Application of Linear Model Fitting in Image Edge Fast Detection

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Abstract A linear model parameter estimation method is proposed based on Bayesian treatment in addition to the linear least squares ,The detail algorithm of one order linear model parameter estimation was introduced in the paper , it can also be used to the parameter estimation of multi-order linear model .we estimate the linear model parameter of image edge use this method .experiment result show that it can finish the parameters fast estimation and the detection of image edge depend on few observed data with a high detect precision.

Keywords:linear model; parameter estimation ; edge detection

1 Introduction

Linear model fitting is the method that parameter estimation of some model function by observed data ,it has a large application in the image processing and analysis such as high speed currency recognize , and other edge detection area ,it is the foundation of image analysis.

Mostly we use the method of linear least squares, many author present some method to the accurency edge detection ,but the algorithm is complex and the amount of calculate and observed data is very large[1][2]. we proposed a linear model parameter estimation method based on Bayesian treatment. estimate the linear model parameter of image edge use this method. it can finish the parameters fast estimation and the detection of image edge depend on few observed data with a high detect precision. Use the high speed currency edge detect as the example to discuss the detail of the application of Linear model fitting.

2 Linear model and parameter estimation

2.1 Linear model

Linear model mean that the model is linear with respect to the model parameters, not(necessarily)with respect to the indicator variables. For example

$$y_i = A_0 + A_1 x_i + A_2 x_i^2 + \cdots \quad (1)$$

Where $A_0, A_1, A_2 \cdots$ are the Linear model parameters of y_i .and x_i is the independent (indicator)variable .Although y_i is not linear with respect to x_i ,there are many similar example in the application ,for example:

$$y_i = A_1 \cos \omega t_i + A_2 \sin \omega t_i \, .$$

we proposed a linear model parameter estimation method based on Bayesian treatment. it can also be introduce to other area .

2.2 Parameter estimation

We assume that we have N observed data values $d_i, i=\{0,1\cdots,n\}$, that are related to N values of the function y_i ,according to

$$d_i = y_i + n_i \qquad (2)$$

Where n_i represents an unknown error component in the measurement of y_i ,described by Gaussian distribution ,the distribution for each n_i to be independent of the value of the other errors and have a common standard deviation σ .mean value 0.

By a linear model ,we mean that y_i can be written as a linear superposition of M functions $g_{i\alpha}$,Denoting the coefficients of the known functions by A_α ,we thus have

$$y_i(A) = \sum_{\alpha=1}^{M} A_\alpha g_{i\alpha} \qquad (3)$$

Where { A_α }is the linear model parameters that we want to estimate ,we will sometimes denote collectively with an unadorned *A* ,for example

$$y_i = A_1 + A_2 x_i + A_3 x_i^2 + \cdots + A_M x_i^{M-1} = \sum_{\alpha=1}^{M} A_\alpha g_{i\alpha} \qquad (4)$$

Then $g_i = \{1, x_i, x_i^2, \cdots, x_i^{M-1}\}$,called basis functions. According to Bayes's theorem, the purpose of linear model fitting is work out a set of parameters { A_α },which can make the joint posterior distribution $p(\{\mathrm{A}_\alpha\} | \{d_i\})$ gets the maximum value.

Because n_i follow gaussian distribution， $n_i = d_i - y_i$ ， so the observed data d_i follow gaussian distribution too， the joint distribution of $\{d_i\}$ is likelihood function：

$$p(\{\{d_i\}|\{A_\alpha\})=\frac{1}{\sigma^n(2\pi)^{\frac{n}{2}}}\exp[-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(d_i-y_i)^2] \qquad (5)$$

Assumed:

$$Q=\sum_{i=1}^{n}(d_i - y_i)^2] \qquad (6)$$

We insert formula（3）into formula（6）, Obviously , Q is the function of { A_α }：

$$Q(\{A_\alpha\})=\sum_{i=1}^{n}(d_i-\sum_{\alpha=1}^{M}A_\alpha g_{i\alpha})^2]$$
$$=\sum_{i=1}^{n}d_i^2+\sum_{i=1}^{n}\sum_{\alpha=1}^{M}\sum_{\beta=1}^{M}A_\alpha A_\beta g_{i\alpha}g_{i\beta}-2\sum_{i=1}^{N}d_i\sum_{\alpha}^{M}A_\alpha g_{i\alpha} \qquad (7)$$

Assumed
$$\vec{d}=[d_1,d_2,\cdots,d_N]^T$$
$$\vec{y}=[y_1,y_{2,}\cdots,y_N]^T$$
$$\vec{g}_\alpha=[g_{1\alpha},g_{2\alpha},\cdots,g_{N\alpha}]^T$$
$$\vec{n}=[n_1,n_2,\cdots,n_N]^T$$

formula（2），（3），（7） can be expressed as matrix and vector:

$$\vec{d}=\vec{y}+\vec{n} \qquad (8)$$

$$\vec{y}=\sum_{\alpha=1}^{M}A_\alpha\vec{g}_\alpha \qquad (9)$$

$$Q(\{A_\alpha\})=(\vec{d}-\vec{y})^2$$
$$=\vec{d}^2+\vec{y}^2-2\vec{d}\cdot\vec{y}$$
$$=\vec{d}^2+\sum_{\alpha\beta}A_\alpha A_\beta\vec{g}_\alpha\cdot\vec{g}_\beta-2\sum_{\alpha}A_\alpha\vec{d}\cdot\vec{g}_\alpha$$
$$=\vec{d}^2+\sum_{\alpha}A_\alpha^2\vec{g}_\alpha^2+2\sum_{\alpha\neq\beta}A_\alpha A_\beta\vec{g}_\alpha\cdot\vec{g}_\beta-2\sum_{\alpha}A_\alpha\vec{d}\cdot\vec{g}_\alpha \qquad (10)$$

In order to make formula（5）get the max，we calculate the Partial Derivative of formula（10）and assumed it equal to 0：

$$\frac{\partial Q(\{A_\alpha\})}{\partial A_\alpha}=2\sum_{\beta=1}^{M}A_\beta \vec{g}_\beta\cdot\vec{g}_\alpha-2\vec{g}_\alpha\cdot\vec{d}=0 \qquad (11)$$

formula（11）can be express as linear equations:

$$\sum_{\beta=1}^{M}A_\beta\vec{g}_\beta\cdot\vec{g}_\alpha=\vec{g}_\alpha\cdot\vec{d} \qquad (12)$$

Solve this linear equations, we can obtain the linear model parameters $\{A_1,A_2\cdots,A_M\}$

3 Application in line parameter fitting

We assumed that the line equation need fitting is $y_i=A_1+A_2x_i$, observed data vector $\vec{d}$, base function is $\vec{g}_1=[1,1,\cdots 1]^T$, it include N element，$\vec{g}_2=[x_i,x_i,\cdots x_i]^T$ include N element too，linear equations is：

$$\begin{cases}A_1\vec{g}_1\cdot\vec{g}_1+A_2\vec{g}_2\cdot\vec{g}_1=\vec{g}_1\cdot\vec{d}\\ A_1\vec{g}_2\cdot\vec{g}_1+A_2\vec{g}_2\cdot\vec{g}_2=\vec{g}_3\cdot\vec{d}\end{cases} \qquad (13)$$

where

$$\vec{g}_1\cdot\vec{g}_1=\sum_{i=1}^{N}1=N$$

$$\vec{g}_1\cdot\vec{g}_2=\vec{g}_2\cdot\vec{g}_1=\sum_{i=1}^{N}x_i$$

$$\vec{g}_2\cdot\vec{g}_2=\sum_{i=1}^{N}x_i^2$$

$$\vec{g}_1\cdot\vec{d}=\sum_{i=1}^{N}d_i$$

$$\vec{g}_2\cdot\vec{d}=\sum_{i=1}^{N}d_i\cdot x_i$$

insert to formula（13）：

$$\begin{pmatrix}N & \sum_{i=1}^{N}x_i\\ \sum_{i=1}^{N}x_i & \sum_{i=1}^{N}x_i^2\end{pmatrix}\begin{pmatrix}A_1\\ A_2\end{pmatrix}=\begin{pmatrix}\sum_{i=1}^{N}d_i\\ \sum_{i=1}^{N}d_i\cdot x_i\end{pmatrix} \qquad (14)$$

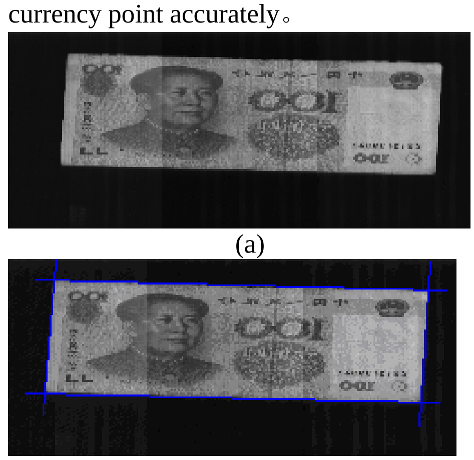
slove the linear equations：

$$A_1=\frac{\sum_{i=1}^{N}x_i^2\sum_{i=1}^{N}d_i-\sum_{i=1}^{N}x_i\sum_{i=1}^{N}d_i\cdot x_i}{N\sum_{i=1}^{N}x_i^2-(\sum_{i=1}^{N}x_i)^2} \qquad (15)$$

$$A_2=\frac{-\sum_{i=1}^{N}x_i\sum_{i=1}^{N}d_i+N\sum_{i=1}^{N}d_i\cdot x_i}{N\sum_{i=1}^{N}x_i^2-(\sum_{i=1}^{N}x_i)^2} \qquad (16)$$

4 experiment result and analysis

In high speed currency recognition , The currency image is obtained by contact image sensor。The currency move speed is 550-600 sheet/min，because the high speed movement，image quality is poor and time for processing is very limited，In order to locate the currency image, we must calculate the for currency point accurately。



(a)

(b)

Figure 1 Currency image edge detect result

We use formula（15）、（16）calculate the four edge line equation , obtain the four currency point coordinate by the four line equation。Through the large amount currency experiment , image resolution is 256*110， we sampling each 10 pixel in horizon , each 5 pixel in vertical to fitting， experiment result show in figure 1,we can obtain each point coordinate by two edge line equation .figure（a） is observed image，figure（b）is the edge detect result by linear model fitting。We label 5300 currency point coordinate manually contrast to the result with this method，as table 1.

5 conclusion

A linear model parameter estimation method is proposed based on Bayesian treatment. It can also be used to the parameter estimation of multi-order linear model. The method can be used to the image edge detection and other area ..experiment result show that it can finish the parameters fast estimation and the detection of image edge depend on few observed data with a high detect precision.

References

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Table 1 error analysis of currency edge detect by linear model fiting

Unit：pixel

	Horizontal max error $\|\Delta x_{\max}\|$	Vertical max error $\|\Delta y_{\max}\|$	Point distance max error $\|D_{\max}\|$
LUL	2	3	3.2
LLL	2	2	2.3
RUL	3	3	3.6
RLL	2	2	2.1

note：because the Horizontalmax error and the vertical max error are not happen in same currency,so point distance max error $|D_{\max}|$are no equal to $\sqrt{\Delta x_{\max}^2 + \Delta y_{\max}^2}$.