Application of Generative Artificial Intelligence in Linear Algebra Teaching

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Abstract. This paper explores the application of generative artificial intelligence (GAI) in teaching linear algebra. With its powerful generative capabilities and creativity, GAI brings new possibilities to educational instruction. This paper first introduces the concept of GAI and its application background in the field of education, then discusses various specific application scenarios of GAI in teaching linear algebra, including: assisting teachers in efficient text processing and teaching design, generating personalized learning resources, promoting deep inquiry through smooth human-computer dialogue, constructing interactive learning platforms, real-time evaluation of learning progress and effectiveness, dynamic updating and optimization of teaching content, playing the role of virtual teachers, and developing intelligent evaluation and feedback systems. These applications can improve teaching effectiveness and student interest, and also provide new ideas and methods for innovation in the field of education.

Keywords: Generative artificial intelligence, linear algebra, teaching methods, personalized learning, intelligent assessment.

1   Introduction

With the rapid development of information technology and the advent of the digital era, the field of education is experiencing unprecedented changes. Artificial Intelligence (AI), as one of the core drivers of this transformation, has penetrated into various aspects of educational instruction, bringing revolutionary changes to traditional teaching methods[1]. Generative Artificial Intelligence (GAI), as an important branch of AI, provides new possibilities for educational instruction with its powerful generative capabilities and creativity[2].

Linear algebra, as a fundamental branch of mathematics, not only has a wide range of applications in the field of mathematics, but also plays an important role in physics, engineering, computer science, economics, and other fields. However, the teaching content of linear algebra is often abstract and complex, making it difficult for students to understand and leading to less than ideal teaching effectiveness. Therefore, exploring new teaching methods and means to improve the quality of linear algebra teaching has important practical significance and application value.

In recent years, scholars both domestically and internationally have conducted a lot of research and practice in the field of generative artificial intelligence and educational instruction. On the one hand, generative artificial intelligence technology has achieved significant results in natural language processing, image generation, music creation, and other fields, providing new ideas and methods for educational instruction[3][4]; on the other hand, more and more educators are trying to apply artificial intelligence technology to educational instruction, exploring new teaching models and strategies[5][6][7]. However, there is relatively little research on the application of generative artificial intelligence in linear algebra teaching.

Therefore, this paper aims to explore the application of generative artificial intelligence in linear algebra teaching, analyze its teaching potential and implementation methods through in-depth analysis, and provide new ideas and methods for the innovation of linear algebra teaching. The significance of this study lies in expanding the application scope of generative artificial intelligence in the field of education, enriching educational teaching methods and means; improving the teaching quality and learning effectiveness of linear algebra, helping students better understand and master knowledge of linear algebra; providing references for innovative teaching in other subjects, promoting the reform and development of educational instruction.

2   Overview of Generative Artificial Intelligence

Generative artificial intelligence is an artificial intelligence technology that uses complex algorithms, models, and rules to learn from large datasets to create new original content. This technology is capable of creating text, images, sounds, videos, and code, among other types of content, possessing strong generative capabilities and creativity. In the field of education, generative artificial intelligence can simulate the teaching process of human teachers by generating teaching content that meets student learning needs, achieving personalized teaching and adaptive recommendation of learning resources.

3   Current Status of Linear Algebra Teaching

As a basic course in the mathematics major, linear algebra covers multiple aspects such as vectors, matrices, and systems of linear equations. However, the traditional linear algebra teaching method often has the following problems: the teaching content is abstract and difficult to understand; the teaching method is single and lacks innovation; student participation is not high, making it difficult to stimulate interest in learning. To address these issues, more and more educators are beginning to try new teaching methods and means to improve the teaching quality of linear algebra.

4   Application of Generative Artificial Intelligence in Linear Algebra Teaching

4.1   Assist Teachers in Efficient Text Processing and Optimize Teaching Design

Utilizing generative artificial intelligence to carry out tasks such as retrieval of teaching resources, automatic generation of lesson plans, and evaluation of lesson plans can reduce the workload of teachers and improve the quality of teaching designs.

For example, using ChatGPT to design a lesson plan for the definition of eigenvalues in linear algebra. First, the teaching objectives are set: (1) Understand the definition and importance of eigenvalues and eigenvectors; (2) Calculate eigenvalues and eigenvectors; (3) Master the geometric and applied significance of eigenvalues and eigenvectors in linear algebra. Next are the teaching steps: (1) Introduce the concept of eigenvalues; (2) Explain the definition of eigenvalues; (3) Calculate eigenvalues and eigenvectors; (4) Geometric interpretation and practical application of eigenvalues; (5) Summary and reflection.

4.2   Generation of Personalized Learning Resources

Generative AI is capable of creating personalized learning resources based on students' learning situations and needs. By collecting information such as students' learning history, grades, and feedback, it generates learning content that matches their cognitive levels and interests. Such personalized learning resources can stimulate students' interest in learning and improve learning outcomes.

For example, the AI system records data such as students' learning history, completed exercises, and online test scores through an online learning platform. This enables the generation of the following personalized learning resources: (1) Customized exercise sets, which generate a set of linear algebra exercises of varying difficulty and types based on students' learning history and ability levels. (2) Case studies and application scenarios, which generate linear algebra cases related to practical applications based on students' interests and academic backgrounds.

4.3   Smooth Human-computer Dialogue to Assist Students in Deep Exploration

By building an intelligent teaching assistant, continuous dialogue communication between students and artificial intelligence is realized, guiding students to conduct personalized inquiry and speculative learning. Students can interact with the AI system using natural language, ask their own questions and doubts. The AI system can quickly understand students' questions, automatically generate answers and suggestions. This kind of intelligent tutoring can not only help students solve problems in time but also guide students to think deeply and improve autonomous learning abilities.

For example, the eigenvalue of linear algebra is the root of the characteristic equation. For a third-order matrix, the characteristic equation is a univariate cubic equation,

which is difficult to solve. Design post-class exploration through ChatGPT for students to solve univariate cubic equations, allowing students and ChatGPT to converse, and explore on their own. ChatGPT provides the following methods: factorization method; root-finding formula; Newton method and bisection method numerical solutions; algebraic methods for further simplification or finding other roots for specific forms of equations, such as known roots using long division or solving quadratic equations. Through this human-computer dialogue, students' innovative thinking and speculative abilities can be cultivated.

4.4   Construction of an Interactive Learning Platform

Generative AI can assist in building interactive learning platforms, providing students with a more vivid and interesting learning experience. By introducing technologies such as Virtual Reality (VR) and Augmented Reality (AR), the AI system can generate three-dimensional linear algebra models and scenes, allowing students to engage in practical operations and exploration in a virtual environment. This interactive learning method can help students better understand and master concepts and methods of linear algebra, enhancing learning effectiveness and interest.

For example, through a generative AI-constructed interactive learning platform, students can realize vector operations in a VR environment. Students put on VR headsets, enter a virtual space where they can intuitively see how vectors move, rotate, and scale in space. They can directly manipulate these vectors through gestures or controllers, gaining a deeper understanding of operations such as the linear combination of vectors, dot products, and cross products.

4.5   Assessment of Learning Progress and Effectiveness

Generative AI can track students' learning progress and effectiveness in real-time, providing targeted teaching suggestions for teachers. This can not only help teachers timely understand students' learning situations but also guide students to adjust learning strategies and methods to improve learning efficiency.

4.6   Playing the Role of a Virtual Teacher

Generative AI can also play the role of virtual teachers, providing students with around-the-clock online tutoring and support. Virtual teachers can not only answer students' questions but also provide personalized learning suggestions and resources based on students' learning situations and needs. This playing of the virtual teacher role can make up for the shortage of teacher resources in traditional teaching, providing students with more convenient and efficient learning support.

For example, using agents to make generative AI play the role of virtual teachers can realize the planning of students' personalized learning paths. Virtual teachers can assess their level of understanding by interacting with students and asking questions about the content of linear algebra instruction. Based on the student's answers and performance, the AI can quickly analyze the student's strengths and weaknesses. Based on the

assessment results, virtual teachers can recommend specific learning paths and re-sources, such as recommending specific linear algebra concepts or problem sets, to help students strengthen their understanding and skills.

4.7 Development of Intelligent Assessment and Feedback Systems

Generative artificial intelligence can also be used to develop intelligent evaluation and feedback systems to provide real-time evaluation and feedback on students' learning outcomes. By automatically correcting and scoring exercises and test questions submitted by students, generative artificial intelligence can promptly identify students' errors and deficiencies, providing corresponding guidance and suggestions. At the same time, generative artificial intelligence can also generate personalized learning reports and suggestions based on students' learning data and performance, helping students better understand their learning situations and directions for improvement. Generating artificial intelligence can also develop test score analysis systems.

For example, designing an intelligent entity for score analysis using generative artificial intelligence allows for automatic score analysis after entering linear algebra scores. The distribution of scores includes statistical data such as average, median, and standard deviation to assess the overall academic level of the class. Analyze the specific scores of each student to understand their performance on different question types or knowledge points, so as to identify strengths and areas for improvement. Present the results of score analysis in the form of charts, reports, or visualizations, providing analysis results that allow teachers and managers to intuitively understand and compare students' academic performance. Through comprehensive score analysis, student needs and challenges can be understood, providing targeted support and improvement measures.

5 Conclusions

The application of generative artificial intelligence in linear algebra teaching significantly enhances teaching quality and learning effectiveness. By automatically generating lesson plans and personalized learning resources, the AI system effectively meets students' personalized learning needs, stimulating their interest in learning, and helping them gain a deeper understanding of complex mathematical concepts. The interactive learning platform provides an immersive learning experience through virtual reality and augmented reality technology, allowing students to explore and apply knowledge in practice, thereby greatly improving the vividness and interest of learning. In addition, AI's personalized learning support and real-time assessment functions effectively alleviate teachers' workload, allowing them to focus more on deepening the content of courses and innovating teaching strategies.

In the future, there is still further development space for the application of generative artificial intelligence in linear algebra teaching. It is anticipated that AI systems will adjust teaching strategies more intelligently based on student learning performance and feedback to provide more accurate and personalized learning support. Meanwhile, the

effective collaboration mode between generative AI and human teachers will be further explored, promoting closer interaction and cooperation between teachers and students. Additionally, with technological advancements, the application of AI in other mathematics and interdisciplinary teaching will gradually expand, bringing new possibilities and opportunities for overall innovation in the field of education.

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