Single Allocation Hub-Spoke Network Design Considering Capacity Selection Under Competitive Environment

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Abstract. Hub-and-spoke network is an important network structure in modern logistics field, which can effectively improve the transportation efficiency of logistics industry. Hub congestion and failure cannot be ignored in hub-and-spoke network, affecting the operation efficiency of hub-and-spoke network. At the same time, with the vast market prospect attracting the continuous influx of capital, there is excessive competition in the industry, leading to the irrational use of resources. In this context, the design problem of single allocation hub-and-spoke network considering capacity selection under competitive environment is studied. To solve this problem, a two-layer programming model is constructed. The upper layer is the leader's mixed integer programming model about the hub layout scheme, and the lower layer is the followers' mixed integer programming model about their own hub layout scheme after observing the leader's behavior. Due to the complexity of solving the two-layer programming model, in order to reduce the computational complexity and ensure the quality of the solution, a variable neighborhood search algorithm is designed to solve the two-layer programming model. The results show： the number of hub, the order of decision making, and whether to take into account the behavior of competitors will affect the hub location scheme, capacity decision and market share of market competition participants.

Keywords: Competition, Capacity Selection, Single Allocation.

1 INTRODUCTION

In recent years, the rapid development of e-commerce has promoted the increasingly vigorous logistics industry. According to the data of the China Federation of Logistics and Purchasing, as an important industry supporting the development of the national economy, the total amount of social logistics in 2022 is 347.6 trillion yuan, an increase of 3.4 percentage year-on-year, indicating that the scale of logistics demand has reached a new level and achieved stable growth. In December 2018, the National Development and Reform Commission and the Ministry of Transport jointly issued the National Logistics Hub Layout and Construction Plan, emphasizing that logistics hubs are the core infrastructure of the logistics network system and play a vital role in the logistics and

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transportation of regional networks. At present, the transportation network structure of most logistics enterprises adopts the hub-and-spoke network proposed by O 'Kelly [1]. Compared with the straight-through network, the hub-and-spoke network realizes the scale effect through the aggregation of goods at the hub node, thus reducing the network operation cost.

Although the use of economies of scale is a major advantage of hub networks, it can also lead to commodity overload in a few hubs, so the issue of hub capacity has been more comprehensively discussed in hub-and-spoke network design. Azizi proposed a hub-and-spoke network design model under random demand and congestion conditions [2]. The effects of random demand and congestion on the location and capacity of pivot points are studied. In addition, hub facilities are prone to unpredictable failures caused by inclement weather conditions, natural disasters, labor disputes and intentional sabotage, the capacity and reliability of hub nodes are often considered together in the study of hub-and-spoke network design. Azizi studied the single assignment reliable hub location problem with multiple capacity levels and traffic-related discount factors [3]. Huang studied a design problem of hub spoke network for container shipping hubs that considered both hub failure and hub congestion, and defined congestion cost as a power function of container flow exceeding port service capacity [4]. In the above research, the competition factor in the design of hub-and-spoke network is missing. The first study on the location of competitive hubs was first proposed by Marianov[5]. From the perspective of followers, the study assumed that in a hub network where the leader has made optimal layout, followers participate in market competition by building their own hub nodes, with the goal of maximizing the market share of followers.

To sum up, the current hub location model design still has two aspects of less attention:(1) The combination of competition and failure; (2) The combination of competition and capacity. Therefore, this study focuses on single allocation hub-spoke network design considering capacity selection under competitive environment.

2 PROBLEM DESCRIPTION AND MODEL

2.1 Problem Description

In the logistics industry market, the timing of logistics enterprises is not equal to the competition. Logistics enterprises make decisions according to the order of entering the industry market, and the different order of decision will make them in different positions. In a market divided by geographical region, there are two enterprises providing homogeneous logistics and distribution services, among which the first one to enter the market is called the leader, and the one who enters the market after the leader is the follower. Both companies provide logistics and transportation services to all consumers in the region through their own hubs, and their goal is to gain the most market share. The problem consists of three stages: the first stage is the leader chooses the location and capacity level of the hub node, and decides the allocation scheme of non-hub nodes under normal conditions and failure conditions. In the second stage, followers choose the location and capacity level of hub nodes according to the leader's hub layout, and decide the distribution plan of non-hub nodes under normal conditions and failure conditions. In the third stage, customers assign probabilities according to the combined service cost provided by the leaders and followers.

2.2 Mathematical Notations

Table 1. Mathematical notations and definitions of various parameters and variables.

Var.	definition
N	Set of nodes in hub-and-spoke network
H^p	Set of the potential hub nodes for leader
H^r	Set of the potential hub nodes for follower
L	Set of the hub capacity levels
w_{ij}	The flow between node i and node j
c_{ij}	The transportation cost between node i and node j
F_k^l	The cost for installing a hub with capacity level l at node k
Q_k^l	The capacity level l at node k
a	Discount factor between hubs;
p	The number of hub nodes for leader
r	The number of hub nodes for follower
β_{ij}	The service cost between node i and node j provided by the leader
γ_{ij}	The service cost between node i and node j provided by the follower
S_{ij}	The share that the leader gets between node i and node j
s_{ij}	The share that the leader gets between node i and node j
h_k^l	1 if capacity level l is decided for hub k and 0 otherwise
X_{ikmj}	1 if flow from node i to node j routed via hubs located at nodes k and m of the leader and 0 otherwise
x_{ikmj}	1 if flow from node i to node j routed via hubs located at nodes k and m of the follower and 0 otherwise
Z_{ij}	1 if node i is allocated to the hub k of leader and 0 otherwise
z_{ij}	1 if node i is allocated to the hub k of follower and 0 otherwise
U_{ikn}	1 if n is the backup hub of leader for node i assigned when hub k is disrupted and 0 otherwise
u_{ikn}	1 if n is the backup hub of leader for node i assigned when hub k is disrupted and 0 otherwise

2.3 Single Allocation Under Competition Model

The service cost between node i and node j provided by the leader (including transportation costs and additional service charges depending on the level of capacity) can be expressed as:

$$\beta_{ij} = \left(c_{ik} + ac_{km} + c_{mj}\right)X_{ikmj} + \frac{\sum_{k\in H^p}\sum_{l\in L}F_k^l h_k^l}{\sum_{i\in N}\sum_{j\in N}w_{ij}} \tag{1}$$

The same can be obtained：

$$\gamma_{ij} = \left(c_{ik} + ac_{km} + c_{mj}\right)X_{ikmj} + \frac{\sum_{k\in H^r}\sum_{l\in L}F_k^l h_k^l}{\sum_{i\in N}\sum_{j\in N}w_{ij}} \tag{2}$$

After the leaders and followers have make their respective hub location decision, the share obtained by the leaders for the flow between node i and node j can be calculated by the following formula:

$$S_{ij} = \frac{\frac{1}{\beta_{ij}}}{\frac{1}{\beta_{ij}}+\frac{1}{\gamma_{ij}}} = \frac{\beta_{ij}\gamma_{ij}}{\beta_{ij}(\beta_{ij}+\gamma_{ij})} \tag{3}$$

The same can be obtained：

$$s_{ij} = \frac{\frac{1}{\gamma_{ij}}}{\frac{1}{\beta_{ij}}+\frac{1}{\gamma_{ij}}} = \frac{\beta_{ij}\gamma_{ij}}{\gamma_{ij}(\beta_{ij}+\gamma_{ij})} \tag{4}$$

Lower-level follower hub location model.

$$max\sum_i\sum_j s_{ij}w_{ij} \tag{5}$$

$$s.t$$

$$\sum_k z_{ik} = 1\ \forall i \tag{6}$$

$$\sum_k z_{kk} = r \tag{7}$$

$$z_{ik} \le z_{kk}\ \forall i,k \tag{8}$$

$$\sum_m x_{ikmj} = z_{ik}\ \ \forall i,j,k \tag{9}$$

$$\sum_k x_{ikmj} = z_{jm}\ \ \forall i,j,m \tag{10}$$

$$\sum_i\sum_j\sum_m s_{ij}w_{ij}x_{ikmj} \le \sum_{l\in L} Q_k^l h_k^l\ \ \forall k \tag{11}$$

$$\sum_{l\in L} h_k^l = z_{kk}\ \forall k \tag{12}$$

$$\sum_{n\neq k} u_{ikn} = z_{ik}\ \ \forall i \neq k \tag{13}$$

$$u_{ikn} \le z_{nn}\ \forall k,n,i \neq k \tag{14}$$

$$\sum_i\sum_j\sum_m s_{ij}w_{ij}z_{in}z_{kk} + \sum_{i\neq k}\sum_{j\neq k} s_{ij}w_{ij}u_{ikn} \le \sum_{l\in L} Q_k^l h_k^l\ \ \forall k,n \tag{15}$$

$$x_{ikmj}, z_{ik}, h_k^l, u_{ikn} \in \{0,1\} \tag{16}$$

Upper-level leader hub location model

.
$$max\sum_i \sum_j s_{ij} w_{ij} \tag{17}$$

$$s.t$$

$$\sum_k z_{ik} = 1 \ \forall i \tag{18}$$

$$\sum_k z_{kk} = r \tag{19}$$

$$z_{ik} \le z_{kk} \ \forall i, k \tag{20}$$

$$\sum_m x_{ikmj} = z_{ik} \ \ \forall i, j, k \tag{21}$$

$$\sum_k x_{ikmj} = z_{jm} \ \ \forall i, j, m \tag{22}$$

$$\sum_i \sum_j \sum_m s_{ij} w_{ij} x_{ikmj} \le \sum_{l \in L} Q_k^l h_k^l \ \forall k \tag{23}$$

$$\sum_{l \in L} h_k^l = z_{kk} \ \forall k \tag{24}$$

$$\sum_{n \ne k} u_{ikn} = z_{ik} \ \ \forall i \ne k \tag{25}$$

$$u_{ikn} \le z_{nn} \ \forall k, n, i \ne k \tag{26}$$

$$\sum_i \sum_j \sum_m s_{ij} w_{ij} z_{in} z_{kk} + \sum_{i \ne k} \sum_{j \ne k} s_{ij} w_{ij} u_{ikn} \le \sum_{l \in L} Q_k^l h_k^l \ \forall k, n \tag{27}$$

$$x_{ikmj}, z_{ik}, h_k^l, u_{ikn} \in \{0,1\} \tag{28}$$

The symbolic parameters in the model are shown in Table 1. The objective function (5) and (17) represents that followers and leaders seek to maximize their own market share. Constraints (6) and (18) ensure nodes are hub nodes or are assigned to hub nodes. Constraints (7) and (19) mean the number of hubs to be built. Constraints (8) and (20) ensure that non-hub nodes can only be assigned to hub nodes. Constraints (9) and (10) ensure that all traffic transportation paths between all nodes pass through the hub. Constraints (21) and (22) are same to constraints (9) and (10). Constraints (11) and (23) indicate that the traffic entering the hub does not exceed the capacity limit of the hub. Constraints (12) and (24) indicate that a hub can only select one capacity level. Constraints (13) and (25) ensure that there is only one backup facility for non-hub nodes. Constraints (14) and (26) ensure that the backup facilities of non-hub nodes are hub nodes; Constraints (15) and (27) ensure that the traffic entering the hub does not exceed the capacity limit of the hub when the hub fails; Constraints (16) and (28) represent binary variable constraints.

3 SOLUTION ALGORITHM

A two-element dual-layer mixed integer programming model is established above. The upper layer is a mixed integer programming problem about the location of the hub where the leader considers that there will be follower competition; the lower layer is a mixed integer programming problem about the location of the hub after the follower observes the leader's decision. Its upper constraint is closely related to the solution of the lower programming, and the model is a non-convex and non-smooth complex optimization problem, belonging to the NP-hard problem. However, for the NP-hard problem whose solution difficulty increases exponentially with the expansion of the problem scale, if accurate solution is used, it may fall into the dilemma of slow solution and high solution cost. Therefore, this paper will use the variable neighborhood search algorithm to solve.

The algorithmic step are as follows: first a leader's hub location scheme is randomly generated, and then the followers choose the hub location scheme with the maximum market share as the target based on the leader's layout scheme. Finally, the leaders reselect the hub location scheme with the maximum market share as the target based on the followers' hub location scheme. Until the market share of the leader no longer increases. In the variable neighborhood search algorithm proposed in this paper, four kinds of neighborhood structures are considered. In the first neighborhood structure, a new solution is generated by exchanging identities between a hub node and a non-hub node assigned to that pivot point. In the second neighborhood structure, the distribution of two non-hub node is exchanged to generate a new solution. In the third neighborhood structure, a non-hub node is randomly selected to change its assignment. In the fourth neighborhood structure, two hub nodes are randomly selected and all the non-hub node assigned are exchanged.

4 CASE STUDY

In this section, CAB standard data set is used, and the first 15 nodes are selected for this experiment. By calculating the total traffic to be transmitted in the entire transportation network, six capacity levels are generated, which are 15%, 30%, 45%, 60%, 75% and 99% of the total flow, and the cost for installing a hub are set at 25 million, 50 million, 75 million, 100 million, 125 million and 150 million yuan respectively. The discount factor is set to 0.6 for both leaders and followers.

4.1 The Experimental Result

The problem studied in this section is that when leaders choose their own hub layout plan, they will consider the hub layout plan chosen by followers, and the hub layout plan of followers depends on the hub layout plan of leaders, and the hub layout plan of leaders is to pursue the maximization of their own market share. The purpose of this experiment is as follows: (1) Compare and observe the additional market share occupied by the leader when the follower behavior is not considered by the leader. (2) When the leader does not consider the follower behavior, the improvement degree of the leader hub capacity utilization when the follower behavior is considered is compared and observed. (3) Compare and observe the impact of the leader's consideration of the followers' behavior on the hub allocation schemes of both sides of the hub.

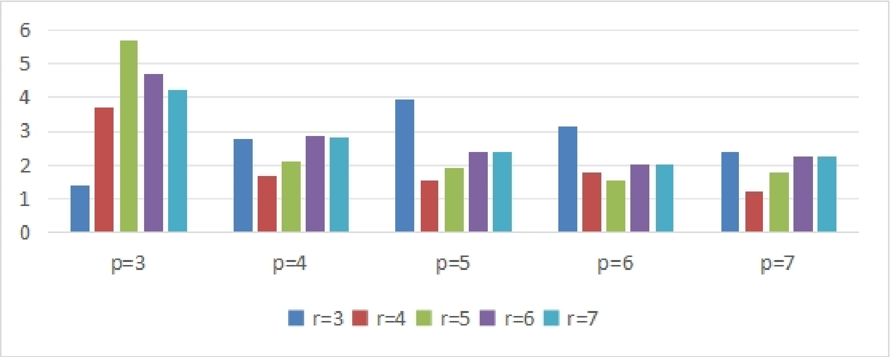

Fig. 1. Market share gains of competitive strategy under different number of hubs for the leader

The benefits of competitive strategy for the leader in a single-distributed hub-spoke network is outlined in Fig.1 under various circumstances. In this network, regardless of the number of hubs, leaders who consider the follower competition behavior can achieve an additional market share ranging from 0.77% to 5.68%, compared to leaders who do not consider follower competition behavior. It is important to note that the total market share gains obtained through competitive strategies decrease as the number of hubs increases, indicating that an increase in the number of hubs can diminish the impact of followers on their market share. This result is attributed to the fact that the decision-making space for leaders is determined by the number of hubs, and therefore, changes in a leader's strategy may have relatively limited effects.

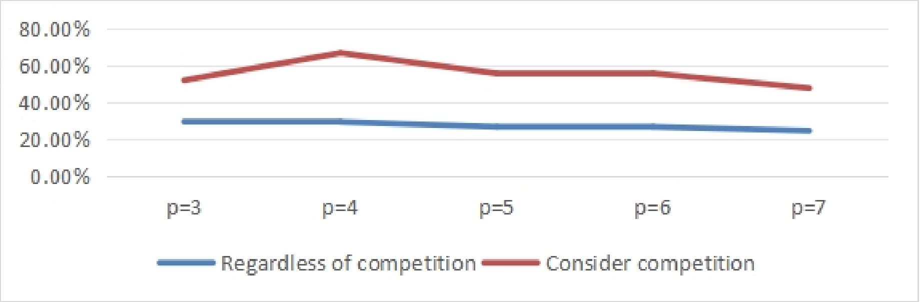

Fig. 2. Comparison of average capacity utilization under different number of hubs for the leader.

In the single allocation hub-spoke network, considering the followers' behavior can lead to a significant improvement in the capacity utilization rate of the leader's hub. Specifically, as shown in Fig.2, this improvement ranges from at least 20.97% to as much as 39.05%. In contrast to when the leader does not consider the followers' behavior. The observed increase in capacity utilization can be attributed to both a reduction in build costs for the leader's hub and an expansion of market share.

Table 2. Hub allocation scheme for Leader and Follower.

p	r	Regardless of competition		Competitive considerations	
		Leader location scheme	Follower location scheme	Leader location scheme	Follower location scheme
3	3	4,7,8	4,7,9	4,9,11	4,9,11
4	4	1,4,7,8	4,7,8,9	1,4,7,9	1,4,7,9
5	5	1,4,7,8,9	1,4,7,8,9	1,4,7,8,9	1,4,7,8,9
6	6	1,4,7,8,9,11	1,4,7,8,9,11	1,2,4,7,8,9	1,2,4,7,8,9
7	7	1,3,4,7,8,9,11	1,4,6,7,8,9,11	1,2,4,5,7,8,9	1,2,4,5,7,8,9

Table 2 records the hub location results in a single allocation hub-and-spoke network with the leader considering and not considering the competitive behavior of the followers. By comparing the observations, whether the leader considers the impending competition will affect his or her own decision and, in turn, the decision of the followers. Combining Figure.3 and observing the number of times each node is selected in Table 2, it can be found that in all cases, nodes 4 and 9 are chosen by the leader and the followers the most. This is because, relative to the other nodes, nodes 4 and 9 have a relative advantage in traffic and location. On the contrary, nodes 10, 12, and 14 have never been chosen, as they are too far away from other nodes and have poor traffic flow, leading the leader and the followers to be unwilling to choose them as hub sites.

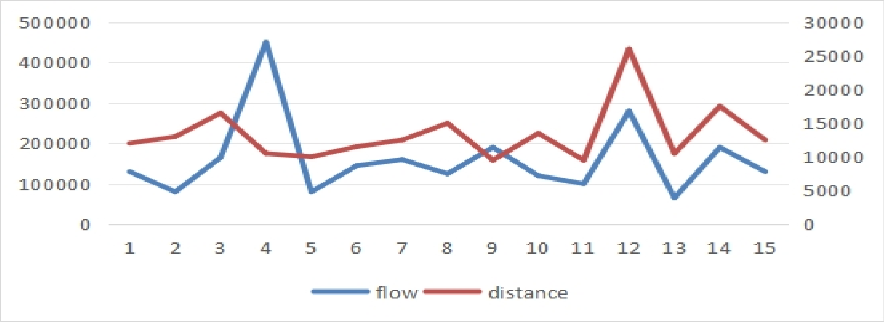

Fig. 3. The flow of each node and its distance from the rest of the nodes.

5 CONCLUSIONS

Building upon existing literature, this study extends the classical single allocation hub-and-spoke network model to the hub competition location model that incorporates capacity decisions and reliability. Subsequently, a variable neighborhood search algorithm is designed for solving the problem. Computational experiments using the CAB dataset demonstrate the necessity of researching hub location problems considering competition.

By observing and summarizing the experimental data, this study ultimately concludes:(1) The number of hubs affects both leader's hub location plan and overall service cost in the single allocation hub-and-spoke networks. An increase in the number of hubs does not completely change the original hub location plan. Generally, it adds other hubs to the original plan. However, an increase in the number of hubs significantly reduces overall service costs, thereby facilitating market share acquisition. (2) Competitive strategies significantly impact leader's hub location plans, capacity decisions, and market share in the single allocation hub-and-spoke networks. Hub location plans considering competitive strategies differ greatly from those without consideration of competitive strategies; furthermore, considering competitive strategy can greatly improve capacity utilization rates and increase market share. (3) Node selection as a hub point exhibits significant correlation with its flow level as well as geographical position. Nodes with higher flow levels and more central geographical positions are associated with higher priority for selection as hubs points.

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