The Rapid Construction Path of Null Hypothesis and Alternative Hypothesis

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Abstract. One crucial and challenging step in parameter hypothesis testing is formulating the null hypothesis and the alternative hypothesis based on the actual problem to be tested. This paper presents a simple and efficient method: for the actual problem to be tested, write down two opposing events, consider the event that includes the equality situation as the null hypothesis, and the opposing event as the alternative hypothesis. This method, which can be applied to different types of actual test situations, helps beginners master the method of hypothesis formulation effectively, having significant practical implications. The method's simplicity allows for widespread application in various fields, from industrial production to scientific research, and provides a clear pathway for conducting hypothesis tests, thereby enhancing the accuracy and reliability of statistical inferences. By systematically addressing the complexities involved in hypothesis testing, this approach offers a robust framework for both novice and experienced statisticians.

Keywords: hypothesis testing, null hypothesis, alternative hypothesis.

1 Introduction

Hypothesis testing is an active research area in statistics and an indispensable tool in practical applications. Correct understanding and application of hypothesis testing are crucial for scientific research and decision analysis [1]. Hypothesis testing is an essential part of statistical inference, aiming to infer certain characteristics of a population based on sample observations, whether the population distribution function type is known but parameters are unknown, or the distribution function is entirely unknown. This paper discusses hypothesis testing of population parameters [2].

For instance, an electric bicycle manufacturer traditionally takes 8 hours on average to produce a bicycle. Due to market competition, a new production technique is needed to shorten production time without compromising product quality. Under new production technology, 100 bicycles were randomly inspected, with an average production time of 7.5 hours. It seems that the new technology has improved production efficiency. The question now is whether the new technology has genuinely had an effect. We need to statistically verify this from a technical perspective.

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In practical applications of hypothesis testing, the first step is to clearly state what we hope to prove, followed by sampling to conduct the test. To obtain strong support for a statement based on sample observations, the statement itself is usually taken as the alternative hypothesis, and its negation as the null hypothesis. However, beginners often do not know what they hope to prove and thus cannot correctly write the null and alternative hypotheses [3].

2 Definition of Null Hypothesis and Alternative Hypothesis

2.1 The Principle of Hypothesis Testing

Hypothesis testing, also known as significance testing, is based on the principle that in a single random experiment, low-probability events are unlikely or almost impossible to occur. In hypothesis testing of mathematical statistics, an event is constructed as the alternative hypothesis under the condition that the null hypothesis (also known as the zero hypothesis) is true. If the probability of the event occurring under the null hypothesis is very low (significance level), and if the event happens in a single experiment, the null hypothesis is rejected. This rejection is based on the principle that if the null hypothesis is reasonable, the low-probability event should not occur in a single experiment. However, if the event does occur, it contradicts the principle of low-probability events, leading to the rejection of the null hypothesis.

2.2 Formulating Null Hypotheses and Alternative Hypotheses

For each hypothesis testing problem, two opposing hypotheses are typically proposed: the null hypothesis (denoted as H_0) and the alternative hypothesis (denoted as H_1). Formulating these hypotheses is crucial, particularly in one-sided hypothesis testing [4].

In the previous example, assuming the sample standard deviation s=3 and the significance test level $\alpha=0.05$, if the hypothesis is made:

H_0: $\mu \geq \mu_0 = 8$,

H_1: $\mu < \mu_0 = 8$.

Using the T-test method, $t=\frac{\overline{x}-\mu_0}{s/\sqrt{n}}=\frac{7.5-8}{3/\sqrt{100}}=-1.667$, the test statistic does not fall within the rejection region $(-\infty, -1.96)$, so we should accept the null hypothesis H_0 and reject the alternative hypothesis H_1. This means the new technology has not improved production efficiency.

However, if the hypothesis is made:

H_0: $\mu \leq \mu_0=8$,

H_1: $\mu > \mu_0=8$.

The test statistic does not fall within the rejection region $(1.96, +\infty)$, so we should accept the null hypothesis H_0 and reject the alternative hypothesis H_1. This means the new technology has not reduced production efficiency.

This example demonstrates that null hypothesis and alternative hypotheses cannot be proposed arbitrarily. The most important and challenging step in hypothesis testing is formulating the null hypothesis and alternative hypotheses correctly [5].

In these two hypotheses, which one should we actually select as the null hypothesis? And which one as the alternative hypothesis? It is important to clearly distinguish between these two hypotheses and ensure that they are mutually exclusive in order to conduct effective statistical inference [6].

2.3 Techniques for Writing Null Hypotheses and Alternative Hypotheses

For beginners, a simple and effective way to write the null hypotheses and alternative hypotheses is: for the actual problem to be tested (mean or variance), write down two opposing events, consider the event that includes the equality situation as the null hypothesis H_0, and the opposing event as the alternative hypothesis H_1. This allows selecting the test statistic, and calculating the observed value, and determining whether it falls within the rejection region to conclude whether to accept or reject the null hypothesis [7].

3 Application of the Method

This paper aims to illustrate the application of hypothesis testing in a single normal population, where both the mean (μ) and variance (σ^2) are of interest. Through a series of examples, we will demonstrate how to formulate null and alternative hypotheses when dealing with questions related to both population means and variances. It is important to note that the following examples will solely focus on stating the null and alternative hypotheses. Detailed steps such as choosing the appropriate test statistic, defining the rejection region, and interpreting the results will not be discussed in depth. These examples are intended to provide a clear understanding of how to articulate the hypotheses in the context of a normal distribution with both mean and variance considerations.

3.1 Application in Two-Sided Tests

Example 1: A food company uses a packaging machine to pack sugar with a rated standard of 0.5 kg per bag. Suppose the weight of each bag follows a normal distribution with a standard deviation $\sigma = 0.015$. On a particular day, nine bags were randomly selected and weighed as follows (kg):

0.497, 0.506, 0.518, 0.524, 0.498, 0.511, 0.520, 0.515, 0.512.

Is the packaging machine working normally? (Significance test level α=0.05) [8].

Analysis: The problem is "Is the packaging machine working normally?" So, the problem divides into two opposing situations: "the packaging machine works normally" and "the packaging machine does not work normally." Here, "normal" means the population mean $\mu = 0.5$ and "not normal" means $\mu \neq 0.5$. Therefore, we make the hypothesis:

H_0: $\mu = \mu_0 = 0.5$,

H_1: $\mu \neq \mu_0 = 0.5$.

Conduct a two-sided test of the mean, namely the U-test, by calculating the sample mean and comparing it with the original population mean. If the calculated U-value falls within the rejection region, then the null hypothesis is rejected, indicating that the packaging machine is not working normally. Otherwise, if the U-value does not fall within the rejection region, the null hypothesis cannot be rejected, suggesting that the packaging machine is functioning normally.

Example 2: An electronics company produces an electronic component whose lifespan follows a normal distribution. Based on historical data, the average lifespan of the electronic component is 100 hours, with a standard deviation of 10 hours. The company has made some process improvements and hopes that these improvements will not affect the variance of the component's lifespan. To verify this, the company randomly selected 20 electronic components to conduct lifespan tests, and the data obtained are as follows (in hours):

92, 105, 98, 110, 102, 95, 100, 108, 103, 107, 101, 104, 106, 96, 99, 97, 109, 111, 94, 93.

Do these data support the conclusion that "the process improvements have not affected the variance of the electronic component's lifespan"? (Significance test level α=0.05)

Analysis: The problem is "Whether the process improvements have affected the variance of the electronic component's lifespan?" The problem divides into "affected the variance" and "not affected the variance", "affected the variance" means $\sigma^2 \neq 100$ and "not affected the variance" means $\sigma^2 = 100$. So, we make the hypothesis:

H_0: $\sigma^2 = \sigma_0^2 = 100$,

H_1: $\sigma^2 \neq \sigma_0^2 = 100$.

3.2 Application in One-Sided Tests

Example 3: A pharmaceutical company produces a drug that is supposed to reduce cholesterol levels. The mean reduction in cholesterol levels for this drug is known to be μ= 30 mg/dL. A new formulation of the drug has been developed, and the cholesterol reduction in 12 patients using the new formulation was recorded as follows:

32, 34, 29, 28, 27, 33, 31, 30, 29, 26, 28, 32.

Can these data support the conclusion that "the new formulation has a greater effect in reducing cholesterol levels"? (Significance test level α=0.01) [9].

Analysis: The problem is "Can these data support the conclusion? " So, the problem divides into supporting the conclusion that "the new formulation has a greater effect" and not supporting the conclusion that "the new formulation has a greater effect." Here, "support" means the population mean $\mu > 30$ and "not support" means $\mu \leq 30$. So, we make the hypothesis:

H_0: $\mu_0 \leq 30$,

H_1: $\mu_0 > 30$.

Example 4: A factory produces metal wires, with the breaking force as the product index. The variance of the breaking force represents the production accuracy, and the smaller the variance, the higher the accuracy. The factory has maintained the variance at 64 (kg^2) or below. Recently, ten wires were sampled, and the breaking force (kg) measured as follows:

578, 572, 570, 568, 572, 570, 572, 569, 584, 570.

The sample variance $s^2 = 83.83$. The factory suspects that the variance of the breaking force has increased, indicating reduced production accuracy. If true, it is necessary to scrutinize production processes to pinpoint the problems. (Significance level $\alpha=0.005$) [6].

Analysis: The problem is "Has the variance of the breaking force increased? " So, the problem divides into "variance increased" and "variance not increased" Here, "variance increased" means $\sigma^2 > 64$ and "variance not increased" means $\sigma^2 \leq 64$. So, we make the hypothesis:

H_0: $\sigma^2 \leq \sigma_0^2 = 64$,

H_1: $\sigma^2 > \sigma_0^2 = 64$.

Example 5: An electronic component requires an average lifespan of no less than 2000 hours and a standard deviation not exceeding 130 hours. Twenty-five samples were taken, with a mean lifespan of 1950 hours and a sample standard deviation of 148 hours. Determine whether this batch of components is qualified at $\alpha=0.05$ significance level (assuming lifespan follows a normal distribution) [5].

Analysis: The problem is " whether this batch of components is qualified at $\alpha=0.05$ significance level " In fact, the problem tests two parameters: population mean and variance. For the population mean, the problem divides into "qualified" and "not qualified", "qualified" means $\mu \geq 2000$ hours and "not qualified" means $\mu < 2000$ hours. And for the variance, the problem divides into "qualified" and "not qualified", "qualified" means $\sigma \leq 130$ hours and "not qualified" means $\sigma > 130$ hours. So, we make the hypothesis:

H_0: $\mu \geq \mu_0 = 2000$,

H_1: $\mu < \mu_0 = 2000$.

And

H_0: $\sigma^2 \leq \sigma_0^2 = 130^2$,

H_1: $\sigma^2 > \sigma_0^2 = 130^2$.

4 Conclusion

For beginners, understanding the hypothesis testing process and simplifying it into basic steps is very helpful. This method is straightforward and easy to use for those new to hypothesis testing. In practical applications, one can write only the null hypothesis and then judge whether to accept or reject it based on observed values. Although null and alternative hypotheses are not entirely oppositional, this can be further explored. This method can also be used for parameter hypothesis testing in double normal populations [1]. By this approach, even beginners can gradually master the basic methods of hypothesis testing and apply them in practical data analysis and decision-making. Of course, with deeper understanding, more details and complex situations need consideration [10].

In practical applications, attention should be paid to the following aspects.

One-Sided vs. Two-Sided Tests: In practical scenarios, decide whether a one-sided or two-sided test is appropriate based on the research question.

Parameter Hypothesis Testing in Double Normal Populations: When dealing with two normal populations, you might use a two-sample t-test to compare means or variances.

Complexities: As you gain more experience, you'll learn about the assumptions underlying different tests, the importance of sample size, the interpretation of p-values, and the concept of statistical power.

While this simplified approach can be a good starting point, a deeper understanding of the nuances and underlying principles of hypothesis testing is crucial for accurate and meaningful results in data analysis and decision-making.

References

1. Hogg R. V., Craig A. T., McKean J. W. Introduction to Mathematical Statistics. Pearson 2019: 512. https://minerva.it.manchester.ac.uk/~saralees/statbook2.pdf.
2. Casella, G., & Berger, R. (2024). Statistical Inference. CRC Press 2002: 660. DOI: 10.1201/9781003456285
3. Lehmann, E. L., Romano, J. P., & Casella, G. (1986). Testing statistical hypotheses (Vol.3). New York: Springer 2005: 784. https://sites.stat.washington. Edu / jaw/ COURSES/ 580s/582/HO/Lehmann and Romano-Testing Statistical Hypotheses. pdf.
4. Mood A. M., Graybill F. A., Boes D. C. Introduction to the Theory of Statistics. McGraw-Hill 1974: 564. https://sistemas.fciencias.unam.mx/~misraim/Mood.pdf.
5. Tiande Zhang. Guidance and Exercise Solutions for Probability Theory and Mathematical Statistics. Beijing: China Society Press, 2007: 292. DOI: 10.1007/0-387-28276-9_1.
6. Meicun Wu. Basic Principles and Methods of Mathematical Statistics. Chengdu: Southwestern University of Finance and Economics Press, 2006: 198. DOI: 10.1007/978-1-4757-2739-5_1.
7. Jinguo Liu, Zhi Zhou, Kewen Cha. Probability Theory and Mathematical Statistics. Beijing: China Science and Technology Press, 2001: 145. DOI: 10.1515/9783111325064-008.
8. Dianwu Yang, Lian Miao. Probability Theory and Mathematical Statistics. Beijing: Science Press, 2022: 132. DOI: 10.1515/9783110466195-009.
9. Zou Sheng, Xie Shiqian, Pan Chengyi. Probability Theory and Mathematical Statistics (Fourth Edition) Concise Edition. Beijing: Higher Education Press, 2009: 174-176. DOI: 10.4135/9781071878835.n4.
10. Wasserman L. All of Statistics: A Concise Course in Statistical Inference. Springer 2004: 442. https://doi.org/10.1111/j.1467-985X.2004.00347_18.x.