Intuitionistic Fuzzy Multi-attribute Decision-making Based on the New Entropy and Improved TOPSIS

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Abstract. Given the limitations of current research on intuitionistic fuzzy entropy, which often overlook the hesitancy and uncertainty degrees, this paper introduces a novel intuitionistic fuzzy entropy that accounts for both deviation and hesitancy degrees. Subsequently, a multi-attribute decision-making model is developed, incorporating this new entropy and an enhanced TOPSIS method. The attribute weights are derived using both the entropy weight method and an optimal model that minimizes entropy. To improve the TOPSIS method, grey relational analysis is employed instead of the traditional distance from the positive-negative ideal solution, measuring the closeness of alternatives to these ideal solutions. Finally, two examples are provided to demonstrate the effectiveness of our proposed method.

Keywords: intuitionistic fuzzy sets; intuitionistic fuzzy entropy; improved TOPSIS; grey correlation.

1 INTRODUCTION

Comparing to real numbers, fuzzy sets (FS) introduced by Zadeh has proved to be a better tool for describing the fuzziness of objective things. The fuzzy set falls short in providing a comprehensive portrayal of all relevant information for a specific decision problem in an uncertain environment[1]. To resolve this issue, Atanassov devised the membership, non-membership, and hesitation functions, representing the decision maker's levels of support, opposition, and hesitation, respectively. These innovations led to the broader concept of intuitionistic fuzzy sets (IFS)[2]. In order to measure uncertain information in IFS, intuitionistic fuzzy entropy (IFE) based on entropy has been proposed by Szmidt. Due to the hesitant information in IFS, the IFE should contain uncertainty degree and unknown degree while expressing the fuzziness of IFS. An approach to IFE is centered on the subtraction of the non-membership degree from the membership degree (MD)[3]. Exponential function is utilized to establish the measure of IFE respectively[4] . However, these entropy measures become ineffective when the difference between the MD and the non-MD equals that of the other. For example, $A = \{\langle x_i, 0.5, 0.3\rangle\}$ and $B = \{\langle x_i, 0.6, 0.4\rangle\}$ are two IFSs, the entropy comparison result calculated by[3] is: $E(A) = E(B)$. But, apparently, the fuzziness of A is larger than B.

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In order to consider the comprehensive information in IFS, MD, non-MD and hesitancy degree (HD) are defined by the IFE[5]. Trigonometric function is applied to IFE in[6]. However, these entropy still cannot measure some IFSs. For example, $A = \{\langle x_i, 0.23,0.54\rangle\}$ and $B = \{\langle x_i, 0.14,0.522\rangle\}$are two IFSs, the entropy comparison result calculated by[7] is: $E(A) < E(B)$, which proves that the fuzziness of A is less than B. Nonetheless, this outcome appears illogical since a closer proximity between the MD and non-MDs of IFSs indicates a higher degree of fuzziness. Thus $E(A) > E(B)$ is the correct result. For example, $A = \{\langle x_i, 0.17,0.58\rangle\}$ and $B = \{\langle x_i, 0.06,0.53\rangle\}$are two IFSs, the entropy comparison result calculated by[6] is: $E(A) > E(B)$. The result which shows A has high fuzziness degree than B is unreasonable. From the view of difference between MDs, non-MDs and HDs, $E(A) < E(B)$ is the correct result.

Therefore, considering the drawbacks of history research, this paper proposes a novel IFE which not only takes the difference between the MD and non-MD into account, but also includes the HD in the entropy measure.

As to decision making models with entropy, lots of achievements have been made. A new algorithm is presented for analyzing large and complex datasets based on fuzzy entropy and support vector machine[8]. An iterative Clustering around Latent Variables-based objective entropy-weighted TOPSIS method is created to benchmark building energy performance considering multiple factors[9]. Shannon entropy is used to obtain weights and ranked the alternatives with fuzzy TOPSIS[10]. Intuitionistic fuzzy TOPSIS given by[11] is a decision making method with flexible entropy, and applied to supplier selection problem.

From the above research, we can find that TOPSIS and entropy are effective tools for fuzzy decision making. However, the existing TOPSIS method can overlook the common issue of multicollinearity, potentially leading to misleading decisions[9]. So, this paper utilizes grey theory to improve TOPSIS method. And then, we construct a multi-attribute decision making (MADM) model based on the new entropy and improved TOPSIS method. Two illustrative examples and a comparative analysis are provided to demonstrate the effectiveness of the proposed approach. The research pipelines are shown in Fig.1 below.

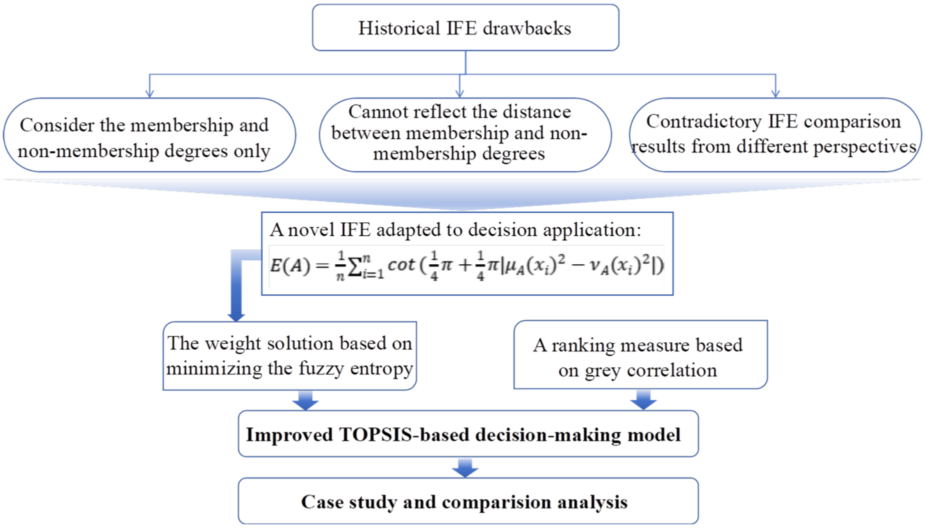

Fig. 1. The research pipelines

2 PRELIMINARIES

Definition 2.1. Consider X as a fixed finite universe set. An IFS defined on X can be denote as $A=\{\langle x_i,\mu_A(x_i),\nu_A(x_i)\rangle | x_i\in X\}$ where $\mu_A(x_i): X\to[0,1]$, $\nu_A(x_i): X\to[0,1]$ denote the MDs and non-MDs of $x_i\in A$, respectively, and satisfy the condition that $0\le\mu_A(x_i)+\nu_A(x_i)\le 1$ for $\forall x\in X$. Besides, $\pi_A(x)=1-\mu_A(x_i)-\nu_A(x_i)$ is declared as the HD.

The complimentary set of A is $A^c=\{\langle x_i,\nu_A(x_i),\mu_A(x_i)\rangle | x_i\in X\}$. If there is only one element in X, then A can be called the intuitionistic fuzzy number (IFN) , which is often denoted in bracketed form: $A=(\mu_A,\nu_A)$.

Definition 2.2. Consider A, B as two IFSs on X. Some oprators and operations below can be given for the IFSs.

(1) $A\subset B \Leftrightarrow \forall x\in X, \mu_A(x_i)\le\mu_B(x_i), \nu_A(x_i)\ge\nu_A(x_i)$

(2) $A=B\Leftrightarrow A\subset B$ and $B\subset A$

(3) $A\cap B=\{\langle x,\mu_A(x_i)\wedge\mu_B(x_i),\nu_A(x_i)\wedge\nu_B(x_i)\rangle | x\in X\}$

(4) $A\cup B=\{\langle x,\mu_A(x_i)\vee\mu_B(x_i),\nu_A(x_i)\vee\nu_B(x_i)\rangle | x\in X\}$

(5) $A+B=\{\langle x,\mu_A(x_i)+\mu_B(x_i)-\mu_A(x_i)\mu_B(x_i),\nu_A(x_i)\nu_B(x_i)\rangle | x\in X\}$

(6) $A\cdot B=\{\langle x,\mu_A(x_i)\cdot\mu_B(x_i),\nu_A(x_i)+\nu_B(x_i)-\nu_B(x_i)\cdot\nu_A(x_i)\rangle | x\in X\}$

Definition **2.3.** Define $E: IFS(X)\to[0,1]$ as an IFE if it satisfies

(1) $E(A)=0$ if A is a crisp set;

(2) $E(A) = 1$ if $\mu_A(x_i) = \nu_A(x_i)$ for $\forall x \in X$;

(3) $E(A) = E(A^c)$ for $\forall x \in IFS(X)$;

(4) $E(B) \leq E(A)$ if $\mu_B(x_i) \leq \mu_A(x_i)$ and $\nu_A(x_i) \leq \nu_B(x_i)$ for $\mu_A(x_i) \leq \nu_A(x_i)$ or $\mu_B(x_i) \geq \mu_A(x_i)$ and $\nu_A(x_i) \geq \nu_B(x_i)$ for $\mu_A(x_i) \geq \nu_A(x_i)$.

3 EXISTING IFE MEASURES AND A NEW ENTROPY

Example 1.The IFE defined by[12] is

$$E^1(A) = \frac{1}{n}\sum_{i=1}^{n}\left[(\sqrt{2}\cos\frac{\mu_A(x_i)-\nu_A(x_i)}{4}\pi - 1) \times (\sqrt{2}+1)\right] \tag{1}$$

The IFE defined by[13] is

$$E^2(A) = 1 - \frac{1}{n}\sum_{i=1}^{n}|\mu_A(x_i) - \nu_A(x_i)| \tag{2}$$

The IFE defined by[4] is

$$E^3(A) = \frac{1}{n(\sqrt{e}-1)}\sum_{i=1}^{n}\left[\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}e^{1-\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}} + \frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}e^{1-\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}} - 1\right] \tag{3}$$

Where $x_i \in X, i = 1,2,\ldots,n$, the same below.

It is unreasonable for those measures that just consider the MDs and non-MDs. The entropy values of $A_1 = (0.5,0.3)$ and $A_2 = (0.3,0.1)$ obtained by Eqs.(1) to (3) respectively are the same, which is unreasonable because on condition that the differences between the MDs and non-MDs are the same, larger HD means higher uncertainty and fuzziness degree, namely, $\mathrm{E}(A_2) > \mathrm{E}(A_1)$.

Example 2. The IFE defined by[3] is

$$E^4(\mathrm{A}) = \frac{1}{n}\sum_{i=1}^{n}(\mu_A(x_i)\log_2\frac{\mu_A(x_i)}{\frac{1}{2}[\mu_A(x_i)+\nu_A(x_i)]} + (1-\mu_A(x_i))\log_2\frac{1-\mu_A(x_i)}{1-\frac{1}{2}[\mu_A(x_i)+\nu_A(x_i)]}) \tag{4}$$

The IFE proposed by[7] is

$$E^5(\mathrm{A}) = \frac{1}{n}\sum_{i=1}^{n}\frac{1-|\mu_A(x_i)-\nu_A(x_i)|^2+\pi_A^2(x_i)}{2} \tag{5}$$

Let $\mathrm{A}_1 = (0.25,0.55)$ and $\mathrm{A}_2 = (0.18,0.53)$ be two IFSs. By Eq. (4) and (5), we obtain $E_{JZ}(\mathrm{A}_1) = 0.071 < E_{JZ}(\mathrm{A}_2) = 0.107$ and $E_G(\mathrm{A}_1) = 0.475 < E_G(\mathrm{A}_2) = 0.4808$, which indicates that the fuzziness of A_1 is less than A_2. Unfortunately, the calculation result is unreasonable because the closer the distance between MDs and non-MDs of IFSs, the higher the fuzziness degree. Thus $E(\mathrm{A}_1) > E(\mathrm{A}_2)$ is the correct result.

Example 3. The IFE defined by[6] is

$$E^6(\mathrm{A}) = \frac{1}{n}\sum_{i=1}^{n}\cos(\frac{\mu_A(x_i)-\nu_A(x_i)}{2(1+\pi_A(x_i))}\pi) \tag{6}$$

The IFE defined by[14] is

$$E^7(\mathrm{A}) = \frac{1}{n}\sum_{i=1}^{n}\frac{1-\max\{\mu_A(x_i),\nu_A(x_i)\}}{1-\min\{\mu_A(x_i),\nu_A(x_i)\}} \tag{7}$$

Where the max{} and min{} function can output the maximum and minimum element in the given set.

Consider two IFSs $A_1 = (0.14, 0.54)$ and $A_2 = (0.08, 0.525)$. By Eqs. (6) to (7), we obtain, $E^6(\mathrm{A}_1) = 0.888 > E^6(\mathrm{A}_2) = 0.877$, $E^7(\mathrm{A}_1) = 0.534 > E^7(\mathrm{A}_2) = 0.516$. The result is unreasonable because it shows A_1 has high fuzziness degree than A_2. However, from the perspective of difference between MDs, non-MDs and HDs, $E(\mathrm{A}_1) < E(\mathrm{A}_2)$ is the correct result.

From the above analysis, it can be seen that existing IFE measurements are difficult to meet the needs of practical decision application, especially in the multi-attribute decision problem considering uncertainty. In this paper, we define a new IFE.

Definition 3.1. For an IFS $\mathrm{A} = \{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle | x_i \in X\}$, the IFE of A is given below.

$$E(A) = \frac{1}{n}\sum_{i=1}^{n} cot(\frac{1}{4}\pi + \frac{1}{4}\pi|\mu_A(x_i)^2 - \nu_A(x_i)^2|) \tag{8}$$

We use an example to prove the new IFE has overcome the disadvantages in history research.

Example 4. Let $A = \{\langle 5,0.1,0.7\rangle, \langle 6,0.3,0.6\rangle, \langle 7,0.4,0.5\rangle, \langle 8,0.7,0.1\rangle, \langle 9,0.9,0\rangle\}$ be an IFS on $X = \{5,6,7,8,9\}$ for risk level assessment. $\sqrt{A}$, A, A^2, A^3 and A^4 may be viewed as "light", "severe", "farely severe", "very severe" and "extremely severe" respectively. Therefore, the IFS entropy order for them should be: $\mathrm{E}(\sqrt{A}) > \mathrm{E}(\mathrm{A}) > \mathrm{E}(A^2) > \mathrm{E}(A^3) > \mathrm{E}(A^4)$.

Table 1. The results from different entropy measures

	$\sqrt{A}$	A	A^2	A^3	A^4
E^1	0.6578	0.6660	0.4963	0.3993	0.3565
E^2	0.5220	0.5	0.33	0.3014	0.2740
E^3	0.9298	0.9351	0.8656	0.8226	0.8029
E^4	0.2185	0.2642	0.1975	0.4454	-0.7775
E^5	0.3398	0.348	0.2691	0.2262	0.2102
E^6	0.6856	0.7190	0.5578	0.4509	0.4011
E^7	0.4518	0.4342	0.2767	0.2797	0.2610
E this paper	0.5071	0.5058	0.3514	0.3296	0.3074

As depicted in Table 1, solely E^2 and E adhere to the logical correlation. In contrast to E^2, the intuitionistic fuzzy entropy (IFE) advocated in this study incorporates not just the membership degrees (MDs) and non-membership degrees (non-MDs), but also the hesitant degree (HD), providing a more nuanced portrayal of fuzziness and yielding a more rational outcome.

4 THE IMPROVED TOPSIS-BASED DECISION-MAKING MODEL

The TOPSIS method ranks alternatives based on their closeness degrees, determined by the distance between the alternatives and ideal solutions. Although widely used across various fields, this method has two significant limitations[15]. Firstly, the need for predefined attribute weights restricts its applicability and appears unreasonable. Secondly, the closeness degree accounts only for the distance between alternatives and ideal solutions, ignoring their correlation. Additionally, if one attribute's distance is disproportionately large, the influence of other attributes on the ranking order diminishes.

To address these shortcomings, we propose an enhanced TOPSIS-based decision-making model, incorporating the novel IFE detailed in Eq. (8). Fig. 2 presents the model framework, which consists of four steps explained in the following sections.

4.1 Initial Decision Matrix Construction

Let assume a MADM problem that consists of the sets of m alternatives $X = \{x_1, x_2, \dots, x_m\}$ and n criteria $C = \{c_1, c_2, \dots, c_n\}$. The evaluation value of alternative x_i regarding to c_j is represented by an IFN $a_{ij} = (\mu_{ij}, \nu_{ij})$, where μ_{ij} and ν_{ij} indicate the satisfactory and dissatisfactory degrees of x_i regarding to c_j respectively, based on the condition $0 \le \mu_{ij}, \nu_{ij} \le 1$ and $0 \le \mu_{ij} + \nu_{ij} \le 1$. Then the initial decision matrix $D = (a_{ij})_{m \times n}$ according to the evaluation information of experts can be obtained, the weight vector $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ with the condition $\omega_j \ge 0, \sum_{j=1}^{n} \omega_j = 1$.

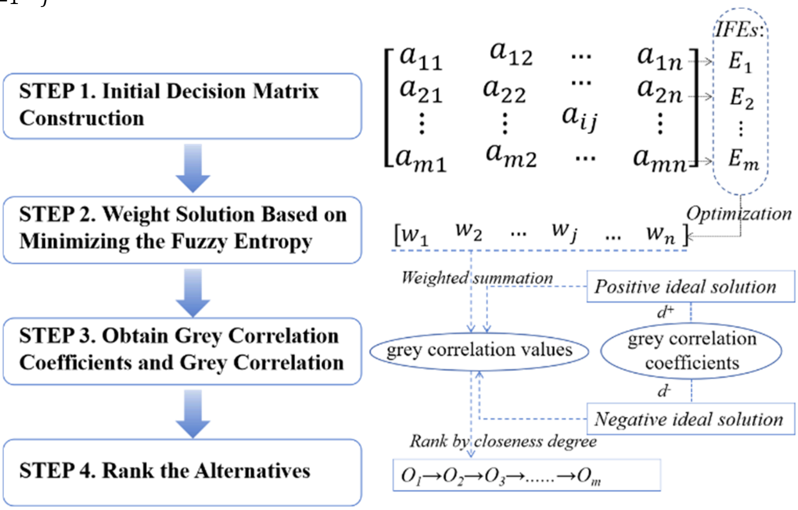

Fig. 2. An improved TOPSIS-based decision-making model framework

4.2 Solving Weights Based on Minimizing the Fuzzy Entropy

Due to the objective environment and subjective cognition, the evaluation values are represented by IFNs. A smaller IFE value indicates more qualitative information and less fuzziness, signifying a better alternative. Therefore, we construct the weight optimization model with the objective function aimed at minimizing the fuzzy entropy.

(1) Solving the weight with constraints

$$\begin{cases} min\, E = \sum_{i=1}^{m}\sum_{j=1}^{n}\omega_j\, cot(\frac{1}{4}\pi + \frac{1}{4}\pi|\mu_{ij}^2 - v_{ij}^2|) \\ s.t\sum_{j=1}^{n}\omega_j = 1, \omega_j \in H \end{cases} \tag{9}$$

where H is the constraint information of weights.

(2) Solving the weight without constraints

$$\omega_j = (1 - \frac{1}{m}\sum_{i=1}^{m}E(a_{ij}))/(n - \frac{1}{m}\sum_{j=1}^{n}\sum_{i=1}^{m}E(a_{ij})) \tag{10}$$

The attribute weights are calculated according to Eq. (9) and (10).

4.3 Obtain Grey Correlation Coefficients and Grey Correlation

This paper applies Deng's grey correlation which calculates the grey correlation coefficient of alternatives to the ideal solutions with respect to criteria and then obtains the closeness degree. The grey correlation represents the relevancy between alternatives and ideal solutions more directly than the distance measure and identification coefficient ρ reduces the influence of exceptional attributes.

The grey correlation coefficients of x_i to the positive and negative ideal solutions regarding to c_j are represented as ξ_{ij}^+ and ξ_{ij}^-, respectively. The positive ideal solution (PIS) and negative ideal solution (NIS) are calculated as follows.

$$\xi_{ij}^+ = \frac{\min_i\{\min_j\{d(a_{ij},a_j^+)\}\} + \rho\max_i\{\max_j\{d(a_{ij},a_j^+)\}\}}{d(a_{ij},a_j^+) + \rho\max_i\{\max_j\{d(a_{ij},a_j^+)\}\}} \tag{11}$$

$$\xi_{ij}^- = \frac{\min_i\{\min_j\{d(a_{ij},a_j^-)\}\} + \rho\max_i\{\max_j\{d(a_{ij},a_j^-)\}\}}{d(a_{ij},a_j^-) + \rho\max_i\{\max_j\{d(a_{ij},a_j^-)\}\}} \tag{12}$$

Where $a_j^+ = (\max_i\{\mu_{ij}\}, \min_i\{v_{ij}\})$ and $a_j^- = (\max_i\{v_{ij}\}, \min_i\{u_{ij}\})$ are the PIS and NIS. Usually, $\rho = 0.5$. $d(a_{ij}, a_j^+)$ and $d(a_{ij}, a_j^-)$ are the Hamming distances between alternatives and ideal solutions denoted by

$$d(a_{ij}, a_j^+) = \frac{1}{2}(|\mu_{ij} - \mu_j{}^+| + |v_{ij} - v_j{}^+| + |\pi_{ij} - \pi_j{}^+|) \tag{13}$$

$$d(a_{ij}, a_j^-) = \frac{1}{2}(|\mu_{ij} - \mu_j{}^-| + |v_{ij} - v_j{}^-| + |\pi_{ij} - \pi_j{}^-|) \tag{14}$$

We obtain the grey correlation values of x_i to the ideal solutions described as follows.

$$\gamma_i^+ = \frac{1}{n}\sum_{j=1}^{n} \xi_{ij}^+ \omega_j \quad (15)$$

$$\gamma_i^- = \frac{1}{n}\sum_{j=1}^{n} \xi_{ij}^- \omega_j \quad (16)$$

4.4 Rank the Alternatives

Using Eq. (17), we can obtain the closeness degree between x_i and the PIS. Therefore, the alternatives are ranked according to the closeness degrees and the alternative with largest closeness degree is the optimal one.

$$o_i = \frac{\gamma_i^+}{\gamma_i^+ + \gamma_i^-}, i = 1,2,\dots,m \quad (17)$$

5 CASE STUDY AND COMPARISON ANALYSIS

Example 5.1. In this paper, we analyze and discuss the example described in[16]. Assume the issue of hiring a teacher for an educational institution. Five attributes are considered in the decision making problem, including attitude c_1, communication skills c_2, moral character c_3, experience c_4 and teaching ability c_5. The intuitionistic fuzzy evaluation information of four candidates x_1, x_2, x_3, x_4 through questionnaire is shown in Table 2. The constraints of five attributes are $\omega_1 > 0.15$, $\omega_2 > 0.2$, $\omega_1 < 0.15\omega_5$, $\omega_4 > 0.2\omega_2$, $0.1 < \omega_3 - \omega_4 < 0.3$ and $\omega_3 - \omega_1 < \omega_5 - \omega_2$.

Table 2. The evaluation information

	c_1	c_2	c_3	c_4	c_5
x_1	(0.7,0.2)	(0.6,0.4)	(0.5,0.4)	(0.3,0.4)	(0.4,0.5)
x_2	(0.6,0.1)	(0.8,0.1)	(0.6,0.2)	(0.7,0.1)	(0.5,0.4)
x_3	(0.7,0.1)	(0.7,0.2)	(0.8,0.1)	(0.6,0.3)	(0.8,0.1)
x_4	(0.6,0.2)	(0.6,0.3)	(0.7,0.1)	(0.8,0.1)	(0.7,0.2)

According to Eq. (8), we have the entropy values of five attributes. $E(c_1) = 3.0451$, $E(c_2) = 3.1323$,$E(c_3) = 3.1909$, $E(c_4) = 3.2731$, $E(c_5) = 3.4955$.

Construct the weight solution model.

$$min = 3.0451\omega_1 + 3.1323\omega_2 + 3.1909\omega_3 + 3.2731\omega_4 + 3.4955\omega_5$$

$$st.\begin{cases} \omega_1 > 0.15 \\ \omega_2 > 0.2 \\ \omega_1 < 0.15\omega_5 \\ \omega_4 > 0.2\omega_2 \\ 0.1 < \omega_3 - \omega_4 < 0.3 \\ \omega_3 - \omega_1 < \omega_5 - \omega_2 \\ \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 = 1 \end{cases}$$

Through MATLAB programming, we have $\omega_1 = 0.15$, $\omega_2 = 0.3214$, $\omega_3 = 0.1643$, $\omega_4 = 0.0643$ and $\omega_5 = 0.3$. Then calculate the Hamming distance and obtain the grey correlation matrix according to Eqs. (13) to (16). The grey correlations of alternatives are calculated and the results are $\gamma_1^+ = 0.4647$, $\gamma_2^+ = 0.7021$, $\gamma_3^+ = 0.8795$, $\gamma_4^+ = 0.6816$, $\gamma_1^- = 0.9571$, $\gamma_2^- = 0.5835$, $\gamma_3^- = 0.5049$, $\gamma_4^- = 0.6120$. By Eq. (17) the closeness degrees are $o_1 = 0.3268$, $o_2 = 0.5461$, $o_3 = 0.6352$, $o_4 = 0.5268$. Thus the order of alternatives is $x_3 \succ x_2 \succ x_4 \succ x_1$. The ranking result aligns with that of [16], confirming the feasibility and effectiveness of the proposed model. It is noteworthy that, unlike in[17] where the decision makers' risk attitude is neutral, this paper does not take risk attitude into account. Hence it is totally reasonable that the ranking order is identical with[16].

For further comparison analysis, this paper utilizes the example in[18].

Example 5.2. An equipment manufacture company intends to choose a supplier. In the decision-making process, the five attributes considered are: overall cost of the product c_1, quality of the product c_2, service performance of supplier c_3, supplier's profile c_4 and risk factor c_5. The information of attribute's weights is incomplete. The constraints are $\omega_1 \le 0.3$, $0.1 \le \omega_2 \le 0.2$, $0.2 \le \omega_3 \le 0.5$, $0.1 \le \omega_4 \le 0.3$, $\omega_5 \le 0.4$, $\omega_3 - \omega_2 \ge \omega_5 - \omega_4$, $\omega_4 \ge \omega_1$ and $\omega_3 - \omega_1 \le 0.1$. The alternative suppliers are x_1, x_2, x_3, x_4, x_5. The evaluation information is illustrated in Table 3.

Table 3. The evaluation information of suppliers

	c_1	c_2	c_3	c_4	c_5
x_1	(0.449,0.370)	(0.565,0.162)	(0.705,0232)	(0.730,0.170)	(0.646,0354)
x_2	(0.719,0.188)	(0.630,0.232)	(0.448,0.378)	(0.557,0.160)	(0.597,0.192)
x_3	(0.546,0.192)	(0.727,0.182)	(0.641,0.322)	(0.399,0.200)	(0.658,0.192)
x_4	(0.520,0.337)	(0.630,0.100)	(0.539,0.271)	(0.679,0.188)	(0.708,0.198)
x_5	(0.727,0.128)	(0.520,0.299)	(0.619,0.318)	(0.618,0.229)	(0.609,0.120)

Construct the linear optimization model according to Eq. (9). Then we have $\omega_1 = 0.1$, $\omega_2 = 0.1$, $\omega_3 = 0.2$, $\omega_4 = 0.25$, $\omega_5 = 0.35$. According to the step 2 to 5 and Eq. (17), we can obtain the grey correlations and the closeness degrees. Table 4 shows the ranking results of the Xu's method [18], Liu's method [19], and the proposed method in this paper. .

Table 4. The comparison of results

	$c(x_1)$	$c(x_2)$	$c(x_3)$	$c(x_4)$	$c(x_5)$	The ranking order
This paper	0.5327	0.4720	0.4986	0.5550	0.5688	$x_5 \succ x_4 \succ x_1 \succ x_3 \succ x_2$
Liu	0.5	0.3408	0.4708	0.8604	0.7791	$x_4 \succ x_5 \succ x_1 \succ x_3 \succ x_2$
Xu	0.3674	0.3483	0.3302	0.3851	0.3964	$x_5 \succ x_4 \succ x_1 \succ x_2 \succ x_3$

The worst alternative in Xu's method is different from ours because Xu constructed the weight optimization model by the score function $\mu_{ij} - \nu_{ij}$ without consideration of the influence of the HD, thus the obtained weights are different. However, the weight optimization model in this paper takes both the subjective and objective weight information into account and accords with the practical condition. The weights calculated by Liu's method are the same as ours but the ranking order is different because this paper introduced the improved TOPSIS method which reduced the influence of exceptional attributes with extreme values. Thus, the proposed method is more reliable.

From the above comparison analysis, we can see that

(1) The grey correlation coefficient is more reasonable than the distance to see the relevancy between alternatives and ideal solutions;

(2) The grey correlation is effective to reduce the influence of some exceptional attributes which enlarge the effect to the ranking results due to the extreme values if distance measure is utilized.

6 CONCLUSION.

To measure the fuzziness degree of IFS more precisely, this paper introduces a novel intuitionistic fuzzy entropy (IFE) that considers not only the difference between the MDs and non-MDs but also incorporates the HD into the entropy measure. This new entropy is thus better equipped to describe the uncertainty and unknown degrees of IFS, addressing the limitation of most existing research that only considers the MD and non-MD difference. Building on this, we establish a multi-attribute decision-making method based on the new entropy and an improved TOPSIS method. The attribute weights are determined by minimizing the IFE values. Additionally, we utilize grey correlation instead of the traditional distance between alternatives and ideal solutions to characterize the closeness degrees of alternatives. Finally, the ranking of alternatives is obtained based on the comprehensive closeness degrees.

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