Parametric Study of Geometric Patterns in the Grid Heart of Ancient Chinese Architectural Doors and Windows

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Abstract. In recent years, parametric design and its associated computer technologies, as well as three-dimensional digital imaging and printing techniques, have opened up new avenues for the development of traditional patterns. They also provide a new model for the application of decorative patterns in the small wooden doors and windows of ancient architecture. Study uses parametric analysis to explore intricate woodcarvings in ancient Chinese architecture, showcasing artistic charm and offering innovative applications through modern design techniques.

Keywords: Traditional Chinese carpenter; Parameterization; Geometric Patterns.

1 Introduction

Chinese traditional patterns have high aesthetic value.[1] Geometric patterns in the doors and windows of ancient Chinese architecture are not only reflected in small carpenter but also widely used and studied in areas such as architectural decoration, landscape gardens, and layout design, holding significant value for design research.[2] Window and door wooden grilles typically feature geometric patterns such as squares, hexagons, octagons, tortoiseshell, cloud patterns, etc. These patterns are creatively employed within the central area of the top, bottom, and side limits, undergoing abstract transformations based on mathematical topology, resulting in a unique decorative effect in architecture.

The width-to-height ratio of the window and door grids in architectural wooden constructions should be determined based on the size of the building structure. Thus, when creating grid patterns, traditional craftsmen needed to establish the form of the grid organization and the size of the grid openings beforehand. The continuous patterns had to fit within specified grid boundaries to achieve coherence, fluidity, and aesthetic appeal.

However, due to material limitations, craftsmen often chose flexible materials like wood for the window grids. The constraints of the mortise and tenon joint structure

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sometimes led to errors in craftsmanship, which over time caused finished doors and windows to suffer from drying cracks and varying degrees of damage due to exposure to sunlight and moisture.

Therefore, through the following parametric algorithm of mathematical geometry, the geometric logical control of the patterns in ancient wooden doors and windows can be established. Using parameterized algorithms can control the geometric logic of the lattice pattern in the small wooden doors and windows of ancient buildings to avoid wood cracking and damage. This approach resolves the errors in the tenon-and-mortise structure, ensuring precise production processes and enhancing the durability and aesthetics of doors and windows.

2 Generating Geometric Algorithms Based on Traditional Door and Window Lattice Patterns

2.1 Relationship between Traditional Lattice Heart Patterns and Geometric Parametrization

Parametric algorithms can generate a variety of regenerated patterns based on different design parameter values, inspired by traditional patterns of ancient architectural doors and windows. However, not all regenerated patterns are suitable for practical application; overly dense patterns may cause visual fatigue. Therefore, it is necessary to apply logical constraints such as size, boundaries, repetition rules, etc., to ensure that the generated patterns meet practical requirements and enhance the usability and aesthetics of the design.

By filtering through effective conditions, we can obtain optimized parameter values. Such regenerated patterns are not only aesthetically pleasing but also correspond to the architectural environment and style. They represent an "experiential" pattern that fulfills functional needs while embodying a cultural sentiment. This "experience" consists of two levels: first, sensory experience, primarily focusing on visual perception and tactile awareness; second, emotional experience, which encompasses awareness of the environment and architectural styles.

Currently, due to the continuous development of computer parametric technology, the traditional lattice patterns of ancient wooden doors and windows are evolving in composition and form. They no longer solely focus on conventional artistic methods of "form conveying meaning" or "interpreting form for significance." The introduction of new technologies has broadened the realms that traditional patterns engage with, catalyzing heterogeneous transformations and multidimensional chaos effects. Traditional patterns are no longer immutable "standard components"; instead, the inexhaustible creativity brought by parametric design reflects the idea of "recreating a universe within reach." This ideological framework, imbued with cultural sentiment, marks a new era, showcasing a trend of cultural development. As renowned architect I.M. Pei stated, "In China, the doors and windows of ancient architecture are like picture frames, always accompanied by gardens."

2.2 Analysis of the Patterns on Traditional Architecture Doors and Windows Grids

The geometric patterns found in ancient architectural doors and windows encompass a variety of styles, such as the "one horse three arrows" style, myriad character patterns, spiral patterns, ice crack patterns, work or sub-character styles, step-by-step brocade designs, cloud patterns, rhombus flower motifs, and various ball patterns.[3]

These patterns stem from basic geometric shapes like squares, rectangles, polygons, circles, and semicircles.

These basic elements, through formal aesthetic principles like continuity, rhythm, and repetition, come together to form a cluster of patterns. This cluster can be infinitely repeated and expanded in four or eight cardinal directions. Sometimes, in the vertical central axis of the lattice, the clusters composed of these basic elements can exhibit different styles, giving rise to lively, varied patterns that resonate with the cultural essence of traditional Chinese heritage.

3 Research on the Geometric Parameterization of Window Grille Patterns

3.1 Algorithm of K-means Clustering

In ancient Chinese architecture, lattice size is determined by the proportion of wooden beams and columns. When designing lattice clusters, consider the width-to-height ratio as the starting point. For a lattice with width A and height B (both positive real numbers), the ratio is k = A/B. Regardless of the lattice shape, starting with a circle of diameter (r), spaced by interval used symbol (x), and consisting of n circles.

The vertical arrangement is consistent with the horizontal arrangement, with the only difference being that the number of grid centers forming a circle is now m, where n and m are both positive integers. This leads to the formulation of two fundamental equations:

$$\mathrm{rn} + (\mathrm{n} - 1)\mathrm{x} = \mathrm{A} \tag{1}$$

$$\mathrm{rm} + (\mathrm{m} - 1)\mathrm{x} = \mathrm{B} \tag{2}$$

Results can be obtained:

$$\mathrm{r} = \frac{\mathrm{B} - \mathrm{A}}{\mathrm{m} - \mathrm{n}} - \mathrm{x}$$

In theory, it is better for the value of x to be as small as possible:

$$\lim_{x \to 0} r = \lim_{x \to 0} \left(\frac{B-A}{m-n} - x \right) = \frac{B-A}{m-n} \qquad (\mathrm{m} \neq \mathrm{n}) \tag{3}$$

$$\text{When } \mathrm{x} = 0, \mathrm{r} = \mathrm{A}/\mathrm{n} = \mathrm{B}/\mathrm{m} \tag{4}$$

The relationship between m and n can be deduced. $\mathrm{m} = \mathrm{n}/\mathrm{k}$

However, as m can be any real number, not limited to positive integers, rounding m and substituting it into formula (3) may yield a different r compared to formula (4). Using numerical approximation methods with n in a natural number sequence enables efficient comparison to identify an r that meets the requirements within an acceptable range (as shown in Table 1).

Table 1. Example of K-means Clustering Algorithm

n	M=Round (m)	A (mm)	B (mm)	k	$r1 = \frac{B-A}{M-n}$	$r2 = \frac{A}{n}$	$\lvert r1 - r2 \rvert$
1	3	500	1700	5/17	600.0	500.0	100.0
2	7				240.0	250.0	10.0
3	10				171.42	166.66	4.76
4	14				120.0	125.0	5.0
5	17				100.0	100.0	0.0
6	20				85.71	83.33	2.38
7	24				70.58	71.42	0.84
8	27				63.15	62.5	0.65
9	31				54.54	55.55	1.01
10	34				50.0	50.0	0.0

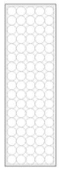
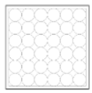
In traditional Chinese architectural lattice windows, the number of lattice clusters in the width direction generally does not exceed 10, and the width of the lattice clusters is usually within 600mm. For convenience, let's assume the width A is 500mm and the height is 1700mm. Values of A and B can be real numbers in Rhino's Grasshopper visual programming software. When A and B are 500 and 700 respectively, with n as 5 or 10, the difference is 0. In this case, a diameter of 100 or 50 for the lattice cluster's perfect circle is most suitable. If the calculated difference for other A and B values is not 0, the minimum difference can be chosen as the diameter for the lattice cluster's perfect circle. In actual production of lattice windows, a difference of less than 0.5mm is acceptable. Once the diameter of the lattice cluster is determined, the lattice cluster grid is then established. This algorithm ensures that when the lattice cluster reaches the edge, the perfect circle of the lattice cluster is tangent to the boundary.

3.2 Method for Parameterized Implementation of Grid Cluster Network

Parametric design software enables the creation of geometric lattice cluster patterns using algorithms. This method has the advantage of generating lattice clusters regardless of A and B values, important for studying ancient architectural window geometries. Start by setting real number sliders for A and B, calculating ratio k (k = A/B). Generate a sequence of natural numbers starting from 1 to establish basic parameters.

This determines the diameter r of the output grid cluster and the number m of grid clusters in the height direction, ultimately drawing the grid cluster network (as shown in Table 2). It can be observed from the table that even when A=B, this algorithm can still be used to identify a suitable grid cluster network.

Table 2. Example of K-means Clustering Algorithm

Type	A. B is a positive integer and B is an integer multiple of A	A. B is a positive integer	A. B is a positive real number	A=B is a positive number
Example	A=500 B=1500	A=510 B=1735	A=562.587 B=1692.011	A=B=575
Grid center cluster net				
r,m,n values	r=50 m=15、n=5	r=51 m=17、n=5	r=46.88225 m=18、n=6	r=47.92 m=6、n=6
Difference value	0	0.08333	0.3542	0

3.3 Parametric Design of Geometric Cluster Patterns

Using the "three-intersecting six-petal rosette" as an example, the basic configu-ration consists of an isosceles triangle and its inscribed circle. The base and legs of the isosceles triangle are of length r, where r is the radius of the circumscribed circle, and the radius of the inscribed circle is half of r. Arranging two basic configurations side by side forms a paral-lelogram. From the form of the three-way intersection hexagonal pattern, it can be seen that the cluster pattern of the lattice center repeats left and right and up and down along the four orientations. Therefore, within the control range of the lattice center scale, the number of repetitions from bottom to top should be m/2 times, and the number of repetitions from left to right should be n times. The part exceeding the lattice center scale limit should be deleted.

4 Conclusion

With computational power, parametric architecture has unlocked new realms of possibilities.[4] This study provides a parametric approach reference for building facade design.[5] By using similar methods as described above, it is possible to analyze the geometric patterns of lattice windows and doors in historical architecture through parameterized algorithms, allowing for the examination of the cluster patterns of the lattice network. China's culture showcases patterns with spiritual depth, enhancing creativity, reducing costs. For the geometric patterns of the lattice windows and doors in ancient Chinese architecture, using parametrically designed lattice geometric patterns represents an innovative, scientific, and artistic approach. These "digital creative patterns" can be widely applied and carry rich cultural significance.

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