The Application of Portfolio Theory in Practice

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Abstract. With the increasing growth of the capital market, investment is particularly important in the application of the market. On this premise, this paper aims to explore the application of portfolio theory in practical investment activities, and take BYD Co., Ltd. and Bank of China as examples for in-depth analysis. Firstly, the paper reviews the basic framework and core principles of portfolio theory, including the risk diversification effect of portfolio, the trade-off between expected return of assets and risk, and the construction of effective frontier. Subsequently, the article takes BYD and Bank of China as examples to elaborate on how to use portfolio theory to make investment decisions. As the leader of new energy vehicles in China, BYD has high growth potential and large market volatility, while Bank of China, as a large state-owned commercial bank, has stable returns and low risks. By incorporating the two into the same portfolio, the balance of risk and return can be achieved. The article further evaluates the performance of BYD and Bank of China in their portfolios through quantitative analysis. Finally, the paper summarizes the effectiveness and challenges of portfolio theory in practical application, and puts forward targeted suggestions in order to provide investors with more scientific and effective investment reference.

Keywords: Investment Decision Daking; Portfolio Theory; Risk Management; Equity Investment.

1 Introduction

With the increasing complexity of the global financial market, investors face many challenges when seeking suitable investment strategies. Investment involves the risks and returns of the capital market as well as the preferences and objectives of the investors. Some are risk-averse, while others are risk-averse, so finding the balance between risk and return is necessary. Investors often hope to achieve satisfactory returns while reducing risks.

In order to balance risk and return optimally, investors opt for equity, bond, and fixed-income, which makes a portfolio an ideal investment strategy. In investment, portfolio risk management and control have always been the focus of attention for investors. As is well known, the risk of an investment portfolio is closely related to the proportion of its asset allocation. In other words, by adjusting the proportion of various assets in the investment portfolio, investors can achieve control over the level of risk.

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Portfolio theory is an essential branch of modern finance, and the core idea is to reduce risk by diversifying investments. Investors should fully understand the risk characteristics of various assets, adjust asset allocation flexibly based on the market environment and their risk tolerance, and then achieve a balance between investment portfolio risk and return. Risk control is always the top priority in the investment process. Investors can steadily move into a capital market by completing an excellent job in risk management.

Since the risk of an investment portfolio depends not only on the risk of individual assets but also on the effect of correlation between each asset, diversified investment strategies have become the secret to reducing investment risk. When making an investment, risk and return are often proportional. In order to achieve higher returns while reducing risks, investors have adopted a diversified investment strategy, also known as diversification. This word comes from French and means "diversity".

Diversified investment strategy is a widely used risk management method in the investment field. By allocating the proportion of assets, investors can diversify risks among different asset categories, industries, and regions, thereby reducing the overall risk of the investment portfolio. In short, diversified investment strategies are a practical risk management approach. However, implementing a diversified investment strategy is difficult, as investors need specific investment knowledge, risk awareness, and long-term investment concepts. In practical operation, investors can utilise professional investment tools such as funds and wealth management products to achieve diversified asset allocation.

This article will use two practical cases to analyse portfolio theory's deep application in investment practice. The research method adopted in this article is mainly portfolio theory. Through this theory, the research will analyse how to construct an effective investment portfolio based on investors' risk tolerance and return needs. The samples in this article will come from various investors worldwide to demonstrate the application of portfolio theory in different contexts.

This article aims to provide investors with a practical tool to help them build a suitable investment portfolio, achieving their return goals while meeting their risk tolerance.

The structure of this article is as follows: Firstly, the article will outline the basic principles of portfolio theory. Secondly, the article will provide a detailed analysis of two practical cases, demonstrating how to apply portfolio theory to make investment decisions. Finally, the article will summarise the main points of this article and propose future research directions.

2 Methodology

This paper mainly uses portfolio theory to calculate the optimal result of two portfolio assets. In the 1950s, Harry Markowitz pushed forward several changes in the thinking of stock prices in modern portfolio theory. [1] In 1952, Markowitz put forward the portfolio theory for the first time and conducted systematic, in-depth and fruitful research. The classical Markowitz portfolio selects parametric representations based on

the first and second moments, assuming no predictable time change. [2] From a narrow point of view, a portfolio is a portfolio of securities that specifies the proportion of investments. Of course, a single security can also be used as a particular portfolio.

Essentially, people invest in uncertain returns and risk choices. Typical financial entities have different kinds of risks — operation risk, liquidity risk, market risk, solvency risk and credit risk. [3] Portfolio theory uses mean variance to characterise these two key factors. The notion of exchangeable random variables plays an essential role in the optimality of the expected utility theory of the naive diversification principle, a particular case of the law of significant numbers diversification principle. [4]

When applied, it is necessary to calculate the mean, variance, standard deviation, and covariance and construct them to obtain the portfolio frontier of the investment portfolio.

Suppose that there is a random portfolio of assets. $\tilde{x}$ and asset $\tilde{y}$, the values of them are x_1, x_2, x_3, ... , x_n and y_1, y_2, y_3, ... , y_n, their probabilities of occurrence at different states are p_1, p_2, p_3, ... , p_n, where the sum of probability is 1.

Mean is the expected return of a portfolio, which is the expected return of a single stock weighted average, and the relative weight of the investment ratio can be calculated with Equation (1).

$$\mathrm{E}[\tilde{x}] = \bar{x} = \sum_{i=1}^{n} p_i \cdot x_j \tag{1}$$

Variance is the degree of dispersion of return on a portfolio, which can be calculated as Equation (2).

$$\mathrm{Var}[\tilde{x}] = \sigma_x^2 = E[(\tilde{x} - \bar{x})^2] = \sum_{i=1}^{n} p_i \cdot \left(x_j - \bar{x}\right)^2 \tag{2}$$

The more dispersed the value, the greater the variance, and the greater the probability of the final result deviating from the mean, the more concentrated the values, the smaller the variance value, and the closer the final result is to the mean.

The standard deviation is another measure of dispersion, the risk measure chosen in the model. [5] The standard deviation of yield is often referred to as volatility, which is used to describe the risk of a portfolio with Equation (3).

$$\mathrm{StD}[\tilde{x}] = \sigma_x = \sqrt{\mathrm{Var}[\tilde{x}]} \tag{3}$$

Covariance is a measure of how two portfolios "vary together", which can be calculated as Equation (4).

$$\mathrm{Cov}[\tilde{x}, \tilde{y}] = \sigma_{xy} = E[(\tilde{x} - \bar{x})(\tilde{y} - \bar{y})] = \sum_{i=1}^{n} p_i \cdot \left(x_j - \bar{x}\right)\left(y_j - \bar{y}\right) \tag{4}$$

Correlation is the degree of correlation between two portfolio investments, which is Equation (5).

$$\mathrm{Corr}[\tilde{x}, \tilde{y}] = \rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y} \tag{5}$$

In general, it can be seen from a scatter plot that two variables have one of the following relationships: positive correlation, negative correlation, and non-correlation. If

one variable is high, then the other is high. Likewise, if the low value is low, then there is a positive correlation. The value of the correlation factor must lie between -1 and 1. The two random portfolios are perfectly positively correlated if correlation factor is 1, thus, are perfectly negatively correlated if correlation factor is -1. If correlation factor is 0, they are uncorrelated.

The expected yield is measured by the average return, and the risk is measured by the variance of the expected yield. The portfolio is built on the basis of these two factors. [6] After calculating the expected returns of each two assets, the expected returns of their portfolio investments can be further calculated. Their respective investment proportions are represented by w_1 and w_2(the sum of them equals to 1). The expected portfolio return is Equation (6).

$$\tilde{r}_p = w_1 r_1 + w_2 r_2 , \quad w_2 = 1 - w_1 \tag{6}$$

Variance represents the degree of dispersion of data. Therefore, the more considerable the variance, the more dispersed the data, and the higher the risk. In finance, it is believed that as long as the actual situation is different from the expected rate of return, it is a risk. So it is calculated with Equation (7).

$$\mathrm{Var}[\tilde{r}_p] = \sigma_p^2 = E\left[(\tilde{r}_p - \bar{r}_p)^2\right] = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12} \tag{7}$$

The standard deviation of the investment portfolio is an important indicator for measuring investment risk, which can help investors evaluate the volatility and stability of investment portfolios, thereby better asset allocation and risk management. The Equation (8) below is used to calculate the standard deviation.

$$\mathrm{StD}[\tilde{r}_p] = \sigma_p = \sqrt{\mathrm{Var}[\tilde{r}_p]} \tag{8}$$

Because of the importance of covariance, the risk of a single asset is not the standard deviation of its yield, but the contribution of the asset to the standard deviation of portfolio yield. The larger the standard deviation, the higher the volatility and risk of the investment portfolio; The smaller the standard deviation, the lower the volatility and risk of the investment portfolio.

Efficient Boundary is a fundamental resource allocation problem that requires finding the optimal investment portfolio that maximizes returns at a given level of risk.[7] However, in practice, the parameters of the effective boundary are unknown and, therefore, must be estimated. The estimator replaces parameters with an estimator to obtain the effective boundary of the sample. The distributional properties of the sample efficient frontier were studied by previous researchs.[8] The point located at the minimum risk is the mean-variance frontier portfolio. Draw a dashed line parallel to the x-axis based on the y-axis of this point. Above the dashed line is the effective frontal portfolio, and below the dashed line is the no effective portfolio. The area above the effective frontier is unattainable. An efficient frontier can be understood as the optimal portfolio of risk assets the market can provide as in Figure 1.

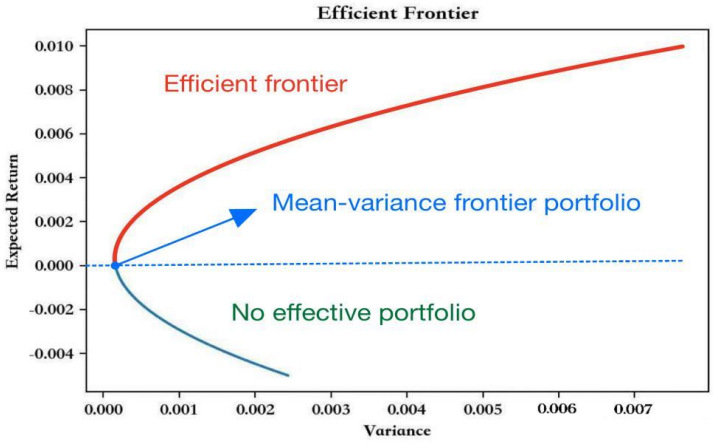

Fig. 1. The chart of an efficient frontier

The above discussion is limited to risk asset portfolios that do not include risk-free assets. Now, introduce a risk-free asset with a return rate of r_f, which is weighted together with a given efficient combination to form a straight line with an intercept term of r_f, passing through the point corresponding to the efficient combination (on the efficient frontier). Taking the risk-free return rate as the y-axis intercept, tangent the Efficient Frontier to the Tangency Portfolio. The tangency portfolio is linked to the risk-free rate as the capital allocation line. The capital allocation line has the Equation (9).

$$r_c = r_f + \frac{r_p - r_f}{\sigma_p} \times \sigma_c \tag{9}$$

The efficient frontier represents the investment portfolio that provides the best risk-return trade-off and integrates risk-free interest rates, promoting the construction of Capital Market Line and optimal risk investment portfolios. The Capital Allocation Line shows the trade-off between risk and return for risk-free assets and risk investment portfolios, with the slope representing the Sharpe ratio.[9] The Sharpe Ratio is the slope of CAL, which represents the risk premium for a portfolio that is evaluated according to the risk that the portfolio carries.[10] The larger the absolute value of the slope, the greater the value that rises or falls in the tangential direction when moving one unit horizontally. As the sharp ratio increases, an increase in one unit's risk rate can lead to a more significant increase in returns. The Sharpe ratio is defined with Equation (10).

$$\text{Sharp ratio} = \frac{\bar{r}_p - r_f}{\sigma_p} \tag{10}$$

where r_f the risk-free rate and σ_p is the standard deviation of p during the investment time. All investors are pursuing the maximum sharp ratio in their investment decisions.

For a portfolio asset of two assets, the expected return and the standard deviation is calculated with Equation (11) and Equation (12).

$$\tilde{r}_{\mathrm{p}} = w_1 r_1 + w_2 r_2 \tag{11}$$

$$\sigma_p = \sqrt{\mathrm{Var}\left[\tilde{\mathrm{r}}_{\mathrm{p}}\right]} = \sqrt{\mathrm{w}_1^2\sigma_1^2 + \mathrm{w}_2^2\sigma_2^2 + 2\mathrm{w}_1\mathrm{w}_2\sigma_{12}} \tag{12}$$

When maximising sharp ratio, the constraint that the sum of combination weights is 1 needs to be met. So, as Equation (13).

$$\mathrm{Max}_{\mathrm{w_i}} S_p = \frac{\tilde{r}_{\mathrm{p}} - \mathrm{r_f}}{\sigma_p} \tag{13}$$

With constraint the sum of weights equals to 1.

In the case of two risky assets, the solution of the optimal risk portfolio is calculated with Equation (14) and Equation (15).

$$\mu = \mathrm{r_f} + \left(\tilde{r}_{\mathrm{p}} - \mathrm{r_f}\right)\mathrm{w}_1 \tag{14}$$

$$\mathrm{W}_2 = 1 - \mathrm{w}_1 \tag{15}$$

3 Analysis

3.1 Introduction of the Financial Assets

The financial assets selected in this paper are the stocks of BYD and Bank of China in the past decade.

BYD is a high-tech company dedicated to "satisfying people's desire for a better life through technology innovation." Founded in Feb. 1995, BYD has developed rapidly for over 20 years and has established over 30 industrial parks around the world. BYD's business plan covers electronics, cars, new energy sources, and rail transport, and has a crucial role to play in those areas. BYD is listed in both Hong Kong and Shenzhen, with a turnover of over 100 billion RMB. BYD has built up technology in the battery sector, which has helped it successfully enter the car industry across borders. After ten years of efforts, BYD has successfully implanted battery technology into cars to create new energy-electric vehicles, covering pure electric private cars, urban public transportation, taxis, forklifts and other fields. BYD's new energy vehicle business has achieved remarkable results and gained a good reputation in the market.

The Bank of China, as one of China's leading financial institutions, has made remarkable progress in many ways in recent years. The Bank of China has been committed to international development, adding many branches worldwide in recent years, especially in Asia, Europe and North America. This not only strengthens the position of Chinese banks in global financial markets but also brings more business opportunities. In order to meet the diversified financial needs of customers, the Bank of China has continuously launched innovative products. For example, various financial products, such as credit cards and loans, have been launched to meet the needs of different customer groups. In recent years, the Bank of China has achieved remarkable development results in internationalisation strategy, digital transformation, product innovation, risk

management and social responsibility, further enhancing its competitiveness and influence in the global financial market.

3.2 Risk and Return of Financial Assets

This paper collects the closing stock prices of the two companies at the end of each year in the past ten years in Table 1, calculates their annual stock price returns through Excel formula, and finally obtains the average annual returns of each company and the overall expected returns are obtained by calculating the price changes at the beginning and end of the period.

The expected return of BYD is 20.93%, while Bank of China is 5.71%.

The variance, standard deviation, covariance of these two assets and correlation between them were then calculated by using formulas is shown in Table 2.

Table 1. The data of the stock prices of each year from 2014 to 2023 (¥)

date		2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	average return	expected return
price	BYD	35.81	62.06	47.70	63.25	49.34	46.22	192.9	266.9	255.83	198	42.6%	20.93%
	BOC	.42	2.47	2.07	2.77	2.59	2.85	2.53	2.60	2.93	3.99	7.07%	5.71%

Table 2. The date of the variance, standard deviation, covariance and correlation corfficient of each stock

	Variance	*Standard deviation*	*Covariance*	*Correlation coefficient*
BYD	1.17	1.08	-0.05923	-0.29725
BoC	0.03	0.18		

Through calculation, it is found that the covariance between BYD and Bank of China is - 0.05923, and the correlation coefficient is - 0.29725, indicating a specific negative correlation between the two, but the correlation is not very close.

3.3 Construct Portfolio Frontier

Operate Excel to set 2000 sets of evenly distributed values as the weights of two assets with the constrain of the sum of the weights equals 1, which can obtain the chart of portfolio frontier as Figure 2.

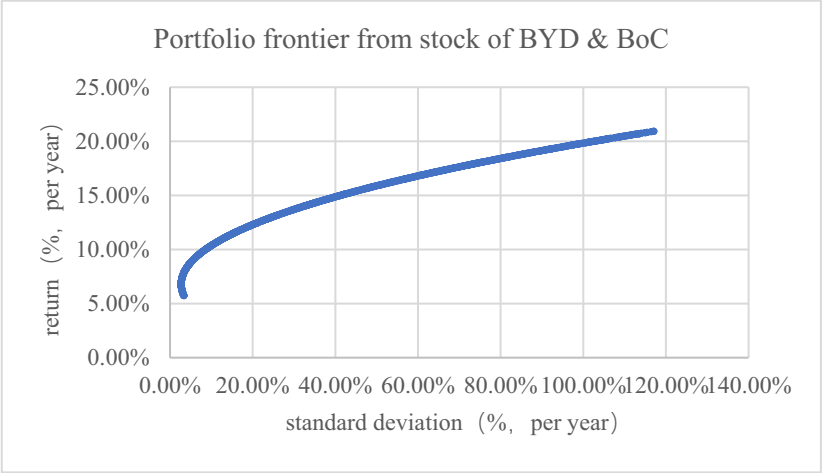

Fig. 2. Portfolio frontier from stock of BYD & BoC

Using Excel solver to find the point with the smallest risk value is the mean-variance frontier portfolio: with 2.73% standard deviation per year and 6.78% in return. The portfolios above this point are efficient frontier as shown in Figure 3.

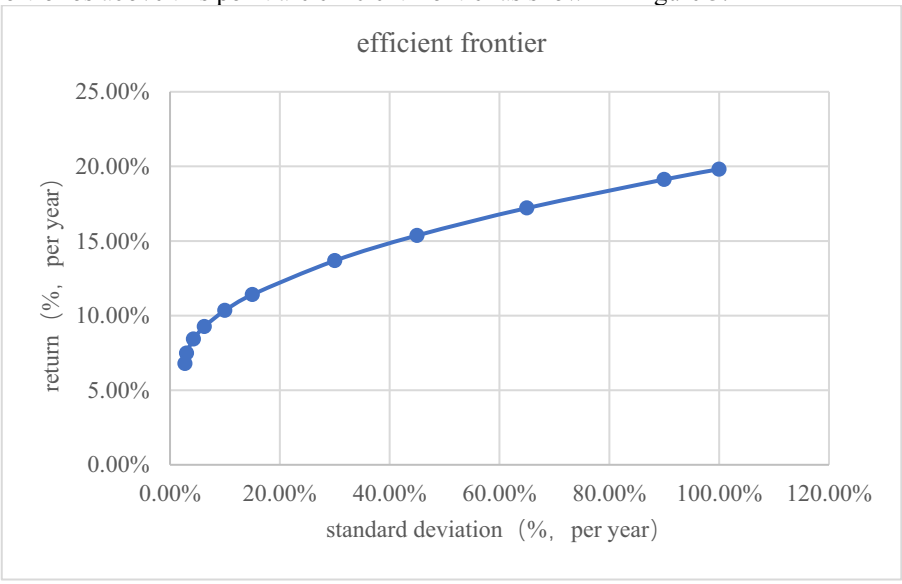

Fig. 3. The efficient frontier of the portfolio

3.4 Construct Risk-free Portfolio

This is to find the capital allocation line. This paper chooses 1-year treasury bonds as zero-risk assets and selects the bond price for the past decade to calculate each year's return which is shown in Table 3.

Table 3. The data of the stock price of risk-free portfolio from 2014 to 2023

Year	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	average return
Return (%)	3.26	2.30	2.65	3.79	2.60	2.36	2.47	2.24	2.10	2.08	2.59%

The return rate of risk-free assets is 2.59% per year.

3.5 The Optimal Portfolio

According to the capital allocation line's Equation (16).

$$R_c = r_f + \frac{r_p - r_f}{\sigma_p} \times \sigma_c \tag{16}$$

When the sharp ratio is the largest, it is the optimal solution and the point tangent to the effective frontier. Based on this theorem, the maximum sharp ratio and the corresponding weight can be found simultaneously with the Excel Solver. In this case, the optimal solution is to buy 14.49% BYD and 85.51% Bank of China, and investors can get a 7.92% expected return while taking 18.63% risk.

In portfolio construction, investors must weigh the relationship between returns and risks. According to the expected rate of return, standard deviation and correlation with other assets, investors can adjust the proportion of BYD's shares in the portfolio to balance risk and return.

4 Conclusion

Based on the above analysis, BYD Company's stock plays an important role in the portfolio as an investment asset with high growth potential and certain risks. Investors can reasonably adjust the proportion of BYD's shares in the portfolio according to their risk tolerance and earnings expectations. Through scientific asset allocation and risk management strategies, investors can achieve the return target and reduce the risk level of the overall portfolio.

It should be noted that the financial market is uncertain and volatile, and investors should fully consider various risk factors when making investment decisions and seek professional investment advice and guidance. At the same time, with the change in the market environment and the company's operating conditions adjustment, investors

need to regularly evaluate and adjust their portfolios to ensure that they meet their investment objectives and risk tolerance.

With the increasing integration of the global economy and the rapid development of financial markets, investors' choices and allocation of financial assets have become increasingly important. Through analysis and discussion, this paper explores the potential value of two financial assets (BYD and Bank of China) in portfolios. Using the analysis method mentioned in the methodology, the portfolio under different proportions can be simulated and evaluated to determine the optimal investment strategy and eventually got the solution to buy 14.49% BYD and 85.51% Bank of China, at 18.63% risk to get 7.92% return.

This paper aims to help people use portfolio investment theory to make sound decisions. The method mentioned in this paper will benefit the selection of other portfolios in future investment strategies. This will also help assess how portfolio investment in different financial products may affect expected returns and risk decisions and how these choices, in turn, affect investment returns.

Reference

1. Jewczyn N. Modern portfolio theory, apt, and the capm: The years 1952 to 1986[J]. INTERNATIONAL JOURNAL OF SOCIAL SCIENCE RESEARCH VOLUME 2 (2013), 2013: 74.
2. Jones, C. Kenneth (2017). Modern Portfolio Theory, Digital Portfolio Theory and Intertemporal Portfolio Choice. SSRN Electronic Journal, (), –. doi:10.2139/ssrn.2956060
3. Duho, K. C. T., Duho, D. M., & Forson, J. A. (2023). Impact of income diversification strategy on credit risk and market risk among microfinance institutions. Journal of Economic and Administrative Sciences, 39(2), 523-546.
4. Koumou, Gilles Boevi (2020). Diversification and portfolio theory: a review. Financial Markets and Portfolio Management, (), –. doi:10.1007/s11408-020-00352-6
5. Parkinson, C. S. (2020). Maximizing Returns for Investors Using Modern Portfolio Theory and the Efficient Frontier.
6. Liu, Y. (2022, March). Application of modern portfolio theory in the stock market. In 2022 7th International Conference on Financial Innovation and Economic Development (ICFIED 2022) (pp. 2653-2658). Atlantis Press.
7. Chatigny P, Sergienko I, Ferguson R, et al. Learning the Efficient Frontier[J]. Advances in Neural Information Processing Systems, 2024, 36
8. Bodnar, Taras; Schmid, Wolfgang (2009). Econometrical analysis of the sample efficient frontier. The European Journal of Finance, 15(3), 317–335. doi: 10. 1080/ 135 184 708 02423478.
9. Linqiu Dai.Empirical analysis of optimised portfolio allocation based on Markowitz and index models. Dean&Francis Press, Vol. 1 No. 5 (2024): Issue 5 DOI: https:// doi. org/ 10. 61173/kz021e65
10. Zhang, J. (2023). The Optimal Allocation of Securities for Portfolios. Youth Business Review, 1(1).