Research on the Oligopolistic Market Equilibrium with Two-Sided Network, Vendor-Managed Inventory and Uniform Distributed Demand

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Abstract. In this paper, we construct and analyze an oligopolistic market model involving two-sided network, vendor-managed inventory, and uniform distributed demand. This model aims to explore how the vendor side (i.e., manufacturers) and the buyer side (i.e., demand markets) achieve decentralized market equilibrium through strategic interaction in a complex market environment, and pays special attention, through numerical simulations, to the influence of excess supply/demand penalty weight on market equilibrium results.

Keywords: Oligopolistic Market Equilibrium, Two-Sided Network, Vendor-Managed Inventory (VMI), Uniform Distributed Demand, Excess Supply, Excess Demand.

1 Introduction

Oligopolistic markets are markets dominated by a few producers that control the supply quantity and price of commodities being transacted [1]. Research on oligopolistic markets can be traced back to the classic Cournot model [2], which has fruitful investigations, generalizations and applications. And researchers have often introduced it as basic framework for analyzing market participants' complex behaviors [3].

Participants in oligopolistic markets are typically multi-sided. A participant's behavior influences not only other participants within the same side but also across to other sides. So, with the increasing volume and mix of market participants, finding market equilibrium has become more and more complex and challenging. For example, [1] explored an oligopoly market for equilibrium and stability with the help of tripled fixed points in Banach spaces.

Uncertainty challenges mechanism designs in economic and complex environments [4]. Especially, demand uncertainty challenges the design of oligopolistic markets. Multi-sided oligopolistic markets could be considered as specific supply chains or supply chain networks. So, the research on demand uncertainty in the supply chain field can provide important references. For example, [5] developed a supply chain network model consisting of manufacturers and retailers in which the demands associated with the retail outlets are random.

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Vendor-managed inventory (VMI) is a viable inventory management model under demand uncertainty, which could optimize the use of resources and improvement of production and logistics processes. For example, [6] proposed a vendor-managed inventory model for a three-layer supply chain considering exponential demand, imperfect system, and remanufacturing. And more, network consideration in supply chain literature is not new but expanding [7]. For example, [8] studied a supply chain network with vendor-managed inventory and IoT-related technologies.

In this paper, an oligopolistic market equilibrium model is analyzed. It is assumed a market configuration with a two-sided network, one is the vendor side and another is the buyer side. According to VMI agreements, the vendor side is assumed to decide on the inventory levels and replenish the uniform distributed demand at the buyer side. Description of the demand uncertainty at the buyer side in this paper is a modification or application of seminal model in [5]. And, most of mathematical variables/expressions in this paper applies the corresponding mode in [9]. However, it is precisely because of introducing the uniform distributed demand assumption and the VMI model that this paper significantly differs from [5] and [9].

The structure of this paper is as follows. Section 2 describes the assumptions and modellings on the oligopolistic market. Section 3 describe equilibrium conditions for decentralized oligopolistic market. Section 4 shows the sensitivity analysis for the profit functions and decision variables. Section 5 provides some conclusions and future research lines.

2 Assumptions and Modellings

Assume that the oligopolistic market in this paper is a two-sided supply chain network, composing of the vendor and buyer sides (see Fig. 1). Assume that the commodities that circulate in the market are homogeneous. Assume that the random demand at the buyer side follows a uniform distribution. Assume that the vendor side adopts the VMI model to manage inventory at the buyer side.

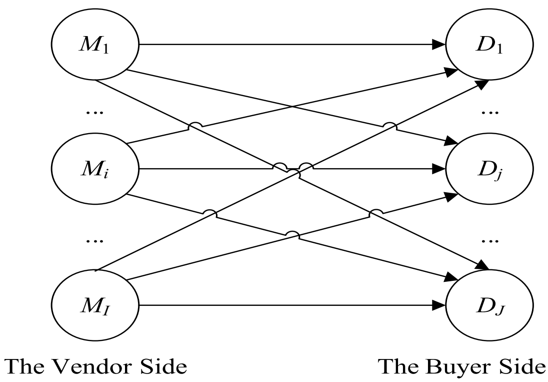

Fig. 1. The oligopolistic market (I, J)

2.1 The Two-sided Network in the Oligopolistic Market

The oligopolistic market is denoted by (I,J), with the vendor side composing of I manufacturers and the buyer side composing of J demand markets, where $I,J \in \mathbb{N}$. Arrows between the vendor and buyer sides represent commodity circulations in the market (see Fig. 1). More specifically, the two-sided network (I,J) will be described by two sets as follows: (1) the *set of all manufacturers* $\mathcal{M} \triangleq \{M_1, M_2, \cdots, M_I\}$, where M_i denote a specific manufacturer, $i \in \{1,\cdots,I\}$; and , (2) the *set of all demand markets* $\mathcal{D} \triangleq \{D_1, D_2, \cdots, D_J\}$, where D_j denote a specific buyer side, $j \in \{1,\cdots,J\}$. And more, let $\mathcal{M}_{-i} \triangleq \{M_1,\cdots,M_{i-1},M_{i+1},\cdots,M_I\}$ denote the set of all manufacturers except for M_i, then $\mathcal{M} = \mathcal{M}_{-i} \cup \{M_i\}$. Let $\mathcal{D}_{-j} \triangleq \{D_1,\cdots,D_{j-1},D_{j+1},\cdots,D_J\}$ denote the set of all demand markets except for D_j, then $\mathcal{D} = \mathcal{D}_{-j} \cup \{D_j\}$.

2.2 The Commodity Logistics in the Oligopolistic Market

Commodity Transaction Quantities in the Oligopolistic Market. The commodity transaction quantities in (I,J) will be described by a quantity matrix $\boldsymbol{q}_{\mathcal{MD}} \triangleq \left(q_{M_iD_j}\right)_{I\times J}$ and a feasible space $\mathcal{K}_{\mathcal{MD}} \triangleq \left\{\boldsymbol{q}_{\mathcal{MD}} \middle| \boldsymbol{0} \le \boldsymbol{q}_{\mathcal{MD}} \le \boldsymbol{q}^U_{\mathcal{MD}}\right\}$. Here, $q_{M_iD_j} \in \left[0, q^U_{M_iD_j}\right]$ denotes the transaction quantity between M_i and D_j. $q^U_{M_iD_j}$ denotes the corresponding finite upper bound. $\boldsymbol{q}^U_{\mathcal{MD}} \triangleq \left(q^U_{M_iD_j}\right)_{I\times J}$ denotes the matrix composed of the corresponding upper bounds.

Commodity Acceptance Quantities at the Buyer Side. The total commodity quantity that D_j accepts from the vendor side is $q_{\mathcal{M}D_j} \triangleq \sum_{i=1}^{I} q_{M_iD_j}$. And, the total quantity that D_j accepts from the vendor side except for M_i is $q_{\mathcal{M}_{-i}D_j} \triangleq q_{\mathcal{M}D_j} - q_{M_iD_j}$.

Commodity Supply Quantities at the Vendor Side. The supply quantity strategy of M_i will be described by a vector $\boldsymbol{q}_{M_i\mathcal{D}} \triangleq \left(q_{M_iD_1}, q_{M_iD_2}, \cdots, q_{M_iD_J}\right)^T$ and a feasible space $\mathcal{K}_{M_i} \triangleq \left\{\boldsymbol{q}_{M_i\mathcal{D}} \middle| \boldsymbol{0} \le \boldsymbol{q}_{M_i\mathcal{D}} \le \boldsymbol{q}^U_{M_i\mathcal{D}}\right\}$. Then, the supply quantity strategy combination of the vendor side will be described by a vector $\left(\boldsymbol{q}_{M_i\mathcal{D}}\right)_{i\in\{1,\ldots,I\}} \triangleq \left(\boldsymbol{q}_{M_i\mathcal{D}}, \boldsymbol{q}_{M_{-i}\mathcal{D}}\right)$ and a feasible space $\mathcal{K}_{\mathcal{M}} \triangleq \mathcal{K}_{M_1} \times \mathcal{K}_{M_2} \times \cdots \times \mathcal{K}_{M_I}$. Here, $\boldsymbol{q}_{\mathcal{M}_{-i}\mathcal{D}} \triangleq \left(\boldsymbol{q}_{M_i\mathcal{D}}, \cdots, \boldsymbol{q}_{M_{i-1}\mathcal{D}}, \boldsymbol{q}_{M_{i+1}\mathcal{D}}, \cdots, \boldsymbol{q}_{M_I\mathcal{D}}\right)$ denotes the supply quantity strategy combination of $\mathcal{M}_{-i}$.

2.3 The Commodity Prices in the Oligopolistic Market

Commodity Demand Prices at the Buyer Side. The commodity demand prices at the buyer side will be described by a *J*-dimensional vector $\boldsymbol{\rho}_{\mathcal{D}} \triangleq \left(\rho_{D_1},\cdots,\rho_{D_j},\cdots,\rho_{D_J}\right)^T$ and a feasible space $\mathcal{K}_{\boldsymbol{\rho}_{\mathcal{D}}} \triangleq \left\{\boldsymbol{\rho}_{\mathcal{D}} \middle| \boldsymbol{0} \le \boldsymbol{\rho}_{\mathcal{D}} \le \boldsymbol{\rho}_{\mathcal{D}}^U\right\}$. Here, $\rho_{D_j} \in \left[0,\rho_{D_j}^U\right]$ denotes the commodity demand price of buyers at D_j. $\rho_{D_j}^U$ denotes the corresponding finite upper bound. $\boldsymbol{\rho}_{\mathcal{D}}^U \triangleq \left(\rho_{D_1}^U,\cdots,\rho_{D_j}^U,\cdots,\rho_{D_J}^U\right)^T$ denotes the vector of finite upper bounds.

Commodity Supply Prices at the Vendor Side. The commodity supply prices at the vendor side will be described by a price matrix $\boldsymbol{\rho}_{\mathcal{MD}} \triangleq \left(\rho_{M_iD_j}\right)_{I\times J}$ and a feasible space $\mathcal{K}_{\boldsymbol{\rho}_{\mathcal{MD}}} \triangleq \left\{\boldsymbol{\rho}_{\mathcal{MD}} \middle| \boldsymbol{0} \le \boldsymbol{\rho}_{\mathcal{MD}} \le \boldsymbol{\rho}_{\mathcal{MD}}^U\right\}$. Here, $\rho_{M_iD_j} \in \left[0,\rho_{M_iD_j}^U\right]$ denotes the commodity supply price between M_i and D_j. $\rho_{M_iD_j}^U$ denotes the corresponding finite upper bound. $\boldsymbol{\rho}_{\mathcal{MD}}^U \triangleq \left(\rho_{M_iD_j}^U\right)_{I\times J}$ denote the matrix composed of the corresponding finite upper bounds.

2.4 The Commodity Demand Uncertainty at the Buyer Side

Demand functions at the buyer side. The commodity demand functions at the buyer side will be described by a *J*-dimensional vector $\boldsymbol{x}_{\mathcal{D}}\left(\boldsymbol{\rho}_{\mathcal{D}}\right) \triangleq \left(x_{D_1}\left(\boldsymbol{\rho}_{\mathcal{D}}\right),\cdots,x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right),\cdots,x_{D_J}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)\right)^T$. Here, $x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)$ denotes the random demand function for the commodity at buyer side D_j.

Assume that $x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)$ follows the uniform distribution in $\left[0,b_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)\right]$, where the upper bound $b_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right) \ge 0$ decreases with ρ_{D_j}. Let $\phi_{D_j}\left(x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)\right)$ denote the density function of $x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)$. Let $\mathrm{E}\left[x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)\right]$ denote the expected value of $x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)$.

$$\phi_{D_j}\left(x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)\right) \triangleq \begin{cases} \dfrac{1}{b_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)}, & \text{if } x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right) \in \left[0,b_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)\right] \\ 0, & \text{if } x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right) \notin \left[0,b_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)\right] \end{cases} \tag{1}$$

$$\mathrm{E}\left[x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)\right] \triangleq \frac{b_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)}{2} \tag{2}$$

Excess Supplies/demands at the Buyer Side. With the VMI model, the manufacturers must decide how much to supply commodities in order to cope with the random demand at the buyer side.

Let $\Delta^{+}_{M_iD_j}$ denote the excess supply that M_i supplies to D_j, and $\mathrm{E}\left[\Delta^{+}_{M_iD_j}\right]$ denote the corresponding expected value.

$$\Delta^{+}_{M_iD_j} \triangleq \max\left\{0, q_{M_iD_j} - \left(x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right) - q_{\mathcal{M}_{-i}D_j}\right)\right\} = \max\left\{0, q_{\mathcal{M}D_j} - x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)\right\} \tag{3}$$

$$\mathrm{E}\left[\Delta^{+}_{M_iD_j}\right] \triangleq \int_0^{q_{\mathcal{M}D_j}} \left[\left(q_{\mathcal{M}D_j} - x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)\right)\phi_{D_j}\left(x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)\right)\right] dx_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right) = \frac{q^2_{\mathcal{M}D_j}}{2b_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)} \tag{4}$$

Let $\Delta^{-}_{M_iD_j}$ denote the excess demand that M_i supplies to D_j, and $\mathrm{E}\left[\Delta^{-}_{M_iD_j}\right]$ denote the corresponding expected value.

$$\Delta^{-}_{M_iD_j} \triangleq \max\left\{0, \left(x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right) - q_{\mathcal{M}_{-i}D_j}\right) - q_{M_iD_j}\right\} = \max\left\{0, x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right) - q_{\mathcal{M}D_j}\right\} \tag{5}$$

$$\mathrm{E}\left[\Delta^{-}_{M_iD_j}\right] \triangleq \int_{q_{\mathcal{M}D_j}}^{b_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)} \left[\left(x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right) - q_{\mathcal{M}D_j}\right)\phi_{D_j}\left(x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)\right)\right] dx_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right) = \frac{\left(b_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right) - q_{\mathcal{M}D_j}\right)^2}{2b_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)} \tag{6}$$

Individual Penalty Functions of Having Excess Supply/Demand at the Buyer Side. With the VMI model, the vendor side takes up the responsibility of optimizing the inventory stocks at the buyer side. That is, the vendor side is assumed to bear the penalty of having excess supply and excess demand at the buyer side.

Let $\gamma^{+}_{M_iD_j} \geq 0$ denote the penalty weight (that is, the unit penalty) of having excess supply that M_i supplies to D_j. Let $\gamma^{-}_{M_iD_j} \geq 0$ denote the penalty weight (that is, the unit penalty) of having excess demand that M_i supplies to D_j. Let $\Gamma_{M_iD_j} \triangleq \gamma^{+}_{M_iD_j}\Delta^{+}_{M_iD_j} + \gamma^{-}_{M_iD_j}\Delta^{-}_{M_iD_j}$ denote the total penalty of having excess supply/demand that M_i supplies to D_j. And, let $\mathrm{E}\left[\Gamma_{M_iD_j}\right]$ denote the corresponding expected value.

$$\mathrm{E}\left[\Gamma_{M_iD_j}\right] \triangleq \gamma^{+}_{M_iD_j}\mathrm{E}\left[\Delta^{+}_{M_iD_j}\right] + \gamma^{-}_{M_iD_j}\mathrm{E}\left[\Delta^{-}_{M_iD_j}\right] = \frac{\gamma^{+}_{M_iD_j}q^2_{\mathcal{M}D_j} + \gamma^{-}_{M_iD_j}\left(b_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right) - q_{\mathcal{M}D_j}\right)^2}{2b_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)} \tag{7}$$

2.5 The Cost Functions in the Oligopolistic Market

Production Cost Functions at the Vendor Side. Let $f_{M_i}\left(\boldsymbol{q}_{M_i\mathcal{D}}, \boldsymbol{q}_{\mathcal{M}_{-i}\mathcal{D}}\right) \triangleq f_{M_i}\left(\boldsymbol{q}_{\mathcal{M}\mathcal{D}}\right)$ denote the production cost of M_i, which is fully borne by M_i. In order to reflect the intense competition within the vendor side, assume that the production cost of each manufacturer depends on all elements in the commodity supply quantity matrix $\boldsymbol{q}_{\mathcal{M}\mathcal{D}}$. Assume that $f_{M_i}\left(\boldsymbol{q}_{M_i\mathcal{D}}, \boldsymbol{q}_{\mathcal{M}_{-i}\mathcal{D}}\right)$ is a continuously differentiable convex function. Then, $\partial^2 f_{M_i}\left(\boldsymbol{q}_{M_i\mathcal{D}}, \boldsymbol{q}_{\mathcal{M}_{-i}\mathcal{D}}\right) / \partial q^2_{M_iD_j} \geq 0$.

Transaction Cost Functions at the Vendor Side. Let $c_{M_iD_j}\left(q_{M_iD_j}\right)$ denote the transaction cost between M_i and D_j, which is fully borne by M_i. Assume that $c_{M_iD_j}\left(q_{M_iD_j}\right)$ is a continuous differentiable convex function. Then, $\partial^2 c_{M_iD_j}\left(q_{M_iD_j}\right)/\partial q_{M_iD_j}^2 \geq 0$.

Additional Cost Functions at the Buyer Side. Let $a_{M_iD_j}\left(\boldsymbol{q}_{\mathcal{MD}}\right)$ denote the additional cost that a consumer at D_j spends on purchasing a unit of commodity from M_i, which is fully borne by D_j. To reflect the intense competition within the buyer side, assume that the additional cost of each buyer side depends on $\boldsymbol{q}_{\mathcal{MD}}$. Assume that $a_{M_iD_j}\left(\boldsymbol{q}_{\mathcal{MD}}\right)$ is a continuous differentiable convex function. Then, $\partial^2 a_{M_iD_j}\left(\boldsymbol{q}_{\mathcal{MD}}\right)/\partial q_{M_iD_j}^2 \geq 0$.

2.6 The Profit Functions in the Oligopolistic Market

Individual Supply Revenues at the Vendor Side. With the demand uncertainty at the buyer side, the commodity quantity that M_i could supply to buyers at D_j is no more than $\min\left\{q_{M_iD_j}, x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)-q_{\mathcal{M}_{-i}D_j}\right\}$. Let r_{M_i} denote the individual supply revenue that M_i obtains from the buyer side and $\mathrm{E}\left[r_{M_i}\right]$ denote the corresponding expected value.

$$\min\left\{q_{M_iD_j}, x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)-q_{\mathcal{M}_{-i}D_j}\right\}=x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)-\left(q_{\mathcal{M}D_j}-q_{M_iD_j}\right)-\Delta_{M_iD_j}^{-} \tag{8}$$

$$r_{M_i} \triangleq \sum_{j=1}^{J}\left(\rho_{M_iD_j}\times\min\left\{q_{M_iD_j}, x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)-q_{\mathcal{M}_{-i}D_j}\right\}\right) \tag{9}$$

$$\mathrm{E}\left[r_{M_i}\right] \triangleq \sum_{j=1}^{J}\left\{\rho_{M_iD_j}\left[\mathrm{E}\left[x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)\right]-\left(q_{\mathcal{M}D_j}-q_{M_iD_j}\right)-\mathrm{E}\left[\Delta_{M_iD_j}^{-}\right]\right]\right\} \tag{10}$$

Individual profit functions at the vendor side. The vendor side produce and sell commodities to the buyer side at certain supply prices. In this process, the vendor side obtains the supply revenues (r_{M_i}), and bears the production costs ($c_{M_iD_j}$), the transaction costs, and the penalty ($\Gamma_{M_iD_j}$) of having excess supply/demand at the buyer side. Let $U_{M_i}\left(\boldsymbol{q}_{M_i\mathcal{D}}, \boldsymbol{q}_{\mathcal{M}_{-i}\mathcal{D}}\right)$ denote the individual profit function of M_i, and $\mathrm{E}\left[U_{M_i}\left(\boldsymbol{q}_{M_i\mathcal{D}}, \boldsymbol{q}_{\mathcal{M}_{-i}\mathcal{D}}\right)\right]$ denote the corresponding expected value.

$$U_{M_i}\left(\boldsymbol{q}_{M_i\mathcal{D}}, \boldsymbol{q}_{\mathcal{M}_{-i}\mathcal{D}}\right) \triangleq r_{M_i}-f_{M_i}\left(\boldsymbol{q}_{M_i\mathcal{D}}, \boldsymbol{q}_{\mathcal{M}_{-i}\mathcal{D}}\right)-\sum_{j=1}^{J} c_{M_iD_j}\left(q_{M_iD_j}\right)-\sum_{j=1}^{J}\Gamma_{M_iD_j} \tag{11}$$

$$\mathrm{E}\left[U_{M_i}\left(\boldsymbol{q}_{M_i\mathcal{D}}, \boldsymbol{q}_{\mathcal{M}_{-i}\mathcal{D}}\right)\right] \triangleq \mathrm{E}\left[r_{M_i}\right]-f_{M_i}\left(\boldsymbol{q}_{M_i\mathcal{D}}, \boldsymbol{q}_{\mathcal{M}_{-i}\mathcal{D}}\right)-\sum_{j=1}^{J} c_{M_iD_j}\left(q_{M_iD_j}\right)-\sum_{j=1}^{J}\mathrm{E}\left[\Gamma_{M_iD_j}\right] \tag{12}$$

To ensure the validity of (4) and (6), the following inequalities are required:

$$q_{\mathcal{M}D_j} \le b_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right), \quad j \in \{1,\cdots,J\} \tag{13}$$

Then, under condition (13), we obtain that $\partial^2 \mathrm{E}\left[\Delta^{+}_{M_iD_j}\right] \big/ \partial q^2_{M_iD_j} \ge 0$, and $\partial^2 \mathrm{E}\left[\Delta^{-}_{M_iD_j}\right] \big/ \partial q^2_{M_iD_j} \ge 0$. Then, we obtain that $\partial^2 \Gamma_{M_iD_j} \big/ \partial q^2_{M_iD_j} \ge 0$. Then, we obtain that $\partial^2 \mathrm{E}\left[r_{M_i}\right] \big/ \partial q^2_{M_iD_j} \le 0$. Then, we obtain that $\partial^2 \mathrm{E}\left[U_{M_i}\left(\boldsymbol{q}_{M_i\mathcal{D}}, \boldsymbol{q}_{\mathcal{M}_{-i}\mathcal{D}}\right)\right] \big/ \partial q^2_{M_iD_j} \le 0$.

3 Equilibrium Conditions for the Oligopolistic Market

3.1 Nash Equilibrium Condition at the Vendor Side

The competition model of manufactures at the vendor side are assumed to be a Cournot-type model, which is an output (i.e., *commodity supply quantity* in this paper) competition model rather than a price competition model [10]. i.e., manufactures are assumed to engage in non-cooperative games with each other, independently and synchronously choosing their commodity supply quantities and maximizing their individual profits. According to [11], the Nash equilibrium condition at the vendor side is as follows:

Nash Equilibrium 1. *Determine* $\left(\boldsymbol{q}^*_{M_1\mathcal{D}}, \boldsymbol{q}^*_{M_2\mathcal{D}}, \cdots, \boldsymbol{q}^*_{M_I\mathcal{D}}\right) \in \mathcal{K}_{\mathcal{M}}$, *such that*

$$\mathrm{E}\left[U_{M_i}\left(\boldsymbol{q}^*_{M_i\mathcal{D}}, \boldsymbol{q}^*_{\mathcal{M}_{-i}\mathcal{D}}\right)\right] \ge \mathrm{E}\left[U_{M_i}\left(\boldsymbol{q}_{M_i\mathcal{D}}, \boldsymbol{q}^*_{\mathcal{M}_{-i}\mathcal{D}}\right)\right], \forall\ \boldsymbol{q}_{M_i\mathcal{R}} \in \mathcal{K}_{M_i}, \forall\ i \in \{1,\ldots,I\} \tag{14}$$

According to the Nash Equilibrium conditions at the vendor side (*Nash Equilibrium 1*), equilibrium transaction quantities $\boldsymbol{q}^*_{\mathcal{M}\mathcal{D}}$ satisfy the first-order conditions $\partial \mathrm{E}\left[U_{M_i}\left(\boldsymbol{q}^*_{M_i\mathcal{D}}, \boldsymbol{q}^*_{\mathcal{M}_{-i}\mathcal{D}}\right)\right] \big/ \partial q^*_{M_iD_j} = 0$, expressed as (15).

$$-\frac{\rho^*_{M_iD_j}\left(q^*_{\mathcal{M}D_j} -_{D_j}\left(\boldsymbol{\rho}^*_{\mathcal{D}}\right)\right)}{b_{D_j}\left(\boldsymbol{\rho}^*_{\mathcal{D}}\right)} - \frac{\partial f_{M_i}\left(\boldsymbol{q}^*_{M_i\mathcal{D}}, \boldsymbol{q}^*_{\mathcal{M}_{-i}\mathcal{D}}\right)}{\partial q_{M_iD_j}} - \frac{\partial c_{M_iD_j}\left(q^*_{M_iD_j}\right)}{\partial q_{M_iD_j}} - \frac{\left(\gamma^{+}_{M_iD_j} + \gamma^{-}_{M_iD_j}\right) q^*_{\mathcal{M}D_j} - \gamma^{-}_{M_iD_j} b_{D_j}\left(\boldsymbol{\rho}^*_{\mathcal{D}}\right)}{b_{D_j}\left(\boldsymbol{\rho}^*_{\mathcal{D}}\right)} = 0 \tag{15}$$

3.2 Spatial Price Equilibrium Conditions at the Buyer Side

And more, commodity transaction quantities and prices satisfy the spatial price equilibrium conditions at the buyer side. According to [5, 12], the spatial price equilibrium conditions at the buyer side is as follows:

Spatial Price Equilibrium 1. *Determine* $\left(\boldsymbol{q}_{\mathcal{M}\mathcal{D}}, \boldsymbol{\rho}_{\mathcal{M}\mathcal{D}}, \boldsymbol{\rho}_{\mathcal{D}}\right) \in \mathcal{K}_{\mathcal{M}\mathcal{D}} \times \mathcal{K}_{\boldsymbol{\rho}_{\mathcal{M}\mathcal{D}}} \times \mathcal{K}_{\boldsymbol{\rho}_{\mathcal{D}}}$, *such that* ($i \in \{1,\cdots,I\}$, $j \in \{1,\cdots,J\}$)*:*

$$\rho_{M_iD_j} + a_{M_iD_j}\left(\boldsymbol{q}_{\mathcal{MD}}\right)\begin{cases} = \rho_{D_i}, & if\ q_{M_iD_j} > 0 \\ > \rho_{D_i}, & if\ q_{M_iD_j} = 0 \end{cases} \tag{16}$$

$$\mathrm{E}\left[x_{D_j}\left(\boldsymbol{\rho}_{\mathcal{D}}\right)\right]\begin{cases} = q_{\mathcal{M}D_j}, & if\ \rho_{D_j} > 0 \\ < q_{\mathcal{M}D_j}, & if\ \rho_{D_j} = 0 \end{cases} \tag{17}$$

In the Spatial Price Equilibrium 1, (16) indicates that if the transaction quantity between D_j and M_i is not zero (that is, $q_{M_iD_j} > 0$), the sum of $\rho_{M_iD_j}$ and $a_{M_iD_j}$ will be equal to ρ_{D_j}. (17) indicates that if the demand price at D_j is not zero (that is, $\rho_{D_j} > 0$), the demand quantity at D_j will be equal to $q_{\mathcal{M}D_j}$ in the aggregate, with exceptions of zero probability[5, 13].

According to the buyer side equilibrium conditions (*Spatial Price Equilibrium 1*), the equilibrium transaction quantities $\boldsymbol{q}^*_{\mathcal{MD}}$, and the equilibrium demand price $\boldsymbol{\rho}^*_{\mathcal{D}} \triangleq \left(\rho^*_{D_j}\right)_{J\times 1}$ satisfy (18) and (19).

$$\rho^*_{D_j} = \rho^*_{M_iD_j} + a_{M_iD_j}\left(\boldsymbol{q}^*_{\mathcal{M}D_j}, \boldsymbol{q}^*_{\mathcal{M}D_{-j}}\right) \tag{18}$$

$$\mathrm{E}\left[x_{D_j}\left(\boldsymbol{\rho}^*_{\mathcal{D}}\right)\right] = \sum_{i=1}^{I} q^*_{M_iD_j} \tag{19}$$

3.3 Decentralized Oligopolistic Market Equilibrium Model

Given the Nash equilibrium conditions at the vendor side and the spatial price equilibrium conditions at the buyer side, the definition of the decentralized oligopolistic market equilibrium can be given as follows[5, 13].

Definition 1 (Decentralized Oligopolistic Market Equilibrium) *A decentralized oligopolistic market equilibrium is a simultaneous realization of the Nash equilibrium conditions at the vendor side (i.e., Nash Equilibrium 1) and the spatial price equilibrium conditions at the buyer side (i.e., Spatial Price Equilibrium 1).*

Then, according to Definition 1, the decentralized oligopolistic market equilibrium $\left(\boldsymbol{q}^*_{\mathcal{MD}}, \boldsymbol{\rho}^*_{\mathcal{MD}}, \boldsymbol{\rho}^*_{\mathcal{D}}\right) \in \mathcal{K}_{\mathcal{MD}} \times \mathcal{K}_{\boldsymbol{\rho}_{\mathcal{MD}}} \times \mathcal{K}_{\boldsymbol{\rho}_{\mathcal{D}}}$ derives from the solution to the system of equations (15)-(19).

Proposition 1. *Decentralized oligopolistic market equilibrium* $\left(\boldsymbol{q}^*_{\mathcal{MD}}, \boldsymbol{\rho}^*_{\mathcal{MD}}, \boldsymbol{\rho}^*_{\mathcal{D}}\right)$ *satisfies the system of equations (15)-(19).*

4 Sensitivity Analysis

It is not easy (and not necessary) to obtain the analytical expression of the equilibrium state (i.e., the solution of the system of equations (15)-(19)). Therefore, we conduct sensitivity analysis by numerical simulations, which are implemented in MATLAB 2024a.

Based on [5, 13], basic numerical simulation functions for (I, J) market are listed in Table 1, which reflect the fact that participants within the same side have equal positions. And more, we assume that $\gamma^+_{M_iD_j} = \gamma^+$ and $\gamma^-_{M_iD_j} = \gamma^-$. i.e., the penalty weights of having excess supply (or excess demand) are indistinguishable for every pair (M_i, D_j), where $i \in \{1, \cdots, I\}$, $j \in \{1, \cdots, J\}$.

We assume that the market structure is (2, 2). i.e., both the number of manufacturers and demand markets are 2. With the penalty weight vector (γ^+, γ^-) changes from (1,1) to (10,1) (i.e., given that $\gamma^- = 1$, with γ^+ increasing from 1 to 10), the numerical results at market equilibrium are listed in left part of Table 2. With the penalty weight vector (γ^+, γ^-) changes from (1,1) to (1,10) (i.e., given that $\gamma^+ = 1$, with γ^- increasing from 1 to 10), the numerical results at market equilibrium are listed in right part of Table 2.

Table 1. Basic numerical simulation functions for (I, J).

The production cost at the vendor side: $f_{M_i}\left(\boldsymbol{q}_{\mathcal{MD}}\right) = 2.5 \times \left(\sum_{j=1}^{J} q_{M_iD_j}\right)^2 + \prod_{m=1}^{I}\left(\sum_{j=1}^{J} q_{M_mD_j}\right) + 2 \times \sum_{j=1}^{J} q_{M_iD_j}$
The transaction cost at the vendor side: $c_{M_iD_j}\left(q_{M_iD_j}\right) = 0.5 \times q^2_{M_iD_j} + 3.5 \times q_{M_iD_j}$
The additional cost at the buyer side: $c_{M_iD_j}\left(\boldsymbol{q}_{\mathcal{MD}}\right) = q_{M_iD_j} + 5$
The upper bound of demand distribution interval at the buyer side: $d_{D_j}\left(\boldsymbol{P}_{\mathcal{D}}\right) = -0.5 \times \rho_{D_j} - 1.5 \times \sum_{m=1}^{J} \rho_{D_m} + 1000$

4.1 Equilibrium Commodity Transaction Quantity Between the Vendor and the Buyer Sides

According to data in the 2nd column and the 7th column in Table 2, we find that, (1) $q_{M_iD_j}$ decreases with unilateral increasing γ^+, but increases with unilateral increasing γ^-; and, (2) $q_{M_iD_j}$ with unilaterally increasing γ^+ is always smaller than $q_{M_iD_j}$ with unilaterally increasing γ^-.

4.2 Equilibrium Commodity Supply Prices at the Vendor Side

According to data in the 3rd and 8th columns in Table 2, we find that, (1) $\rho_{M_iD_j}$ increases with unilateral increasing γ^+, but decreases with unilateral increasing γ^-; and, (2) $\rho_{M_iD_j}$ with unilaterally increasing γ^+ is always bigger than $\rho_{M_iD_j}$ with unilaterally increasing γ^-.

4.3 Equilibrium Commodity Demand Prices at the Buyer Side

According to data in the 4th and 9th columns in Table 2, we find that, (1) ρ_{D_j} increases with unilateral increasing γ^+, but decreases with unilateral increasing γ^-; and, (2) ρ_{D_j} with unilaterally increasing γ^+ is bigger than ρ_{D_j} with unilaterally increasing γ^-.

4.4 Equilibrium Individual Profit at the Vendor Side

According to data in the 5th and 10th columns in Table 2, we find that, (1) U_{M_i} decreases with unilateral increasing in the penalty weight vector $\left(\gamma^+, \gamma^-\right)$; and, (2) U_{M_i} with unilaterally increasing γ^+ is always bigger than U_{M_i} with unilaterally increasing γ^-.

Table 2. Sensitivity analysis of the penalty weight vector for having excess supply and demand

(γ^+, γ^-)	Vendor side		Buyer side	Individual profit	(γ^+, γ^-)	Vendor side		Buyer side	Individual profit
	$q_{M_iD_j}$	$\rho_{M_iD_j}$	ρ_{D_j}	U_{M_i}		$q_{M_iD_j}$	$\rho_{M_iD_j}$	ρ_{D_j}	U_{M_i}
(1,1)	9.5838	260.1777	274.7614	991.1647	(1,1)	9.5838	260.1777	274.7614	991.1647
(2,1)	9.5482	260.2538	274.8020	983.7578	(1,2)	9.6193	260.1015	274.7208	979.3608
(3,1)	9.5127	260.3299	274.8426	976.3787	(1,3)	9.6548	260.0254	274.6802	967.4426
(4,1)	9.4772	260.4061	274.8832	969.0273	(1,4)	9.6904	259.9492	274.6396	955.4100
(5,1)	9.4416	260.4822	274.9239	961.7037	(1,5)	9.7259	259.8731	274.5990	943.2631
(6,1)	9.4061	260.5584	274.9645	954.4079	(1,6)	9.7614	259.7970	274.5584	931.0018
(7,1)	9.3706	260.6345	275.0051	947.1399	(1,7)	9.7970	259.7208	274.5178	918.6261
(8,1)	9.3350	260.7107	275.0457	939.8996	(1,8)	9.8325	259.6447	274.4772	906.1361
(9,1)	9.2995	260.7868	275.0863	932.6872	(1,9)	9.8680	259.5685	274.4365	893.5317
(10,1)	9.2640	260.8629	275.1269	925.5025	(1,10)	9.9036	259.4924	274.3959	880.8130

5 Summary

In this paper, an oligopolistic market model involving two-sided network, vendor-managed inventory, and uniform distributed demand is constructed and analyzed. This model aims to explore how the vendor side and the buyer side achieve decentralized market equilibrium through strategic interaction in a complex market environment, and pays special attention, through numerical simulations, to the influence of excess supply/demand penalty weight on market equilibrium results.

Ceteris paribus, with unilaterally increasing the penalty weight of having excess supply, (1) the equilibrium commodity transaction quantity between per manufacturer and per demand market decreases, (2) the equilibrium commodity supply price between per manufacturer and per demand market increases, (3) the equilibrium commodity demand price at per demand market increases, and (4) the equilibrium individual profit per manufacturer decreases.

Ceteris paribus, with unilaterally increasing the penalty weight of having excess demand, (1) the equilibrium commodity transaction quantity between per manufacturer and per demand market increases, (2) the equilibrium commodity supply price between per manufacturer and per demand market decreases, (3) the equilibrium commodity demand price at per demand market decreases, and (4) the equilibrium individual profit per manufacturer decreases.

Ceteris paribus, compared with unilaterally increasing the penalty weight of having excess demand, unilaterally increasing the penalty weight of having excess supply would bring forth (1) smaller equilibrium commodity transaction quantity between per manufacturer and per demand market, (2) bigger equilibrium commodity supply price between per manufacturer and per demand market, (3) bigger equilibrium commodity demand price at per demand market, and (4) bigger equilibrium individual profit per manufacturer.

Contextualizing this paper' findings and deriving detailed policy recommendations with real-world examples or case studies will be the focus in our future researches.

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