

$$u=0,(t,x)\in\mathbb{R}^+\times\partial\Omega,$$

where $g(t,\xi)=\frac{e^{-t-\xi}}{4}$, $\rho(t,\xi)=t+\xi-\frac{3\pi}{2}$, $\eta(\xi)=-\xi$, $a(t)=1$, $h(u)=u^2$, $b_1(t)=e^t$, $h_1(u)=u^2$, $\tau_1(t)=t-\frac{\pi}{2}$, $q(t,x,\zeta)=(x^2+1)e^{t-\zeta}$, $f(u)=ue^{u^2}$, $\sigma(t,\zeta)=t-\zeta$, $t_k=2^{k+1}$. It is easy to verify that the conditions (H1)-(H3) and the condition of theorem 2.1 are satisfied. Hence all solutions of the above problem oscillate.

Example 4.2 Consider the equation

$$\frac{\partial}{\partial t}\left[u(t,x)+\int_{\frac{3\pi}{2}}^{\frac{5\pi}{2}}\frac{e^{-t-\xi}}{4}u(t-\xi,x)d(-\xi)\right]=u^2\Delta u+e^t u^2(t-\frac{\pi}{2},x)\Delta u(t-\frac{\pi}{2},x)$$
$$-\int_{\pi}^{2\pi}(x^2+1)e^{t-\zeta}u(t-\zeta,x)e^{u^2(t-\zeta,x)}d\zeta,\quad t>1, t\neq 2^{k+1}, (t,x)\in\mathbb{R}^+\times\Omega=G,$$
$$u((2^{k+1})^+,x)-u((2^{k+1})^-,x)=\frac{1}{2^{k+1}}u(2^{k+1},x),\quad k=1,2,\cdots,$$

with the boundary condition

$$\frac{\partial u}{\partial n}+t^2x^2u=0,(t,x)\in\mathbb{R}^+\times\partial\Omega,$$

where $g(t,\xi)=\frac{e^{-t-\xi}}{4}$, $\rho(t,\xi)=t-\xi$, $\eta(\xi)=-\xi$, $a(t)=1$, $h(u)=u^2$, $b_1(t)=e^t$, $h_1(u)=u^2$, $\tau_1(t)=t-\frac{\pi}{2}$, $q(t,x,\zeta)=(x^2+1)e^{t-\zeta}$, $f(u)=ue^{u^2}$, $\sigma(t,\zeta)=t-\zeta$, $t_k=2^{k+1}$, $\varphi(t,x)=t^2x^2$. It is easy to verify that the conditions of Theorem 3.1 are satisfied. Hence all solutions of the above problem oscillate.

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