

Thermal Performance of Ground Heat Exchanger based on Infinite-length Linear Heat-source Model

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Abstract. Based on the principle of superposition, the Infinite-length Linear Heat-source Model (*ILM*) is applied to the analysis of the long-term performance of Ground Heat Exchanger for Ground Source Heat Pump. The existing heat transfer models, including the Infinite-length Linear Heat-source Model (*ILM*), the Finite-length Linear Heat-source Model (*FLM*) and the Infinite-length Cylindrical Heat-source Model (*ICM*), are compared. It is found that for deep buried pipe, the results of the *ILM* are exactly the same with that of the *FLM*, while the results of *ICM* show large deviation. Comparing with *FLM*, *ILM* is of much higher calculation speed which may save lots of computer burden when applying to actual projects.

1. Introduction

A lot of researches on the heat transfer model of ground source heat pump were reported in literature home and abroad which can be divided into analytic and numerical methods [1-9]. The underground heat exchange of ground source heat pump is a long-term non-steady-state process. Unless long-term (at least one year) dynamic simulation has carried out, the ground heat exchanger performance analysis can not be predicted. As the temperatures of inlet and outlet water of heat pump vary with the underground temperature change, the traditional method of energy analysis can not be used to predict annual energy consumption. So, dynamic energy simulation software must be coupled with the process of ground heat exchanger to carry on energy analysis.

The results of numerical model which takes into account complex boundary conditions are not general, while that of analytical model whose boundary conditions are simple are common but accuracy is low. However, for long-term progress (month, year or even longer), the accuracy of analytical solutions meets project needs. In actual project the dimensionless criterion $Fo = \frac{at}{r^2} > 20$ is used as criterion for small error [1, 10].

On the basis of the principle of superposition, the existing analytical models are used to analyze the ground heat exchanger performance under variable load and compared. The results show that the Infinite-length Linear Heat-source Model has a lot of advantages when used to analyze the heat transfer performance such as simple formula, higher calculation speed which is worth using in the design and analysis of ground heat exchanger system.

2. Heat transfer model of underground exchanger

There are mainly three heat transfer models of underground exchanger which are the Infinite-length Linear Heat-source Model (*ILM*), the Finite-length Linear Heat-source Model (*FLM*) and the Infinite-length Cylindrical Heat-source Model (*ICM*).

2.1. *ILM*

The soil is assumed to be infinite medium with uniform temperature distribution. The linear heat-source is turned on at zero time. The underground temperature distribution is [10, 11]:

$$T(r, \tau) - T_0 = \frac{q_l}{2\pi\lambda_s} \int_x^\infty \frac{e^{-\beta^2}}{\beta} d\beta = \frac{q_l}{2\pi\lambda_s} Ei(X). \quad (1)$$

Where

$$X = \frac{r}{2\sqrt{a\tau}};$$

T —the underground temperature with a distance of r to the linear heat source, °C;

T_0 —the undisturbed temperature of soil, °C;

q_l —linear heat source heat dissipation, W/m;

r —the distance to linear heat source, m;

λ_s —thermal conductivity of soil, W/(m·K);

a —thermal diffusivity of soil, $a = \lambda_s / \rho_s / c_s$;

ρ_s —the density of soil, kg/m³;

c_s — the specific heat of soil, J/kg/ °C;

τ —the time from the start time, s.

2.2. ICM

Kavanaugh calculated the heat transfer and underground temperature using *ICM*. Assuming a separate column is surrounded by infinite homogeneous medium and the heat transfer process is purely heat conduction. The analytical solution model is as follow [12]:

$$\theta(r, \tau) = \frac{q_l}{\pi^2 \lambda_s} \int_0^\infty (e^{-u^2 \tau} - 1) \frac{J_0(ur) Y_1(ur_b) - Y_0(ur_b) J_1(ur_b)}{u^2 [J_1^2(ur_b) + y_1^2(ur_b)]} du. \quad (2)$$

Where

θ —dimensionless temperature, $\theta(r, \tau) = T(r, \tau) - T_0$;

J_0, J_1, Y_0, Y_1 —Bessel functions of the first kind and Bessel functions of the second kind.

The analytical solution obtained by *ICM* which contains infinite integral of Bessel functions is very difficult to use. It is recommended to use the following approximate formula for calculation:

$$\theta(r, \tau) = \frac{q_l}{2k\pi r_0} \sum_j \frac{V_j}{j} \frac{K_0(\omega_j r)}{\omega_j K_1(\omega_j r_0)}$$

Where

$$\omega_j = \sqrt{\frac{j \ln 2}{a\tau}}$$

$$V_j = \sum_{k=\text{int}\left(\frac{j-1}{2}\right)}^{\min(j,5)} \frac{(-1)^{j-5} k^5 (2k)!}{(5-k)!(k-1)!k!(j-k)!(2k-j)!}$$

2.3. FLM

The borehole where the U-tube heat exchanger is laid which transfers heat with the soil is considered to be a finite linear heat source which exchanges heat with the semi-infinite medium. Green function and linear superposition principle are used. The analytical solution is obtained as follow [4]:

$$\theta(\rho, z, \tau) = \frac{q_l}{4\pi\lambda_s} \int_0^H \left\{ \frac{\text{erfc} \left[\frac{\sqrt{\rho^2 + (z-h)^2}}{2\sqrt{a\tau}} \right]}{\sqrt{\rho^2 + (z-h)^2}} - \frac{\text{erfc} \left[\frac{\sqrt{\rho^2 + (z+h)^2}}{2\sqrt{a\tau}} \right]}{\sqrt{\rho^2 + (z+h)^2}} \right\} dh. \quad (3)$$

Where

H —the depth of the hole, m;
 ρ —the radial distance to the linear heat source, m;
 z —the distance to the ground, m;
 $erfc$ —the complementary error function.

2.4. The analytical model under variable load

The above heat transfer models are proposed for the constant load, which can not be used in practical projects unless modified because the ground heat exchanger load varies with building load and COP of heat pump. The heat transfer equation for the constant-property medium is linear equation. Assuming that the ground heat exchanger load is constant in a small period of time (usually 1h), the underground temperature distribution can be analyzed by the principle of superposition using above three models [13, 14].

Assuming that the ground heat exchanger load is constant in the period of (τ_{i-1}, τ_i) , the underground temperature can be described by the following formula:

$$\theta(r, \tau_n) = \sum_{i=1}^n (q_i - q_{i-1}) g(\tau_n - \tau_{i-1}) + q_n R_b. \quad (4)$$

Where

q_i —the load of i time, W/m;

τ_i —the i time, s;

τ_n —the end time, s;

$g(\tau)$ —the temperature response coefficient;

R_b —the borehole thermal resistance

For the pipe group which is composed by n U-tube, formula (4) is changed as follow on the base of the superposition principle:

$$\theta(r, \tau_m) = \sum_{j=1}^{n-1} \sum_{i=1}^m (q_i - q_{i-1}) g(\tau_m - \tau_{i-1}) + q_m R_b. \quad (5)$$

Considering the borehole thermal resistance, the U-tube loop average temperature can be obtained by:

$$T_f(\tau) = T(r_b, \tau) + q_l(\tau) R_b. \quad (6)$$

Where

r_b —Borehole radius, m.

For each discrete time, Eq. 6 changes as follows:

$$T_f(\tau_i) = T_0 + \sum_{j=1}^i (q_j - q_{j-1}) g(\tau_i - \tau_{j-1}) + q_i R_b. \quad (7)$$

$$T_{f,o}(\tau_i) = 2T_f(\tau_i) - \frac{q}{cm}. \quad (8)$$

$$T_{f,i}(\tau_i) = 2T_f(\tau_i) - T_{f,o}(\tau_i). \quad (9)$$

Where

$T_{f,o}$ —the outlet temperature of U-tube, °C;

$T_{f,i}$ —the inlet temperature of U-tube, °C;

c —the specific heat of the circulating fluid, J/(kg·K);

m —the mass flow of the circulating fluid, kg/s.

3. Comparison of the above three analytical model

According to formula (4) it can be seen that the main difference of the above three analytical models for heat transfer analysis of ground heat exchanger is reflected in the different temperature response coefficient $g(\tau)$. Thus the comparison of the three heat transfer model can be carried out by the comparison of $g(\tau)$.

3.1. The expression of $g(\tau)$

For *ILM*, $g(\tau)$ can be written as:

$$g(\tau) = \frac{1}{4\pi\lambda_s} Ei\left(\frac{r_b^2}{4a\tau}\right). \quad (10)$$

For *ICM*, it is:

$$g(\tau) = \frac{1}{\pi^2\lambda_s} \int_0^\infty (e^{-u^2\tau} - 1) \frac{J_0(ur)Y_1(ur_b) - Y_0(ur_b)J_1(ur_b)}{u^2 [J_1^2(ur_b) + y_1^2(ur_b)]} du. \quad (11)$$

For *FLM*, it is:

$$g(\tau) = \frac{1}{4\pi\lambda_s} \int_0^H \left\{ \frac{\operatorname{erfc}\left[\frac{\sqrt{\rho^2 + (z-h)^2}}{2\sqrt{a\tau}}\right]}{\sqrt{\rho^2 + (z-h)^2}} - \frac{\operatorname{erfc}\left[\frac{\sqrt{\rho^2 + (z+h)^2}}{2\sqrt{a\tau}}\right]}{\sqrt{\rho^2 + (z+h)^2}} \right\} dh. \quad (12)$$

3.2. Comparison

The comparison of $g(\tau)$ for the three analytical models is shown as Fig. 1-3. The different curves of $g(\tau)$ for one year (8760h) of three models are drawn in the same coordinate plane.

The solutions of *ILM* and *ICM* are independent of depth while that of *FLM* not. It can be observed that $g(\tau)$ increases with time for each model. It has a faster increment at the beginning and trends toward constant. With the increase of the borehole depth, the results of *ILM* and *FLM* harmonize, while the result of *ICM* has a greater deviation with the former two.

For the common borehole with depth of 100m, the comparison of $g(\tau)$ for *ILM* and *FLM* is shown as Figure 4. It can be found that, for the borehole with the depth of 100m, *ILM* and *FLM* are exactly the same.

3.3. Computing speed

The computer which has been used to do the calculation is configured as Table 1. To calculate $g(\tau)$ of 8760h for *ILM* it had taken 0.002278s, while *FLM* 7167.406341s.

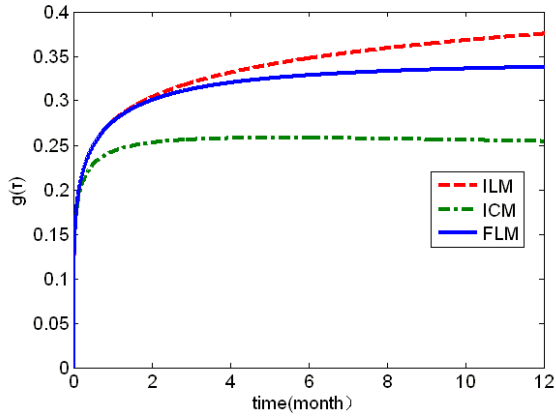


Fig. 1. Comparison of $g(\tau)$ for 10m depth borehole

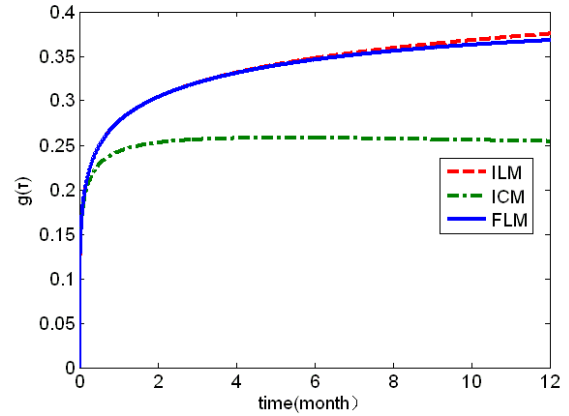


Fig. 2. Comparison of $g(\tau)$ for 20m depth borehole

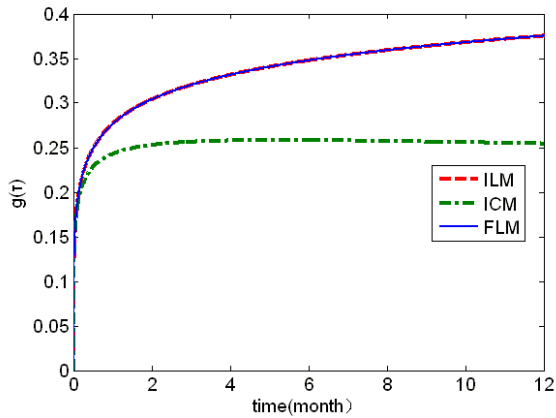


Fig. 3. Comparison of $g(\tau)$ for 30-160m depth borehole

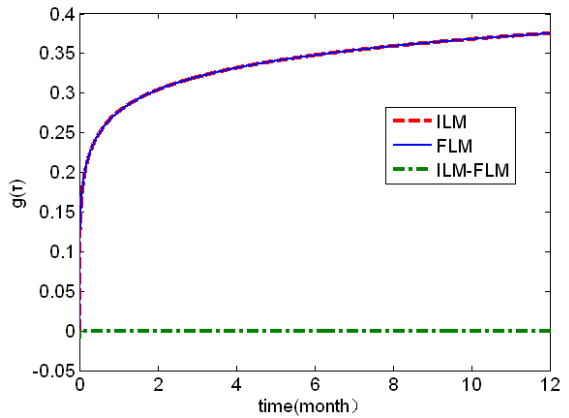


Fig. 4. Comparison of $g(\tau)$ for 100m depth borehole of infinite-length linear heat source model and finite-length linear heat source model

Table 1. Computer configuration

CPU	Intel(R) Core(TM)2 Duo CPU E4500 @ 2.20GHz
BIOS	Award Software International, Inc.; F2
Motherboard	Gigabyte Technology Co., Ltd.;945PL-S3P
Video card	NVIDIA GeForce 7300 GT
Memory	DDR2 SDRAM;1024 MB;DDR2 667(PC5300)
Hard disk	ST3160815AS;7200 rad/min;160.0 GB

To calculate $g(\tau)$ for one year, the speed of ILM is 3.15 million times of FLM which suggests that ILM is excellent when using to analyze the thermal performance of the ground heat exchanger which is deep buried.

4. Summary

- (1) For the ground source heat pump with deep buried pipes, the solution of *ILM* is the same as *FLM*, while *ICM* has larger deviations with them.
- (2) For the annual simulation of 100m depth borehole, the calculation speed of *ILM* is million times of *FLM*.

(3) *ILM* can be applied to the analysis of the thermal performance for ground heat exchangers on the basis of the principle of superposition.

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