The Theory and Practice of Integrating Mathematical Modeling Thought into College Mathematics Based on Constructivism Viewpoint

Jian Du[1,*], Ying Liu[2]

[1]Department of Fundament, Army Academy of Armored Forces, Beijing, 100072, P.R. China
[2]Center for Innovation and Application, China Academy of Aerospace Aerodynamics, Beijing, 100071, P.R. China

*Jiandu2531@163.com

Abstract. College mathematics teaching should enable students to grasp solid theoretical knowledge, understand the process of knowledge construction from a deep level, apply the idea and method of mathematical modeling to practice, and strengthen the cultivation of innovative application ability, which is the main trend of the current mathematics teaching reform. On the basis of analyzing the main teaching viewpoints of constructivism, this paper puts forward the basic strategies of constructivism teaching design. This paper analyzes the learning, cognition and teaching process of integrating mathematical modeling thought into the curriculum. And through the case practice, the effectiveness of the teaching model is tested. The practice shows that in the teaching process, the thought method of mathematical modeling can make students understand the relationship between the real world and mathematics subject, cultivate students' application consciousness, enhance mathematics application ability, and realize the positive role of mathematics curriculum in the training of thinking.

Keywords: Constructivism; Mathematical Modeling; Innovation Ability.

1 Introduction

The mathematical thought method is student's understanding of the essence of mathematical knowledge, it is the generalization of mathematical thinking method and practice method, it contains in the process of the occurrence, development and application of mathematical knowledge. The content of mathematics always reflects two lines, that is, mathematical knowledge and mathematical thought and method, which constitute a dynamic system of knowledge and method[1]. Mathematical knowledge is the carrier of mathematical thought and method, which is the essence of mathematical knowledge and the inner motive force of mathematical development. It is the bridge of knowledge into ability, the group belt of students to form cognitive structure, and the key to cultivate mathematical concept and promote creative thinking. The mathematical thought method is the most active and active factor in the students' mathematical cognitive

J. Yin et al. (eds.), *Proceedings of the 4th International Conference on New Media Development and Modernized Education (NMDME 2024)*, Advances in Intelligent Systems Research 188,
https://doi.org/10.2991/978-94-6463-600-0_54

structure[2]. Under the guidance of constructivism, strengthening the teaching of mathematical thoughts and methods can help students establish scientific mathematical concepts and mathematical spirit, promote the understanding of mathematical knowledge, form the transfer of mathematical knowledge, and improve students' innovative application ability[3].

2 Constructivist View of Education

Constructivism is a theory about knowledge and learning, which emphasizes the initiative of learners and believes that learning is a process in which learners generate meaning and construct understanding based on original knowledge experience, and this process is often completed in social and cultural interaction[4]. Social constructivism believes that learning is a process of cultural participation, and learners construct relevant knowledge by participating in the practical activities of a certain community. Learning is not only the individual's active processing of learning content, but also requires the learners to cooperate and help each other[5].

Constructivism's view of knowledge constructivism makes us realize that knowledge is not only related to the relationship with the external world, but also reflects the relationship between knowledge, subject and the external world, which establishes the subjective status of learners and the basic role of learners' original knowledge experience. Constructivism's view of knowledge provides the epistemological basis for the transformation of Chinese educational view[6]. The constructivism activity view shows that knowledge is the result of mutual construction of subject and object, and individuals need to acquire knowledge through their own activities and through repeated application and practice in different knowledge situations to master knowledge flexibly[7]. New concepts and ideas such as construction, generation, subject, autonomy, activity, inquiry, cooperation, context, collaboration, communication and meaning have emerged, and researchers have conducted in-depth discussions on the nature of knowledge, the relationship between subject and object in education, teaching and learning, direct and indirect experience, knowledge transfer and ability development, and the relationship between teacher and student[8].

Constructivism believes that learning is always connected with a certain social and cultural background, that is, the "situation". Learning in the actual situation can enable learners to assimilate and index the new knowledge they currently learn by using the relevant experience in their original cognitive structure, so as to give the new knowledge some meaning. In mathematical modeling teaching, it is necessary to create problem situations, stimulate students' interest in exploring knowledge, encourage students to raise questions, find problems and try to solve problems. Teachers should design questions carefully and scientifically, and try to create a relaxed atmosphere of positive thinking and exploration for students to learn modeling[9].

3 Constructivism Teaching Design

Constructivism theory points out that learning is a process in which students, under the guidance of teachers, take the original knowledge experience as the starting point and actively construct the meaning of new knowledge[10]. In the teaching process, teachers need to constantly think about how to improve learning enthusiasm, promote students to absorb knowledge quickly and effectively, improve teaching quality, and achieve teaching objectives, which has important guiding significance for the development of learning activities[11].

3.1 Emphasizes the Important Role of Context in Meaning Construction

Constructivism believes that learning is always connected with a certain social and cultural background, that is, the "situation". Learning in the actual situation can enable learners to assimilate and index the new knowledge they have learned by using the relevant experience in their original cognitive structure, so as to give the new knowledge some meaning[12]. If the original experience can not assimilate the new knowledge, it should cause the "adaptation" process, that is, the transformation and reorganization of the original cognitive structure. In short, only through "assimilation" and "adaptation" can the meaning of new knowledge be constructed. In the traditional classroom teaching, because it can not provide the vividness and richness of the actual situation, it will make it difficult for learners to construct the meaning of knowledge.

3.2 Emphasize the Key Role of Collaborative Learning in Meaning Construction

Constructivism believes that the interaction between learners and the surrounding environment plays a key role in understanding the learning content. This is one of the core concepts of constructivism. Under the guidance and organization of teachers, students discuss and communicate together to establish and become a member of a learning group[13]. In such groups, theories, ideas, beliefs, and hypotheses are critically examined together; Consultations and debates, first among themselves and then among themselves. Through such a collaborative learning environment, the thinking and wisdom of the learner group can be shared by the whole group, that is, the whole learning group can complete the learned knowledge together, instead of one or several students completing the meaning construction.

3.3 Emphasize the Ultimate Goal of the Learning Process is to Complete the Meaning Construction

In the learning environment of constructivism, it emphasizes that students are cognitive subjects and active constructors of meaning, so students' knowledge is regarded as the ultimate goal of the whole learning process. Instructional design usually does not start from the analysis of teaching objectives, but from how to create situations conducive

to students' meaning construction. The whole instructional design process closely revolves around the center, whether it is students' independent exploration, collaborative learning or teacher guidance, in short, all activities in the learning process should be subordinate to this center. All of them should be conducive to completing and deepening the meaning construction of the learned knowledge.

4 Integrate the Theoretical Process of Mathematical Modeling into the Course

4.1 The Learning and Cognitive Process of Mathematical Modeling Integrated into the Curriculum

From the analysis of learning process to mathematical modeling, computer system is used as an experimental tool, probability and statistics theory as the principle, mathematical teaching materials as the modeling object, simple human-computer dialogue or complex program as the modeling form, numerical calculation, symbolic calculation or graphic demonstration as the modeling content; With case analysis, simulation and summary as the main modeling methods, it is a mathematical thinking activity process with the purpose of assisting learning mathematics, assisting using mathematics or assisting doing mathematics, and discovering laws and solving practical problems. The teaching of mathematical modeling focuses on enabling students to deeply understand the basic concepts and theories of probability and statistics, familiarize themselves with commonly used mathematical software, cultivate students' ability to establish mathematical models with what they have learned and solve practical problems with computers, and integrate mathematical knowledge, mathematical modeling and computer application.

4.2 Teaching Process Integrating Mathematical Modeling Thought Method

Constructivism learning theory holds that knowledge is acquired by means of meaning construction with the help of others and the use of necessary learning materials under certain circumstances. The application of constructivist learning theory in teaching implies a change in the roles and roles of teachers and students. Constructivism emphasizes linking the learned knowledge with certain real task situations or simulated task situations in teaching, so that students can cooperate to solve situational problems. Learning should focus on solving practical problems in life and should be carried out in specific situations; The learning effect should be evaluated in the context; The teacher puts the focus of teaching in a macroscopic situation, and guides students to find, form and solve problems with the help of the materials in the situation. Students have their learning needs in a complete and real problem situation. Through the communication between teachers, teachers and students, and the cooperative learning between students and students, they can personally experience the whole process from identifying goals to achieving goals.

5 Teaching Practice of Integrating Mathematical Modeling Thought

In order to explore the effectiveness of the constructivism based probability theory and mathematical statistics curriculum integrated with mathematical modeling thought method in teaching practice, this project studies the teaching reform in the class of "Probability theory and mathematical statistics".

Case in point: Airline overbooking strategies

Many people book their tickets long in advance, and there are always passengers who can't get on their flights on time because of various changes. In order to reduce the economic loss of operating empty seats caused by quota tickets, airlines usually use overbooking strategies, selling several more tickets per flight. The Regulations on the Disposal of overbooked Public Air Transport Flights require that volunteers who give up boarding should be found before using the priority flight rule when a flight is overbooked, and free or reduced air transport and miles are provided to the volunteers as compensation.

5.1 Problem Analysis

The aircraft capacity is n ,the number of overtickets is q , and the number $n+q$ of passengers who do not board on time is r (random variable).

2. The probability of each booked passenger not boarding on time is p , that whether they board on time or not is independent of each other. This assumption applies to businessmen and tourists who act alone.

3. The price of each ticket is s_1 , and the amount of compensation for each passenger who is unable to board because the plane is full is s_2 . The cost of the flight has little to do with the number of passengers and is not taken into account.

5.2 Model Building

q : Number of overbooked tickets, r : number of passengers who do not book flights on time, n : aircraft capacity, s_1 : ticket price, s_2 : compensation amount.

- $r \le q$

  $n+q-r \ge n$: Number of passengers boarding on time.

  n : passenger boarding, $s_1 n$: ticket revenue.

  $q-r$ passenger was compensated is $s_2(q-r)$.

  $s_1 n - s_2(q-r)$ Earnings.

- $r > q$

$n+q-r<n$: Number of passengers boarding on time.

$s_1(n+q-r)$: Earnings.

For overbooking q, r : booking the number of passengers who do not board on time flights

Airline earnings

$$s(r,q)=\begin{cases} s_1 n - s_2(q-r) & r \le q \\ s_1(n+q-r) & r > q \end{cases} \tag{1}$$

p : The probability of each booked passenger not boarding on time, and whether they are on time is independent of each other.

$f(r)$: The probability r passenger that one of the booked passengers $n+q$ will not board on time.

$$f(r)=C_{n+q}^{r} p^{r}(1-p)^{n+q-r}, r=0,1,\ldots n+q \tag{2}$$

Binomial distribution

The average revenue of flights

$$\begin{aligned} E(q) &= \sum_{r=0}^{n+q} s(r,q) f(r) \\ &= \sum_{r=0}^{q} \left(s_1 - s_2(q-r)\right) f(r) + \sum_{r=q+1}^{n+q} s_1(n+q-r) f(r) \end{aligned} \tag{3}$$

The number of overbooked tickets s_1, s_2, n, p is known to solve q to maximize the average revenue $E(q)$.

5.3 Model Solving

$$E(q)=\sum_{r=0}^{q}\left(s_1 - s_2(q-r)\right) f(r) + \sum_{r=q+1}^{n+q} s_1(n+q-r) f(r) \tag{4}$$

The paper boy selling model is similar to discrete demand inequation

$$P(r \le q)=\sum_{r=0}^{q} C_{n+q}^{r} p^{r}(1-p)^{n+q-r} \ge \frac{s_1}{s_1+s_2} \tag{5}$$

Make the smallest q, make the largest q.

$s = s_1 / s_2$:The ratio of the compensation amount s_2 to the price of the ticket s_1,

$$P(r \le q) = \sum_{r=0}^{q} C_{n+q}^{r} p^{r} (1-p)^{n+q-r} \ge \frac{1}{1+s} \tag{6}$$

The capacity of the aircraft is $n = 300$, the probability $p = 0.05$ of each booked passenger not boarding on time, the ratio $s = 1/2$ of the compensation amount to the ticket price.

Under the goal of maximum average revenue, for fixed aircraft capacity, the only parameters that affect the number of overtickets are $q(n = 300)$.The calculation results are shown in Table 1.

Table 1. The number of overtickets with the greatest average revenue.

	s=1/3	s=1/2	s=1
p=0.01	4	4	3
p=0.03	11	10	9
p=0.05	18	17	16

q Increased when the probability p of passengers not showing up for boarding becomes greater;

q Decreases when the compensation amount (compared to the ticket price) becomes larger.

5.4 Model Evaluation

Two models of maximum economic benefits of overbooking strategy and social reputation are established, which can be combined in application.

For the maximum number q of overbooking and considering the threshold j of social reputation, if the probability $p_j(q)$ of passengers who cannot board is too large, it can be appropriately reduced q, and the revenue can be sacrificed in exchange for the reduction $p_j(q)$.

The probability p of passengers not boarding on time has a great impact on economic income and social reputation, so it is necessary to adjust the probability p in real time by using statistical data according to different flights and different times (seasons, holidays and other factors) to improve the accuracy of the model.

6 Conclusions

In terms of problem-solving ability, constructivism is result-oriented. In the process of mathematizing actual problems and building models, the functional relationship between variables is used to describe the object relationship of actual problems. The solution of mathematical models can better explain the real world, and mathematical modeling builds a bridge between real problems and mathematical problems. Students improve their ability to solve practical problems in the process of experimental simulation and model building.

In terms of innovation ability, project-based learning based on constructivism takes specific cases as the teaching context, which provides more space for students to play freely. Students can independently build models and design algorithms without being bound by fixed patterns, abandon the only correct answer, open questions and experimental modeling, which can arouse students' desire for knowledge, stimulate students' creativity, and improve students' innovation ability.

References

1. Renan Agtarap,Aldwin T.Miranda: The Mediating Effect of Students' Resiliency on the Relationship of Self-Concept and Mathematics Performance, Asian Journal of Education and Social Studies[J]. Journal 12(3):1-10(2022).
2. Bailey Judy:Mathematical investigations for supporting pre-service primary teachers repeating a mathematics education course,Australian Journal of Teacher Education [J]. Journal 39(2):86-100(2014).
3. Song Xian, Zhu Wenwen: Science education based on Science and Innovation Competition: Learning environment, learning process and training Effect [J]. Global Education Perspectives, Journal 52 (11): 85-97(2023).
4. Mutiawati: Application of Mathematical Models in Science and Engineering,Journal of Asian Scientific Research[J]. Journal 5(6) :269-279(2015).
5. Marita Samantha,Hord Casey,Hendricks Julie:Using visual strategies simultaneously to support students with learning disabilities in mathematics, Support for Learning. Journal 36(3): PP 327-341(2021).
6. Deng Huayu, Sun Yuqin, Zhu Xiaojing et al: Case Teaching of Mathematical Modeling and Talent Training of Energy and Electric Power [J]. Mathematical Modeling and Its Applications, Journal 12 (03): 68-75(2023).
7. Du Jian, Liang Hongmei, Hu Hongjuan: Research on results-oriented teaching reform of "Probability Theory and Mathematical Statistics" [J]. Journal of Higher Education Research, Journal 46 (02): 110-116(2023).
8. Du Jian, Hu Hongjuan, Xia Jing: Exploration and practice of University Mathematical modeling teaching [J]. Heilongjiang Science, Journal 12 (23): 57-58+61(2021).
9. Hu Hongjuan, Du Jian, Ju Guiling et al: Visualization teaching of Central Limit Theorem in MATLAB and Mathematical modeling [J]. Information Systems Engineering, Journal 8: 167-168(2020).
10. Du Jian, Xia Jing, Hu Hongjuan: Point of teaching quality evaluation based on the theory of TQM strategies [J]. Journal of quality education in the west. Journal 6 (01) :26-29(2020).

11. Hu Hongjuan, Du Jian, Li Huizhen et al: Discussion on the teaching of "Probability Theory and Mathematical Statistics" for through-train students [J]. Information Systems Engineering, Journal 3: 175-176(2018).
12. Du Jian, Ding Shiyong, Yao Jian: Research on reform of college talent training model for practical needs [J]. Journal of Higher Education Research, Journal 39 (01): 16-21(2016).
13. Du Jian, Ding Shiyong, Dong Yucai: Research and practice of Mathematical modeling thought integrated into main Mathematics courses in universities [J]. Information Sys-tems Engineering, Journal 3: 148-149(2016).