Catastrophe Insurance for Historical Buildings Based on a Panel Quantile Regression Model

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Abstract. In this paper, we investigate the issue of whether insurance companies should provide coverage for historical buildings under specific catastrophic scenarios, particularly as extreme weather events become increasingly frequent. Initially, we briefly analyze the repair data and budget area ratios for historical buildings across the country over recent years. Subsequently, we selected the economic development level (GDP), population density, and repair efficiency from 2018 to 2021 across 28 provincial-level administrative regions in China as conditioning variables. Using a panel quantile regression model, we estimated the repair costs of historical buildings, thereby establishing a historical building repair value assessment model. Based on fundamental insurance theories and in conjunction with the repair value assessment model, we then employed a risk index method—considering both the occurrence of extreme weather events and their impact on the insured regions—to design a catastrophe insurance scheme specifically for historical buildings (targeting extreme weather). This approach aims to ensure the long-term financial health and stability of insurance companies engaged in this business. Finally, we summarize the strengths and weaknesses of the catastrophe insurance model and offer prospects in light of the rapid advancements in artificial intelligence.

Keywords: Panel Quantile Regression Model, Catastrophe Insurance, Historical Building Repair Value Assessment Model

1 Introduction

In recent years, the frequency of extreme weather events globally has been on the rise, with increasing losses caused by floods, hurricanes, earthquakes, wildfires, and other extreme events, profoundly impacting human production and daily life.

Catastrophic risks primarily include natural disasters and man-made disasters. Different organizations and countries set specific standards, such as the amount of loss, the affected population, and the number of deaths, to determine whether an event qualifies as a catastrophe. From an economic perspective, a natural disaster can be defined as a natural incident that disrupts the functioning of the economic system and has a significant negative impact on assets, production factors, output, employment,

A. K. Draman Mud et al. (eds.), *Proceedings of the 2024 5th International Conference on Big Data and Social Sciences (ICBDSS 2024)*, Advances in Computer Science Research 116,
https://doi.org/10.2991/978-94-6463-562-1_36

or consumption[1]. For example, on August 6, 2023, a magnitude 5.5 earthquake in Pingyuan, Shandong, damaged more than 2,900 houses and caused direct economic losses of 240 million yuan[2]. In April 2024, an unprecedented flood in Qingyuan, Guangdong, left the Wenfeng Pagoda in Yingde stranded in the middle of a river, causing varying degrees of damage to local buildings. Extreme weather has become a dilemma that both policyholders and insurance companies must face. The management of risks following a catastrophe primarily relies on disaster relief. One form of relief is purchasing catastrophe insurance, which, through institutional arrangements, disperses the risk and provides economic compensation for property losses and casualties caused by natural disasters such as earthquakes and floods.

Frequent extreme weather events threaten the profitability of insurance companies, making it crucial to study how they can adjust their underwriting methods and scope to enhance system resilience. By employing economic models and big data analysis, companies can more accurately calculate underwriting data and adjust coverage amounts and scopes, thereby improving risk management. Appropriate insurance policies not only provide economic compensation to disaster-stricken areas but also help disperse disaster risks. However, many insurance companies currently face challenges in addressing frequent weather events: their coverage is limited, leaving many regions and potential policyholders without protection, while high-risk policies may lead to bankruptcy. Therefore, accurately assessing underwriting risks in affected areas is essential. The damage to ancient buildings is increasingly severe, and introducing new forms of insurance for these structures could improve the situation, providing economic protection and promoting cultural preservation and development[3].

On August 6, 2024, following the successful UNESCO World Heritage designation of areas like Beijing’ s central axis, Xi Jinping emphasized the importance of protecting China's cultural and natural heritage, calling for these treasures to thrive and shine in the new era. On November 8, 2021, the State Council issued the "14th Five-Year Plan for Cultural Relics Protection and Technological Innovation," marking the first national-level planning for the development of cultural relics[4]. The plan highlights the severe safety situation of cultural relics and the urgent need to enhance protection management and technological innovation capabilities.

In this context, research on ancient building insurance has become particularly important. With the increasing frequency of extreme weather events, ancient buildings of cultural or economic significance face high risks, significant payouts, and low profitability. Due to their historical age, these buildings' resilience to earthquakes and disasters is gradually weakening, and extreme weather not only directly damages ancient structures but may also alter surrounding environments, exacerbating their deterioration. Moreover, the market-based risk transfer and response mechanisms in the field of cultural relic protection still have shortcomings.

To effectively protect ancient buildings, we propose a value reassessment model that comprehensively considers factors such as building materials, age, and historical significance to evaluate replacement costs. In constructing the model, we employ a panel quantile model to analyze the relationship between replacement costs and various variables, reflecting regional heterogeneity. By incorporating information over time, the panel model can effectively explore individual differences across regions and is robust to outliers in the data.However, traditional panel data models may not adequately reflect how independent variables influence the dependent variable at different levels. To address this, Koenker introduced quantile regression into panel models in 2004, analyzing differences in samples at various quantiles to enhance the model's fit to the data[5]. This integrated approach provides an important theoretical basis and practical guidance for the protection of ancient buildings.

Moreover, statisticians and econometricians have continued to advance the research and application of panel quantile models.In 2010, IVAN A. CANAY introduced a moment estimation method for identifying and estimating panel quantile models when the time axis is short[6]. In 2016, Kato and Galvao show that under an asymptotic framework where both the numbers of individuals and time periods grow at the same rate, the fixed-effects estimator for the smoothed objective function has a limiting normal distribution with a bias in the mean[7]. In 2017, Li Shaomin and Ren Yanyan proposed a two-step panel quantile instrumental variable estimation method (2S-IVFEQR), which outperforms the traditional IVFEQR method in terms of parameter estimation accuracy, especially in small sample sizes or long panel data, and has shorter computation times[8]. In 2024, Ren Yanyan, Li Donglin, and Wang Wenyue focused on panel quantile models with a two-dimensional heterogeneous structure, achieving parameter estimation accuracy close to Oracle estimators[9]. These studies have continuously refined the theoretical framework of panel quantile models.

In practical applications, panel quantile models have been widely used in economics and insurance. For example, in 2015, Jia Liwen et al. used a panel quantile model to study the issue of insurance depth in the non-life insurance market[10]. In 2021, Wang Rui analyzed the development background, process, existing scale, and market mechanisms of China’s carbon finance market and empirically studied the factors affecting carbon trading prices using a panel quantile regression model[11]. However, panel quantile regression models are still in the developmental stage, and a unified standard has not yet been established. The optimal estimation methods still require further research.

Due to the incomplete public disclosure of cultural relics data in China, it is challenging to obtain such data. This study selects restoration project expenditures from 2018 to 2021 in 28 provinces, municipalities, and autonomous regions, including Beijing, Hebei, Shanxi, and Inner Mongolia, as the dependent variable. The explanatory variables include economic development level (GDP), population density, number of professional restoration technicians, and the area occupied by cultural relics. In the panel quantile model, due to the significant differences in the unit magnitudes of the dependent variable, we preprocess the data using logarithmic transformation to adjust the data distribution, making it closer to a normal or uniform distribution. The panel quantile model is built on the framework of quantile regression, considering fixed effects across individual and time dimensions, as well as potential heteroscedastic properties. We use a penalized least squares method to estimate the model parameters, minimizing the residual sum of squares to fit the model. This approach is particularly beneficial when the sample's time length is relatively short, as quantile regression fixed effects estimation can enhance the precision of parameter estimates.

Additionally, when the data exhibits a heavy-tailed distribution, the use of a panel quantile model is more appropriate. Once we obtain the data, we can design an appropriate insurance policy in conjunction with insurance principles. We employ instrumental variable methods and the panel quantile model for analysis to ensure the robustness and accuracy of the model.

2 Historical Building Repair Value Assessment Model

2.1 Panel Quantile Regression Model

China's historical buildings can be classified into state-owned and non-state-owned categories, and their insurance issues should be studied within a framework that combines government regulation and market mechanisms. As social resources, historical buildings, whether open to the public or not, primarily rely on funding from the national and local governments, as well as societal contributions, for their preservation. The value of these buildings is largely economic, with their economic value primarily reflected in their physical structure, which can be determined through Historical Building Repair Value Assessment Model.

2.1.1 Assumptions.

To simplify the analysis, the following assumptions are proposed for the cultural heritage restoration cost model:

A-1: It is assumed that for historical buildings of the same grade within the same region, the protective measures and the degree of gradual losses such as daily weathering are the same.

A-2: It is assumed that the economic value of the building is only related to its physical structure.

A-3: The economic value of historical buildings is measured in cash or its equivalent, without considering other measurement scales.

A-4: The restoration cost of historical buildings refers to the cost of reconstruction using original materials, technologies, and techniques as prescribed by national regulations.

A-5: The protection status of historical buildings varies across provinces, autonomous regions, and municipalities, reflecting regional heterogeneity.

A-6: The restoration cost of historical buildings is related to GDP, population density, and restoration efficiency, with restoration efficiency being defined as the restoration area divided by the number of scientific restoration personnel.

A-7: Ancient building conservation unit is treated as the smallest relevant unit for historical buildings and is considered as a whole in the analysis, without discussing its internal structure.

Based on assumptions A-1 to A-7, this paper establishes Historical Building Repair Value Assessment Model and uses this model to price comprehensive insurance for the related historical buildings.

2.1.2 Principle of the Panel Quantile Model.

Under assumption A-5, the panel quantile regression model is applied to analyze data on historical buildings across different regions. This analysis explores the relationship between the protection status of these buildings, maintenance budgets, maintenance areas, restoration efficiency, regional economic development levels, and population density. The paper posits that GDP is the most significant factor influencing restoration costs.

The panel model can extract individual heterogeneity information and estimate individual differences and the coefficients of independent variables by integrating time-dimensional information, which helps in a detailed analysis of the impact of different factors on the value of historical buildings. Given the potential valuation differences across regions, the standard conditional quantile regression method is employed to estimate these bias effects. In 2004, Koenker discussed the conditional quantile model for pure location shift effects in his study, providing a mathematical expression, as follows:

$$Q_{y_t}(\tau \mid x_{it}) = \alpha_i + x_{it}' \times \beta(\tau), \qquad i = 1,2,\cdots,N; \qquad t = 1,2,\cdots,T \tag{1}$$

This approach allows for a more accurate assessment of the effects of various factors on the restoration costs of historical buildings, accommodating the potential regional disparities and ensuring a robust insurance pricing mechanism.

In this model, the parameter α_i represents the individual fixed effects, which do not vary across quantiles. The parameter $\beta(\tau)$ denotes the coefficients of the independent variable x, which vary with different quantiles τ. Subsequently, Koenker applied the penalized least squares method to solve equation ①, and the solution is given by:

$$\left\{[\hat{\beta}(\tau_j,\lambda)]_{j=1}^{J},\ [\hat{\alpha}_i(\lambda)]_{i=1}^{N}\right\} = \underset{(\alpha,\beta)}{argmin} \sum_{j=1}^{J}\sum_{t=1}^{T}\sum_{i=1}^{N} w_j\, \rho_{\tau_j}\left(y_{it} - \alpha_i - x_{it}' \times \beta(\tau_j)\right) + \lambda \sum_{i=1}^{N}|\alpha_i| \tag{2}$$

Compared to the fixed effects model estimated by the least squares method, the panel quantile regression model offers a more flexible framework. The least squares fixed effects model solves for the conditional mean regression equation, where the right-hand side of the equation represents the expected value when the explanatory variables are held constant. In contrast, in the panel quantile regression model, the right-hand side of the equation represents the conditional quantile value rather than the expected value. This allows the model to better capture regional differences in Historical Building Repair Value Assessment Model, while also analyzing the impact of factors such as GDP on restoration costs. According to assumptions A-4 and A-7, the historical building value assessment and restoration cost model can be expressed as equation ②, where the dependent variable is the restoration cost R, with the specific form as follows:

$$Q_{V_{R_{it}}}(\tau \mid GDP_{it}, z_{it}) = \alpha_0(\tau) + \alpha_1(\tau)\cdot GDP_{it} + z_{it}\cdot\beta(\tau) + \varepsilon_{it} \tag{3}$$

$$Q_{V_{R_{it}}}(\tau \mid GDP_{it}, ME_{it}, MD_{it}) = \alpha_0(\tau) + \alpha_1(\tau)\cdot GDP_{it} + \beta_1(\tau)\cdot ME_{it} + \beta_2(\tau)\cdot MD_{it} + \varepsilon_{it} \tag{4}$$

Where $Q_{V_{R_{it}}}$ denotes the restoration costs of historical buildings, ME_{it} denotes the restoration efficiency, GDP_{it} denotes the regional economic development level, MD_{it}represents the population density, and ε_{it} the error term.

Equation ④ provides the relationship between the restoration costs of historical buildings and factors such as regional economic development, population density, and restoration efficiency. Since restoration costs serve as a proxy for assessing the restoration expenses of historical buildings, when detailed restoration data is available, these costs can be converted according to assumption A-4. Subsequently, based on equation ⑤ and relevant data, the restoration costs of historical buildings can be estimated.

$$Q_{V_a}(\tau \mid GDP, ME, MD) = \alpha_0(\tau) + \alpha_1(\tau) \cdot GDP + ME \cdot \beta_1(\tau) + MD \cdot \beta_2(\tau) + \mu \quad (5)$$

where, V_a denotes the estimated restoration costs of historical buildings based on historical restoration data.

2.2 Historical Building Repair Value Assessment Model

2.2.1 Determining Repair Costs.

The restoration costs (or expenses) of historical buildings are influenced by regional characteristics, local economic conditions (GDP), and other factors. Therefore, this study uses the maintenance expenses of cultural heritage sites as the dependent variable, with population density, restoration efficiency, and GDP as the explanatory variables. The relationship between the restoration expenses of historical buildings and these factors across different regions is examined, and the parameters of the historical building value assessment and restoration cost model are quantified.

(1) Budget Area Ratio

First, conduct a brief analysis based on the budget area ratio indicator. The budget area ratio is defined by the following equation ⑥:

$$Budget\ Area\ Ratio = \frac{Project\ Budget}{Restoration\ Area} \quad (6)$$

Here, project budget refers to the budget for conservation and restoration projects of historical conservation units, representing the restoration expenses of the historical buildings. The restoration area is the area that needs to be protected and restored as part of these projects.

We calculated the budget area ratio based on the restoration project budgets and the areas restored during the year, as provided by 28 provincial-level cultural heritage protection units from 2018 to 2021, as shown in Table 1. These values also reflect the average protection and restoration conditions of national-level cultural heritage sites across the country.

Table 1. Budget Area Ratio of 28 Provincial-Level Administrative Regions from 2018 to 2021.

	Budget Area Ratio (10,000 RMB/m^2)			
Area \ Year	2018	2019	2020	2021
Beijing	0.0413	1.8743	23.1358	5.3869
Hebei	0.2503	0.0231	0.1194	0.0401
Shanxi	1.8160	1.4411	0.6034	4.0428
Inner Mongolia	0.0043	0.0036	0.0152	0.0089
Liaoning	0.3998	0.8443	0.6756	0.2043
Jilin	0.0297	0.0091	0.0186	0.0022
Heilongjiang	0.3001	0.1727	0.0014	0.0009
Jiangsu	0.0042	0.0044	0.0043	0.0065
Zhejiang	0.1853	0.2204	0.3701	1.1902
Anhui	0.0039	0.0722	0.1071	0.0080
Fujian	0.3484	0.2751	0.4203	0.4242
Jiangxi	0.0066	0.0085	0.1339	0.5297
Shandong	0.1413	0.3203	5.9994	0.5146
Henan	0.0345	0.5523	2.0727	1.2798
Hubei	0.0172	0.0347	0.1721	0.2015
Hunan	0.0920	0.0699	0.1081	1.8709
Guangdong	3.2365	1.6108	0.7575	0.5775
Guangxi	0.0481	0.0460	0.0630	0.0924
Chongqing	0.3424	0.5034	0.6052	2.6467
Sichuan	0.3391	0.4144	0.0276	1.0396
Guizhou	0.3567	0.1170	0.1435	0.2707
Yunnan	0.0101	0.1949	0.2331	0.2263
Tibet	5.6154	31.6530	66.1397	65.4212
Shaanxi	0.6491	0.3684	0.6288	26.2633
Gansu	0.6990	0.2629	0.0204	6.5156
Qinghai	0.1998	0.0859	0.0364	0.0238
Ningxia	0.2925	0.3361	0.1405	0.7023
Xinjiang	0.0001	0.1786	0.0212	1.0087

(2) Descriptive Analysis

This study utilizes data obtained from the years 2018 to 2021, covering 28 provinces, autonomous regions, and municipalities in China. The data includes the overall budget for protection and restoration projects of national key cultural heritage sites and cultural heritage sites in various regions, along with regional economic development levels, population density, restoration area, and the number of professional technical staff in cultural heritage protection bureaus. Among these, the project budget (Va) is the dependent variable, representing reconstruction costs (or restoration expenses); the regional economic level (GDP) is the independent variable; population density (MD) and restoration efficiency (ME) are control variables. Information on each variable is Variable Description Table provided in Table 2.

Table 2. Variable Description.

Variable	independent variables	Economic Implication	Data Source and Calculation Method
Va	dependent variables	Historical Building Restoration Project Budget (10,000 RMB)	*China City Statistics Yearbook(2019-2022)*[12-15], *China Cultural and Tourism Statistics Yearbook(2019)*[16], *China Cultural Relics and Tourism Statistics Yearbook(2020-2022)*[17-19].
GDP	control variables	Economic Development Level (100 million RMB)	*China City Statistical Yearbook(2019-2022)*[12-15].
ME	control variables	Maintenance efficiency，the workload per repair personnel(m^2/person)	*China City Statistics Yearbook(2019-2022)*[12-15], *China Cultural and Tourism Statistics Yearbook(2019)*[16], *China Cultural Relics and Tourism Statistics Yearbook(2020-2022)*[17-19].
MD	control variables	Population Density (people per square kilometer)	*China City Statistical Yearbook(2019-2022)*[12-15].

Reasons for Selecting Each Indicator:

1. Project Budget (Va): The project budget is a variable that directly reflects the reconstruction costs (or restoration expenses). When studying restoration costs, it is essential to consider the actual financial investment. A higher project budget generally implies a greater scope and quality of restoration, so we consider the project budget as the replacement cost.

2. GDP: The funding for the protection of ancient buildings mainly comes from national government or regional financial resources and related institutions such as cultural heritage departments. Regions with higher GDP usually have more abundant financial resources, which can provide sufficient funding for the restoration projects of ancient buildings. High-GDP areas also tend to utilize the latest technologies and methods in restoring ancient buildings, thereby improving the quality and efficiency of the projects and reducing long-term maintenance costs.

3. Maintenance Efficiency (ME): It is usually expressed as the ratio of the restoration area to the number of technical restoration personnel, i.e.

$$ME = \frac{\mathit{The\ Area\ Restored}}{\mathit{The\ Number\ of\ Technical\ Personne}}$$

Maintenance efficiency is an important indicator for measuring the workload borne by each maintenance worker in ancient building restoration projects. A higher value implies that each technician is responsible for a larger workload.

4. Population Density (MD): Population density influences building costs and the economic benefits of the project. High-population-density areas usually have more available labor, which can support the construction and maintenance of restoration projects. Additionally, in areas with high population density, community involvement may be higher, which can encourage local residents to actively participate in and support the restoration and protection of ancient buildings.

For the purpose of analysis, this study focuses only on samples from national key historical buildings for empirical research. The descriptive statistics of each variable for the years 2018-2021 are shown in Table 3. From 2018 to 2021, the mean, quartiles, and maximum/minimum value of regional economic level (GDP) all show a gradual increase, with the standard deviation also expanding over time. This indicates that the budgets for ancient building protection and restoration projects across different regions have been steadily growing, suggesting that regional heterogeneity has been increasing during the sample period. Moreover, the means and medians of the variables differ significantly, and there are considerable differences between the upper and lower quartiles and the median, maximum, and minimum values. Additionally, due to the significant differences in the scales of each variable and the large differences in magnitude, the subsequent analysis applies logarithmic transformation to the data.

Table 3. Variable Description.

year	variable	mean	Standard Deviation	max	min	Lower Quartile	Upper Quartile
2018	Va	32694.09	32013.25	119530.20	3229.80	10767.13	45095.43
2018	GDP	30656.60	24181.85	97277.80	14776.30	16039.85	37161.00
2018	MD	320.47	295.56	1304.76	2.88	118.76	405.46
2018	ME	3917.25	6655.02	28266.15	37.82	195.45	3546.05
2019	Va	32128.88	31738.54	115781.20	1677.70	12035.48	44781.75
2019	GDP	33134.45	26114.97	107671.00	1697.82	15980.15	43253.33
2019	MD	321.65	296.50	1303.57	2.94	119.23	406.58
2019	ME	2599.48	6575.84	33205.68	8.47	201.51	1682.80
2020	Va	33996.57	33605.18	115517.00	2661.00	11433.00	46564.25
2020	GDP	34074.95	26827.52	110761.00	1902.74	16469.25	43558.60
2020	MD	322.28	297.54	1302.98	2.98	119.54	407.49
2020	ME	3168.99	7314.88	31539.32	6.88	155.10	1975.42
2021	Va	35345.92	36955.67	129217.00	2784.00	11911.00	44945.25
2021	GDP	38298.51	30166.91	124370.00	2080.20	18685.70	49111.03
2021	MD	322.67	298.13	1302.98	2.98	119.07	407.00
2021	ME	3467.56	7241.48	27298.29	4.09	102.95	1231.22

Data Sources: *China City Statistics Yearbook (2019-2022), China Cultural and Tourism Statistics Yearbook (2019), China Cultural Relics and Tourism Statistics Yearbook (2020-2022).*

When searching for data in the *Chinese Cultural Relics and Tourism Statistics Yearbook* from 2019 to 2022, we found that the restoration data of ancient buildings in

Tianjin, Shanghai, and Hainan were partially missing. This situation may have arisen because the statistical yearbooks did not include data from these regions for certain years or specific indicators, leading to the occurrence of missing values. Due to the incomplete data for these provinces and cities, we reduced the statistical scope from 31 provinces to 28 to ensure analytical accuracy. This adjustment was made to avoid significant randomness in the restoration area data, which could lead to severe bias in the analysis results if inappropriate imputation methods were used. However, this also means that we cannot continue to examine the maintenance status of ancient buildings in these three economically developed regions, which could affect the generalizability of the results to some extent.

In the absence of supplementary data, reducing the sample size is a reasonable approach to ensure the reliability of the results. Therefore, when interpreting the analysis results, it is essential to consider the potential impact of this data gap on the overall conclusions and seek other sources to supplement the missing data in future research.

2.2.2 Estimation of Repair Value Model Parameters.

Based on Koenker's (2004) penalized quantile regression approach, this study establishes a panel quantile model to explore the relationships between the variables. Given the significant heterogeneity in the protection of national cultural heritage sites and regional economic levels across the 28 provinces, municipalities, and autonomous regions in China, and the notable fluctuations in the range of these variables, the study employs a panel quantile model with quantiles set at0.1, 0.25, 0.5, 0.75 and 0.9. The parameter estimation results for the years 2018-2021 are shown in Table 4.

Table 4. Coefficients of the Panel Quantile Regression Model.

variation	0.1	0.25	0.5	0.75	0.9
logGDP	0.1978	0.6929**	0.5726**	0.3873*	0.3980*
P Value	0.5659	0.0270	0.0160	0.0611	0.0824
logMD	0.1289	-0.1280	-0.1261	0.0043	0.0382
P Value	0.6001	0.5148	0.3570	0.9724	0.8002
logME	-0.1083	-0.0527	-0.1744***	-0.2189***	-0.2566***
P Value	0.1654	0.4853	0.0002	0.0007	0.0085
Intercept	12.4851	-0.0121	4.8160	10.2749**	10.5604**
P Value	0.1498	0.9988	0.4412	0.0511	0.0714

Note: "*", "**", and "***" indicate significance levels of 10%, 5%, and 1%, respectively. Standard errors of the coefficients are in parentheses. FE denotes the fixed effects model.

From Table 4 it can be observed that GDP is positive and has the highest value across all quantiles among the variables, indicating that the regional economic indicator GDP is the key factor among these three major variables. This finding supports the rationale of Hypothesis A-3.

By comparing the effects of various influencing factors across different quantiles, the following results were obtained:

(1) GDP:

Except for the 0.1 quantile, GDP has a significant impact on repair costs at the 10% significance level across other quantiles, with all coefficients being positive. Notably, the coefficient for GDP decreases from 0.6929 at the 0.25 quantile to 0.3980 at the 0.9 quantile, indicating a gradual decline. This suggests that the impact of GDP on repair costs diminishes as the repair costs increase. We interpret this as follows: In economically developed regions, when historical buildings require repair, the costs are generally at a medium to high level, and the government is willing to pay these costs. However, as the expenses increase, the government's willingness to pay tends to decrease.

(2) Population Density (MD):

Population density is not significant at any quantile.

(3) Maintenance Efficiency (ME):

Maintenance efficiency significantly impacts repair costs at the 1% significance level at the 0.5, 0.75, and 0.9 quantiles, with all coefficients being negative. As the quantile increases, the negative impact becomes more pronounced. The coefficient for maintenance efficiency shows a gradually increasing negative trend, rising from -0.1744 at the 0.5 quantile to -0.2566 at the 0.9 quantile. This indicates that at medium or high repair costs, improvements in maintenance efficiency significantly reduce repair costs, and this effect becomes more pronounced as repair costs rise. We can explain this in the context of China's unique circumstances: many organizations responsible for preserving historical buildings have a specific staffing structure, where an increase in the area each technician repairs implies a higher level of restoration skill. Consequently, the overall cost decreases due to the reduced total man-hours required. Additionally, efficient work reduces the likelihood of rework, thereby saving material expenses and time costs.

The corresponding formula for the panel quantile regression model is as follows:

$$
\begin{aligned}
&Q_{V_{A_{it}}}(0.1 \mid log\ GDP, log\ ME, log\ MD) \\
&= 12.4851 + 0.1978 \cdot log\ GDP - 0.1083 \cdot log\ ME + 0.1289 \cdot log\ MD \\
&Q_{V_{Al}}(0.25 \mid log\ GDP, log\ ME, log\ MD) \\
&= -0.0121 + 0.6929 \cdot log\ GDP - 0.0527\ \cdot log\ ME - 0.1280\ \cdot log\ MD \\
&Q_{V_{A_{it}}}(0.5 \mid log\ GDP, log\ ME, log\ MD) \\
&= 4.8160 + 0.5726 \cdot log\ GDP - 0.1744 \cdot log\ ME - 0.1261\ \cdot log\ MD \\
&Q_{V_{A_{it}}}(0.75 \mid log\ GDP, log\ ME, log\ MD) \\
&= 10.2749 + 0.3873 \cdot log\ GDP - 0.2189 \cdot log\ ME + 0.0043\ \cdot log\ MD \\
&Q_{V_{A_{it}}}(0.9 \mid log\ GDP, log\ ME, log\ MD) \\
&= 10.5604 + 0.3980 \cdot log\ GDP - 0.2566 \cdot log\ ME + 0.0382 \cdot log\ MD
\end{aligned} \tag{7}
$$

2.2.3 Sensitivity Analysis.

To evaluate the robustness of the panel quantile regression model under different conditions of explanatory variable perturbation, we conducted a sensitivity analysis on all explanatory variables. Specifically, we perturbed the data for the explanatory variables to a certain extent, adjusting the regional economic development level logGDP, population density logMD, and restoration efficiency logME by ±10%. We then examined the output changes of the model regression parameters at the 0.1, 0.25, 0.5, 0.75, and 0.9 quantiles. Fig. 1 shows the comparative changes in sensitivity of the three variables under different quantiles.

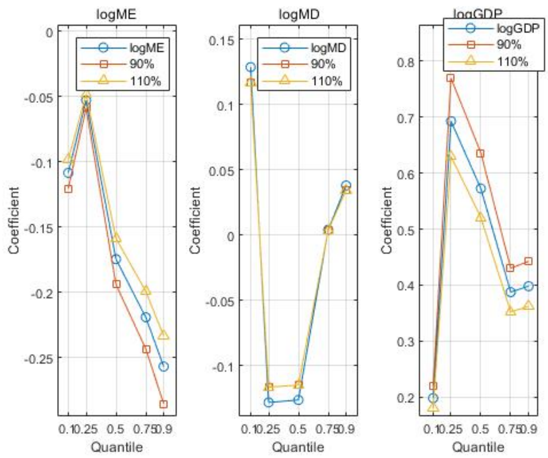

Fig. 1. Sensitivity analysis for three variations.

Table 5. Summary of Panel Quantile Regression Model Parameters After ±10% Variation in Three Variables.

		0.9 times logGDP		1.1 times logGDP	
Quantile	Variation	Coefficient	P Value	Coefficient	P Value
0.1	Intercept	12.4954	0.1116	12.4954	0.1481
0.1	logGDP	0.2198	0.5312	0.1798	0.5611
0.1	logMD	0.1289	0.6026	0.1289	0.5885
0.1	logME	-0.1083	0.1773	-0.1083	0.1735
0.25	Intercept	-0.0071	0.9993	-0.0071	0.9993
0.25	logGDP	0.7699	0.0246	0.6299	0.0322
0.25	logMD	-0.1280	0.5431	-0.1280	0.5367
0.25	logME	-0.0527	0.4898	-0.0527	0.4809
0.5	Intercept	4.8326	0.4380	4.8326	0.4134
0.5	logGDP	0.6362	0.0148	0.5205	0.0114

0.5	logMD	-0.1261	0.3938	-0.1261	0.3964
0.5	logME	-0.1744	0.0001	-0.1744	0.0001
0.75	Intercept	10.2957	0.0773	10.2957	0.0716
0.75	logGDP	0.4303	0.0840	0.3521	0.0835
0.75	logMD	0.0043	0.9743	0.0043	0.9751
0.75	logME	-0.2189	0.0008	-0.2189	0.0021
0.9	Intercept	10.5848	0.0712	10.5848	0.0757
0.9	logGDP	0.4423	0.0771	0.3619	0.0865
0.9	logMD	0.0382	0.7919	0.0382	0.7910
0.9	logME	-0.2566	0.0075	-0.2566	0.0060
		0.9 times logMD		1.1 times logMD	
Quantile	Variation	Coefficient	P Value	Coefficient	P Value
0.1	Intercept	12.4954	0.1903	12.4954	0.1903
0.1	logGDP	0.1978	0.6058	0.1978	0.6058
0.1	logMD	0.1172	0.6453	0.1172	0.6453
0.1	logME	-0.1083	0.1984	-0.1083	0.1984
0.25	Intercept	-0.0071	0.9993	-0.0071	0.9993
0.25	logGDP	0.6929	0.0348	0.6929	0.0348
0.25	logMD	-0.1163	0.5407	-0.1163	0.5407
0.25	logME	-0.0527	0.4709	-0.0527	0.4709
0.5	Intercept	4.8326	0.3827	4.8326	0.3827
0.5	logGDP	0.5726	0.0071	0.5726	0.0071
0.5	logMD	-0.1147	0.3921	-0.1147	0.3921
0.5	logME	-0.1744	0.0000	-0.1744	0.0000
0.75	Intercept	10.2957	0.0539	10.2957	0.0539
0.75	logGDP	0.3873	0.0626	0.3873	0.0626
0.75	logMD	0.0039	0.9718	0.0039	0.9718
0.75	logME	-0.2189	0.0008	-0.2189	0.0008
0.9	Intercept	10.5848	0.0728	10.5848	0.0728
0.9	logGDP	0.3980	0.0833	0.3980	0.0833
0.9	logMD	0.0347	0.7970	0.0347	0.7970
0.9	logME	-0.2566	0.0072	-0.2566	0.0072
		0.9 times logME		1.1 times logME	
Quantile	Variation	Coefficient	P Value	Coefficient	P Value
0.1	Intercept	12.4850	0.1505	12.4850	0.1749
0.1	logGDP	0.1978	0.5661	0.1978	0.5858
0.1	logMD	0.1289	0.6031	0.1289	0.6099

0.1	logME	-0.1203	0.2024	-0.0985	0.1791
0.25	Intercept	-0.0121	0.9988	-0.0121	0.9988
0.25	logGDP	0.6929	0.0294	0.6929	0.0266
0.25	logMD	-0.1280	0.5460	-0.1280	0.5330
0.25	logME	-0.0586	0.4772	-0.0480	0.4667
0.5	Intercept	4.8160	0.4111	4.8160	0.4490
0.5	logGDP	0.5726	0.0112	0.5726	0.0190
0.5	logMD	-0.1261	0.4087	-0.1261	0.4327
0.5	logME	-0.1937	0.0002	-0.1585	0.0002
0.75	Intercept	10.2749	0.0590	10.2749	0.0528
0.75	logGDP	0.3873	0.0721	0.3873	0.0677
0.75	logMD	0.0043	0.9785	0.0043	0.9777
0.75	logME	-0.2432	0.0020	-0.1990	0.0019
0.9	Intercept	10.5603	0.0641	10.5603	0.0630
0.9	logGDP	0.3980	0.0725	0.3980	0.0694
0.9	logMD	0.0382	0.8039	0.0382	0.7974
0.9	logME	-0.2851	0.0066	-0.2333	0.0095

According to the parameter data of the panel quantile regression model after ±10% changes in the three variables in Table 5, the analysis is as follows:

(1) Sensitivity analysis of logGDP

After adjusting logGDP by ±10%, the results show that logGDP still has a significant impact on the estimation of repair costs at various quantiles. Especially at the 0.25 quantile, when logGDP increases by 10%, its coefficient significantly increases from 0.6929 to 0.7699, indicating a significant impact of GDP on the low-to-medium repair cost range. In addition, the significance level of logGDP is almost unaffected at all quantiles, indicating that the model is robust to logGDP.

(2) Sensitivity analysis of logMD

After adjusting logMD by ±10%, the results showed that the coefficient of logMD changed little across all quantiles, and did not reach a significant level in most quantiles, with p-values greater than 0.1. This indicates that population density has little impact on the estimated restoration costs of the model, and the model has low sensitivity to logMD.

(3) Sensitivity analysis of logME

The efficiency of restoration logME is one of the key factors affecting the cost of restoring historical buildings. The sensitivity analysis results show that the coefficient of logME changes significantly with a ±10% change in its value at the 0.5, 0.75, and 0.9 quantiles, especially at the 0.9 quantile, where the coefficient changes significantly from -0.2566 to -0.2333 when the logME value increases by 10%. This indicates that in high-cost repair projects, repair efficiency has a greater impact on repair costs, and the model has a high sensitivity to logME.

(4) Discussion of Results

Overall, the model shows high sensitivity to changes in logGDP and logME data, while the sensitivity to logMD is relatively low. This indicates that the accuracy of logGDP and logME data is crucial when using models for repair cost prediction. Future research should further focus on data collection and quality control of these key variables to improve the accuracy and reliability of model predictions.

3 Insurance Pricing Model

To more effectively design insurance products suitable for ancient buildings, we conducted regression analysis on the restoration costs of ancient buildings in different regions using a panel quantile model. Based on this analysis, we constructed an insurance pricing model under the following assumptions:

3.1 Symbol Assumptions: Symbol Description

Table 6. Symbol Description.

Symbol	Description
W_n	The frequency of extreme weather events in the n-th year
ACC_n	The average claim cost after suffering from extreme weather in the n-th year
R_n	The value at risk of extreme weather events in the n-th year
IPR_n	Insurance premium rate in the n-th year
IC_n	Insurance cost in the n-th year
TCA_n	Total liquid assets of the insurance company in the n-th year
RP_n	The ratio of value at risk to total assets in the n-th year
Rf_n	Risk-free rate of return in the n-th year
Rp_n	Risk premium rate in the n-th year
DR_n	Discount rate in the n-th year
CAI_n	Bank's demand deposit interest rate in the n-th year
IP_n	The insurance premium collected by the insurance company in the n-th year
TIP	Total premium

Based on Table 6, the explanation of symbols is as follows:

1. W_n: The frequency of risk events within the insurance coverage in a given year, typically measured as the number of occurrences per 100 risk events.

2. ACC_n: To simplify the analysis, the average claim cost resulting from different extreme weather events impacting ancient buildings is set as a constant.

3. R_n: The total amount of compensation the insurance company expects to pay out due to extreme weather events in a given year, i.e., the total amount paid out by the insurance company within a year.

4. IPR_n: The proportion of the insurance premium charged by the insurance company, as agreed upon in the insurance contract with the policyholder, based on risk assessment.

5. IC_n: The cost paid by the policyholder to the insurance company to obtain insurance coverage.

6. Rf_n: The rate of return on investments in risk-free assets, such as government bonds.

7. DR_n: In economics, this concept refers to the process of discounting future cash flows to present value, reflecting the time value of money and the compensation for bearing risk. The discount rate is composed of the risk-free rate of return and the risk premium.

3.2 Assumptions

Let's consider region A and assume the insurance period extends over the next n years. The assumptions for the insurance pricing model are as follows:

Assumption 1: The average claim cost (ACC_n) will not significantly change over the next n years and remains consistent with historical data levels.

Assumption 2: The total current assets (TCA_n) of the insurance company accurately reflect its financial status and can be used for risk assessment.

Assumption 3: The insurance company can accurately assess risk and reasonably price the insurance premium rate (IPR_n), with the rate remaining relatively stable over the next n years.

Assumption 4: The risk premium rate can be expressed as the ratio of the risk value to total assets.

Assumption 5: The risk premium rate is proportional to the ratio of the risk to total assets.

Assumption 6: The risk-free rate of return is derived from government bonds.

3.3 Insurance Pricing Explanation

By using historical data to generate expectations of the frequency of extreme weather events in the future, the insurance company calculates the future underwriting costs using the average claim cost (ACC_n). This calculation determines whether to underwrite the region. If the region has an excessive number of claims and the underwriting costs exceed the expected amount, the insurance company may consider not underwriting the region; otherwise, it would proceed with the coverage.

Step 1: Expected frequency of extreme weather events and calculation of average loss amounts

First, by combining historical information on geography and meteorology, we can initially estimate the approximate frequency of various extreme weather events in the region for each of the next n years. Admittedly, due to the lack of extensive historical data, this frequency carries a significant degree of unpredictability. However, with the rapid development of machine learning and deep learning technologies, we can mitigate these challenges as much as possible.

Next, we can determine the average losses caused by extreme weather events within the insurance coverage (such as heavy rainfall, hurricanes, and floods/mudslides) based on the estimated frequency. We can use the mean of these loss samples to calculate the average loss amount.

Step 2: Calculate risk value

Using the risk index method, we calculate the risk value Rn, which combines the probability of risk events occurring with their potential impact. The average cost caused by risk events within the insurance coverage is represented by ACC_n , indicating the degree of impact from weather events. The claim costs are determined by the Historical Building Repair Value Assessment Model established earlier.

The risk value can be expressed as:

$$R_n = W_n \cdot ACC_n \tag{8}$$

Step 3: Calculate premium

When calculating the premium, it is necessary to consider the annual values of R_n and IPR_n for the region. The premium calculation method is based on the risk multiplied by the insurance rate, while the insurance amount represents the maximum limit of compensation or payment responsibility that the insurer assumes, typically specified in the contract or mutually agreed upon. The insurance company's willingness to pay is assumed to be P_n . The basic premium rate is $\frac{R_n}{P_n \cdot TCA_n}$.

The benchmark rate formula incorporates the aforementioned elements. The gross rate of the comprehensive insurance consists of the benchmark rate and the rate adjustment factor. To determine the rate adjustment factor, risk factors must be assessed, and according to the actual condition, the floating proportion of the basic rate must be established. Then, the benchmark rate is adjusted using the rate adjustment factor to obtain the final gross rate IPR_n.

The rate adjustment factor is positively correlated with the risk factor scoring table; as the coverage limit decreases, the rate factor will decrease with higher limits.

$$IC_n = R_n \cdot IPR_n \tag{9}$$

Step 4: Evaluate the risk premium rate

This step involves assessing the relationship between the risk that an insurance company must bear when underwriting a specific region's Historical Building Repair Value Assessment Model and the company's total assets. The risk premium rate is used to discount future insurance revenues or claim costs to their present value, taking into account various risk factors associated with the insurance business.

$$RP_n = \frac{R_n}{TCA_n} \tag{10}$$

Step 5: Calculate the discount rate

The discount rate reflects the minimum return rate required by an insurance company for bearing different risks. It is also a key indicator for evaluating the value of insurance assets and conducting risk management. Let RP_n represent the risk premium rate, then:$Rp_n = f(RP_n)$. This function reflects the impact of the ratio of risk value to total assets on the risk premium rate. Based on the assumptions, we have:

$Rp_n = t_n \cdot RP_n$. Where k_n is a constant that represents the sensitivity of the risk premium rate to the ratio of risk to total assets($0< t_n<1$).

Thus, the discount rate is:

Discount Rate = Risk-free Rate + Risk Premium Rate

$$DR_n = Rf_n + Rp_n = Rf_n + t_n \cdot RP_n \tag{11}$$

Step 6: Calculate expected returns

Considering the time value of money, the expected returns for the next n years are calculated. The expected return for each year is based on the previous year's value:

The expected return for the n-th year in the region:

$$IP_n = \frac{IC_n}{(1+DR_n)^n} \tag{12}$$

Step 7: Evaluate total expected returns

Calculate the total expected returns over n years and compare them with the bank's fixed deposit interest rate. By comparing the total expected returns over n years with the profits the insurance company would earn if it invested its total assets in the bank, a decision can be made on whether to underwrite the region.

If the total expected returns are less than the bank profits, the region is underwritten; otherwise, it is not.

$$TIP = \sum_{k=1}^{n} IP_k \tag{13}$$

If $TIP \geq \sum_{k=1}^{n}(1 + CAI_k) \cdot R_k$, then Underwrite;

If $TIP < \sum_{k=1}^{n}(1 + CAI_k) \cdot R_k$, then do not Underwrite.

4 Summary and Outlook

4.1 Summary

1. Model application: We employed a panel quantile model to accurately regress the restoration costs in different environments. This method effectively captures the heterogeneity of data across different quantiles, providing more detailed and comprehensive analysis results.

2. Insurance policy design: During the insurance policy design process, we thoroughly considered the impact of extreme weather on the restoration costs of ancient buildings while incorporating the unique maintenance cost characteristics of ancient buildings in China. The designed insurance policy is more favorable and practical for policyholders, better meeting their needs.

3. Data Limitations: Due to the difficulty in collecting data related to the restoration of ancient buildings, we could only find a limited sample from libraries. Specifically, in economically developed regions such as Tianjin, Shanghai, and Hainan, there were partial data gaps in the four-year restoration data of ancient buildings. As a result, we had to reduce the sample size, decreasing the number of provinces from 31 to 28 to ensure analytical accuracy. However, this lack of data may impact the generalizability

of the model, although it enhances the accuracy and reliability of the model for the selected provinces. Therefore, future research should focus on more extensive and detailed data collection to improve the model's generalizability.

4. In economically developed regions, the government's willingness to carry out restorations is higher. The relatively favorable conditions in these areas attract more skilled restoration professionals, resulting in higher restoration efficiency. Therefore, to better protect historical buildings in less economically developed regions, the government needs to implement strong policies to allocate specialized restoration talent to these areas, aiming for better preservation of the local historical buildings.

4.2 Outlook

1. Research and Prediction of Extreme Weather: With the development of artificial intelligence and deep learning technologies, significant progress has been made in the study and prediction of extreme weather events, even though these events appear as white noise in time series. For instance, some literature uses ensemble learning methods such as random forests to predict earthquakes or other extreme weather events. Future research can incorporate these advanced prediction technologies into the model, further enhancing the precision and risk management capabilities of insurance companies when issuing insurance products in specific regions.

2. Application of Time Series Analysis: In this study, GDP and population density factors were more accurately estimated through time series analysis. This provides stronger robustness for insurance companies when using the panel quantile model discussed in this paper for insurance policy design. Future research can further analyze the dynamic changes of these economic and population factors to provide more solid data support for insurance product design.

3. Model Improvement and Expansion: This study utilized a fixed-effect panel quantile model. Future research can explore more model improvements and expansions, such as random effect models and mixed regression models. These models can provide more flexible and comprehensive analysis results under different assumptions, further enhancing the breadth and depth of the research.

Through further data collection and model improvements, we are confident that we can achieve greater breakthroughs in the field of ancient building insurance design, providing more scientific and effective protection solutions for China's cultural heritage.

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