Price of Information Asymmetry in Labor Market: Competitive Equilibrium Wage and Total Surplus Loss

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Abstract. Information asymmetry about workers' productivity between workers and employers usually exists and produces total surplus loss in labor market. A new concept — the price of information asymmetry (PoIA) — is proposed to measure the total surplus loss caused by information asymmetry in labor market. Comparative static analysis of the PoIA with respect to key coefficients brings forth a wealth of (but may conflicting) measures to reduce the total surplus loss in labor market.

Keywords: Labor Market, Total Surplus, Competitive Equilibrium, Price of Information Asymmetry (PoIA).

1 INTRODUCTION

To measure the degree to which the efficiency of a system is reduced due to the selfish behavior of its participants, Koutsoupias and Papadimitriou [1], [2] and Papadimitriou [3] proposed the concept of "Price of Anarchy" (PoA), which characterizes the efficiency loss of a system caused by selfish behavior of its participants. The PoA is a counterpart to the notion of approximation ratio in algorithm design and has been used in many scenarios, such as supply chains[4, 5], auction design[6] and transportation networks[7], to measure these systems' performances.

Labor market functions through the interaction of workers who are the suppliers of labor services and employers who are the demanders of labor services[8]. In many real-life situations, employers do not necessarily know how hard workers are working or how productive they are. Drawing on the core idea of the *price of anarchy* (PoA), we propose a new concept —the *price of information asymmetry* (PoIA) — to measure the total surplus loss caused by information asymmetry in labor market.

The paper is organized as follows. In section 2, basic assumptions are introduced. In section 3, the competitive equilibria in labor market under information symmetry and under information asymmetry are characterized. In section 4, the concept of PoIA is introduced. In section 5, comparative static analysis of the PoIA is performed. In section 6, results and discussions are summarized.

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2 BASIC ASSUMPTIONS

All employers in labor market are homogeneous, are always willing to employ workers who accept being employed, are risk neutral and pursue to maximize their own profits. And, all workers in labor market are not homogeneous but heterogeneous, are risk neutral and pursue to maximize their own profits.

All employers are receivers of the price of product (or service) produced by workers in labor market. For simplicity, the price of one unit of product (or service) is assumed to be one unit of currency.

The heterogeneity of a worker is reflected by his productivity level. Let θ denote the productivity of worker. A worker employed by an employer could produce θ units of product (or service) for the employer per unit of time. The coefficient θ is considered as the type of worker and follows the uniform distribution in $\left[\underline{\theta},\bar{\theta}\right]$, where $0<\underline{\theta}<\bar{\theta}<\infty$. Let $f(\theta)$ denote the probability density function of θ in $\left[\underline{\theta},\bar{\theta}\right]$. Let $\mathrm{E}[\theta]$ denote the expected productivity of worker.

$$f(\theta):=\begin{cases}\dfrac{1}{\bar{\theta}-\underline{\theta}}, & \theta\in\left[\underline{\theta},\bar{\theta}\right]\\ 0, & \theta\notin\left[\underline{\theta},\bar{\theta}\right]\end{cases} \tag{1}$$

$$\mathrm{E}[\theta]:=\int_{\underline{\theta}}^{\bar{\theta}}\theta f(\theta)d\theta=\frac{\underline{\theta}+\bar{\theta}}{2} \tag{2}$$

Let $r(\theta)$ denote the opportunity cost of a worker whose productivity is θ. And, the opportunity cost function is assumed to be a positive affine transformation of θ. That is, $r(\theta):=\alpha\theta$, where $\alpha\in(0,1)$ is the augmented coefficient, such that $r(\theta)\le\theta$.

All workers independently choose whether to accept being employed and always accept being employed by employers whose wages are not lower than their opportunity costs. Let w denote the wage paid by an employer to a worker. Let $\Theta(w)$ denote the set of worker's productivities which are employed.

$$\Theta(w):=\left\{\theta\in\left[\underline{\theta},\bar{\theta}\right]\middle|r(\theta)\le w\right\}=\begin{cases}\left[\underline{\theta},\dfrac{w}{\alpha}\right], & w\in\left[\alpha\underline{\theta},\alpha\bar{\theta}\right]\\ \left[\underline{\theta},\bar{\theta}\right], & w\in\left[\alpha\bar{\theta},+\infty\right)\end{cases} \tag{3}$$

Let n denote the total number of workers in labor market. Let $\hat{n}(w)$ denote the number of workers who accepts being employed when the wage paid by employers is w. Based on equ. (3), $\hat{n}(w)$ is a continuous function of the coefficient w.

$$\hat{n}(w):=\begin{cases}\dfrac{\left(\dfrac{w}{\alpha}-\underline{\theta}\right)n}{\bar{\theta}-\underline{\theta}}, & w\in\left[\alpha\underline{\theta},\alpha\bar{\theta}\right]\\ n, & w\in\left[\alpha\bar{\theta},+\infty\right)\end{cases} \tag{4}$$

Let $\mathrm{E}\left[\theta \middle| \theta \in \Theta(w)\right]$ denote the expected productivity of worker who accepts being employed when the wage paid by employers is w.

$$\mathrm{E}\left[\theta \middle| \theta \in \Theta(w)\right] = \mathrm{E}\left[\theta \middle| r(\theta) \le w, \theta \in \left[\underline{\theta}, \bar{\theta}\right]\right] = \begin{cases} \dfrac{\left(\dfrac{w}{\alpha}\right)^2 - \underline{\theta}^2}{2\left(\bar{\theta} - \underline{\theta}\right)}, & w \in \left[\alpha\underline{\theta}, \alpha\bar{\theta}\right] \\ \dfrac{\underline{\theta} + \bar{\theta}}{2}, & w \ge \alpha\bar{\theta} \end{cases} \tag{5}$$

3 COMPETITIVE EQUILIBRIUM IN LABOR MARKET

Labor is a commodity supplied by a worker, in exchange for a wage paid by demanding employers. More specifically, labor of different worker productivity is a different commodity, in exchange for a different wage paid by demanding employers. And, every worker knows his own productivity. However, employers may not know every worker's productivity.

3.1 Competitive Equilibrium in Labor Market Under Information Symmetry

In labor market under information symmetry, it is assumed that productivities (i.e., types) of workers are publicly observable for employers. Then, the wage paid for different worker productivity should be not unified but differentiated.

Definition 1. (Competitive Equilibrium in Labor Market under Information Symmetry)[9] *In a competitive labor market when the productivity of worker is publicly observable, a competitive equilibrium is a vector* $\left(w^{**}, \Theta^{**}\right)$*, where* w^{**} *is the wage paid by employers and* Θ^{**} *is the set of productivities of worker employed by employers, such that*

$$\Theta^{**} = \left[\underline{\theta}, \bar{\theta}\right] \tag{6}$$

$$w^{**} = \theta \tag{7}$$

In a competitive equilibrium under information symmetry, the wage paid by employers to a worker is equal to his productivity. The set of worker productivities employed by employers is $\Theta^{**} = \left\{\theta \in \left[\underline{\theta}, \bar{\theta}\right] \middle| r(\theta) \le \theta\right\} = \left[\underline{\theta}, \bar{\theta}\right]$. This shows that all workers are employed by employers. That is, the number of workers employed by employers is $\hat{n}^{**} = n$. The total surplus of labor market under information symmetry is:

$$TS^{**} = \hat{n}^{**}\int_{\theta \in \Theta^{**}} \left(\theta f(\theta)\right) d\theta + \left(n - \hat{n}^{**}\right)\int_{\theta \notin \Theta^{**}} \left(r(\theta) f(\theta)\right) d\theta = \frac{n\left(\underline{\theta} + \bar{\theta}\right)}{2} \tag{8}$$

3.2 Competitive Equilibrium in Labor Market Under Information Asymmetry

In the case of information asymmetry, it is assumed that employers cannot publicly observe productivities (i.e., types) of workers. That is, employers do not know the productivity of each worker and pay the wage which is independent of the productivity of worker. Therefore, for all types of workers, the wages paid by employers are not differentiated but unified.

Definition 2. (Competitive Equilibrium in Labor Market under Information Asymmetry) [9] *In a competitive labor market when the productivity of worker is unobservable, a competitive equilibrium is a vector* (w^*, Θ^*)*, where* w^* *is the wage paid by employers and* Θ^* *is the set of productivities of worker employed by employers, such that*

$$\Theta^* = \left\{\theta \in \left[\underline{\theta}, \overline{\theta}\right] \middle| r(\theta) \le w^*\right\} \tag{9}$$

$$w^* = \mathrm{E}\left[\theta \middle| \theta \in \Theta^*\right] \tag{10}$$

Under the unified wage w^*, the set of types of workers employed by employers is shown in equ.(9). And the unified wage w^* is equal to the expected productivity of workers employed by the employer, as shown in equ. (10).

Proposition 1. *In the competitive labor market when the productivity of worker is unobservable, (1) if* $\frac{\overline{\theta} + \underline{\theta}}{2\overline{\theta}} < \alpha < 1$*, then there is no competitive equilibrium. (2) if* $0 < \alpha \le \frac{\overline{\theta} + \underline{\theta}}{2\overline{\theta}}$*, then there are two competitive equilibria* (w_1^*, Θ_1^*) *and* (w_2^*, Θ_2^*)*, such that*

$$w_1^* = \alpha^2\left(\overline{\theta} - \underline{\theta}\right) + \alpha\sqrt{\alpha^2\left(\overline{\theta} - \underline{\theta}\right)^2 + \underline{\theta}^2}; \quad \Theta_1^* = \left[\underline{\theta}, \alpha\left(\overline{\theta} - \underline{\theta}\right) + \sqrt{\alpha^2\left(\overline{\theta} - \underline{\theta}\right)^2 + \underline{\theta}^2}\right] \tag{11}$$

$$w_2^* = \frac{\underline{\theta} + \overline{\theta}}{2}; \quad \Theta_2^* = \left[\underline{\theta}, \overline{\theta}\right] \tag{12}$$

Proposition 1 gives possible competitive equilibria in labor market under information asymmetry. Based on Proposition 1, we compute the number of employed workers and the total surplus at possible competitive equilibria.

At the competitive equilibrium (w_1^*, Θ_1^*). According to equ. (4), the number of workers employed by employers is:

$$\hat{n}_1^* = \frac{\left(\frac{w_1^*}{\alpha} - \underline{\theta}\right)n}{\overline{\theta} - \underline{\theta}} = \frac{\left\{\left[\alpha\left(\overline{\theta} - \underline{\theta}\right) + \sqrt{\alpha^2\left(\overline{\theta} - \underline{\theta}\right)^2 + \underline{\theta}^2}\right] - \underline{\theta}\right\}n}{\overline{\theta} - \underline{\theta}} \tag{13}$$

The total surplus in labor market is:

$$TS_1^* = n_1^* \int_{\underline{\theta}}^{\frac{w_1^*}{\alpha}} \left(w_1^* f(\theta)\right) d\theta + \left(n - n_1^*\right) \int_{\frac{w_1^*}{\alpha}}^{\bar{\theta}} \left(r(\theta) f(\theta)\right) d\theta$$
$$= \frac{n\alpha}{\left(\bar{\theta} - \underline{\theta}\right)^2} \left[\frac{w_1^*}{\alpha} \left(\frac{w_1^*}{\alpha} - \underline{\theta} \right)^2 + \left(\bar{\theta} + \frac{w_1^*}{\alpha} \right) \left(\bar{\theta} - \frac{w_1^*}{\alpha} \right)^2 \right] \tag{14}$$

At the competitive equilibrium $\left(w_2^*, \Theta_2^*\right)$. According to equ. (4), the number of workers employed by employers is $\hat{n}_2^* = n$

The total surplus in labor market is:

$$TS_2^* = n_2^* \int_{\theta \in \Theta_2^*} \left(w_2^* f(\theta)\right) d\theta + \left(n - n_2^*\right) \int_{\theta \notin \Theta_2^*} \left(r(\theta) f(\theta)\right) d\theta = \frac{n\left(\underline{\theta} + \bar{\theta}\right)}{2} \tag{15}$$

Obviously, $w_1^* \le w_2^*$ and $\Theta_1^* \subseteq \Theta_2^*$. Comparing equs. (14) and (15), we can find that $TS_2^* \le TS_1^*$. That is,

$$\min\left\{TS^*\right\} = \min\left\{TS_1^*, TS_2^*\right\} = TS_1^* \tag{16}$$

4 PRICE OF INFORMATION ASYMMETRY (POIA) IN LABOR MARKET

In many real-life situations, labor market design is done under information asymmetry. So, the total surplus loss caused by information asymmetry cannot be ignored. Like the core idea of the price of anarchy (PoA)—a concept introduced by Koutsoupias and Papadimitriou [1], [2] and Papadimitriou [3], we propose a new concept —the price of information asymmetry (PoIA) — to measure the total surplus loss caused by information asymmetry in labor market.

Definition 3. (Price of Information Asymmetry in labor market) *The Price of Information Asymmetry* (PoIA) *is the worst-case ratio of the total surplus in labor market under information symmetry and information asymmetry. That is,*

$$\text{PoIA} = \frac{\max\left\{TS^{**}\right\}}{\min\left\{TS^*\right\}} \tag{17}$$

where TS^{**} *is the total surplus in labor market under information symmetry, and* TS^* *is the total surplus in labor market under information asymmetry.*

According to Definition 3, the PoIA is a worst-case bound, and is a proxy for the magnitude of the total surplus loss between under information asymmetry and under information symmetry. The total surplus in labor market under information symmetry is in general no less than that under information asymmetry. That is to say, $TS^{**} \ge TS^*$. Therefore, there always exists that $\text{PoIA} \ge 1$. The further away the PoIA is from 1, the greater the total surplus loss caused by information asymmetry. On the contrary, the closer the PoIA is to 1, the smaller the total surplus loss caused by information asymmetry.

According to equ. (16), the PoIA in labor market can be calculated based on equs. (8) and (14):

$$\text{PoIA} = \frac{TS^{**}}{TS_1^*} = \frac{\left(\underline{\theta}+\overline{\theta}\right)\left(\overline{\theta}-\underline{\theta}\right)^2}{2\alpha\left[\frac{w_1^*}{\alpha}\left(\frac{w_1^*}{\alpha}-\underline{\theta}\right)^2+\left(\overline{\theta}+\frac{w_1^*}{\alpha}\right)\left(\overline{\theta}-\frac{w_1^*}{\alpha}\right)^2\right]} \tag{18}$$

where the equilibrium wage w_1^* is expressed in equ. (11).

5 COMPARATIVE STATIC ANALYSIS OF THE POIA IN LABOR MARKET

For the PoIA in equ. (18), we conduct comparative static analysis for coefficients $\underline{\theta}$, $\overline{\theta}$ and α. And, the numerical analysis algorithm is implemented in MATLAB 2023b.

5.1 Comparative Static Analysis of the Lower Bound on Worker's Productivity

Given $\overline{\theta}=2$ and $\alpha=0.75$, the variation curves of PoIA and $\partial\text{PoIA}/\partial\underline{\theta}$ about $\underline{\theta}\in[1,1.99]$ are plotted in Fig. 1 and Fig. 2, respectively.

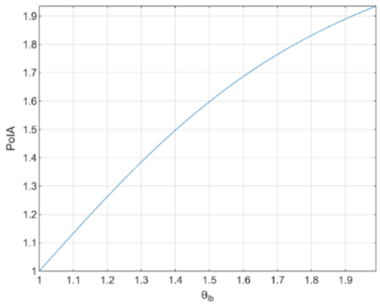

Fig. 1. The curve of PoIA about $\underline{\theta}$

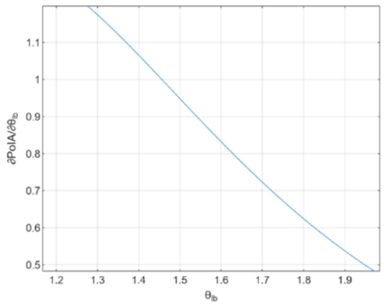

Fig. 2. The curve of $\partial\text{PoIA}/\partial\underline{\theta}$ about $\underline{\theta}$

We find that the lower bound of worker's productivity has diametrically opposed effects on the PoIA and its change rate, respectively. At this point, a dilemma arises. In order to reduce the total surplus loss, it is necessary to reduce the lower bound of worker's productivity. However, in order to reduce the change rate of the total surplus loss, it is necessary to increase the lower bound of worker's productivity.

5.2 Comparative Static Analysis of the Upper Bound on Worker's Productivity

Given $\underline{\theta}=1$ and $\alpha=0.75$, the variation curves of PoIA and $\partial\text{PoIA}/\partial\overline{\theta}$ about $\overline{\theta}\in[1.01,2]$ are plotted in Fig. 3 and Fig. 4, respectively.

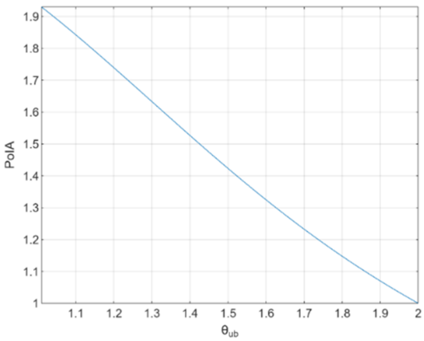

Fig. 3. The curve of PoIA about $\bar{\theta}$

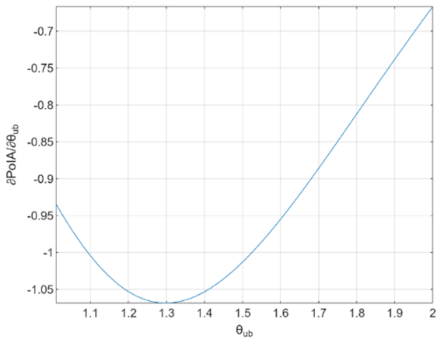

Fig. 4. The curve of $\partial\text{PoIA}/\partial\bar{\theta}$ about $\bar{\theta}$

We find that the upper bound of worker's productivity has different effects on the PoIA and its change rate, respectively. At this point, a dilemma may arise. In order to reduce the total surplus loss, it is necessary to increase the upper bound of worker's productivity. However, in order to reduce the change rate of total surplus loss, it is necessary to take flexible measures according to the range of the upper bound of worker's productivity. When the upper bound of worker's productivity is at a low level, in order to reduce the change rate of total surplus loss, it is necessary to increase the upper bound of worker's productivity. When the upper bound of worker's productivity is at a high level, in order to reduce the change rate of total surplus loss, it is necessary to decrease the upper bound of worker's productivity.

5.3 Comparative Static Analysis of the Augmented Coefficient of Worker's Opportunity Cost Function

Given $\underline{\theta}=1$ and $\bar{\theta}=2$, the variation curves of PoIA and $\partial\text{PoIA}/\partial\alpha$ with respect to $\alpha\in[0.01,0.75]$ are plotted in Fig. 5 and Fig. 6, respectively.

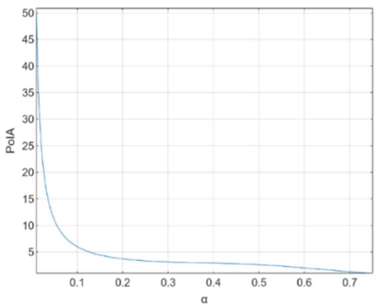

Fig. 5. The curve of PoIA about α

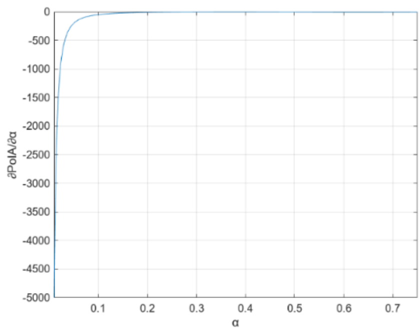

Fig. 6. The curve of $\partial\text{PoIA}/\partial\alpha$ about α

We find that the augmented coefficient of worker's opportunity cost has diametrically opposed effects on the PoIA and its change rate, respectively. At this point, a dilemma arises. In order to reduce the total surplus loss, it is necessary to increase the augmented coefficient of worker's opportunity cost, that is, to increase worker's opportunity cost. However, in order to reduce the change rate of the total surplus loss, it is necessary to reduce the augmented coefficient of worker's opportunity cost, that is, to reduce worker's opportunity cost.

6 SUMMARY

Labor market design must often be done under information asymmetry. To measure the degree to which the total surplus in labor market is reduced due to information asymmetry, a new concept — the Price of Information Asymmetry (PoIA) — is proposed. The PoIA is a worst-case bound and is a proxy for the magnitude of the total surplus loss caused by information asymmetry in labor market. The further away the PoIA is from 1, the greater the total surplus loss caused by information asymmetry. The closer the PoIA is to 1, the smaller the total surplus loss caused by information asymmetry.

Comparative static analysis of the PoIA with respect to key coefficients brings forth a wealth of (but may conflicting) measures to reduce the total surplus loss in labor market. (1) In order to reduce the PoIA, it is necessary to reduce the lower bound of worker's productivity. However, in order to reduce the change rate of the PoIA with respect to the lower bound, it is necessary to increase the lower bound. (2) In order to reduce the PoIA, it is necessary to increase the upper bound of worker's productivity. However, in order to reduce the change rate of the PoIA with respect to the upper bound, it is necessary to take flexible measures according to the range of the upper bound. (3) In order to reduce the PoIA, it is necessary to increase the augmented coefficient of worker's opportunity cost. However, in order to reduce the change rate of the PoIA with respect to the augmented coefficient, it is necessary to reduce the augmented coefficient of worker's opportunity cost.

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