Balance Risk and Return: Application of Markowitz Portfolio Theory in the Investment Field

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Abstract. The capital market has become more active but complex in recent years. Many risk-averse investors find it urgent to find a way to obtain relatively higher returns while suffering the same level of risk. This paper introduces Markowitz's Portfolio Theory, which cares about expected returns and standard deviation of risky assets, as a creditable approach to solving this problem. This paper will first explain the benefit of diversification implied by this theory. Then, it will take gold, the stock of Ford Motor and Manning&Napier High Yield Bond Series(MNHAX) as sample risky assets of commodity, equity and bond, respectively, to form a portfolio that captures the most common investment products. Next, this portfolio will be optimised through Lagrangian based on maximising the Sharpe ratio, which will be conducted by Python, to be seen as an application of the theory in the investment field. The result shows the highest Sharpe ratio and the weight of each asset. This paper is expected to remind investors of the importance of diversification and provide a proper method of balancing risk and return and comparing different assets to improve their portfolio.

Keywords: Markowitz Portfolio Theory, risk-return balance, portfolio optimization.

1 Introduction

In recent years, more and more investors hope to obtain higher returns through capital markets. However, most investments carry certain risks while having returns. Generally, getting higher returns means investors need to suffer higher risks. On the contrary, if investors have to endure higher risks, they usually expect higher returns [1]. According to Markowitz, uncertainty is a risk that cannot be easily ignored when optimising investors' investment behaviour [2]. It is generally believed that many investors are risk-averse in reality, which means they prefer lower risk to higher risk for a given situation [1]. Moreover, they also have different risk aversion or risk preferences. Therefore, they are eager to find a balance between risk and return. This implies that investors will sacrifice some returns in exchange for lower risk. However, the returns cannot be too low; otherwise, it goes against the original purpose of the investment. On this basis, investors should not only invest in assets that have relatively low risk and

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yield, such as bonds and fixed incomes. They should also invest in some equity. Then, at this point, a portfolio is formed as the portfolio is composed of different assets in a particular proportion. The portfolio risk is also related to the risk of these assets. So, the investors can adjust the proportion of each asset to change the portfolio risk. This process is also called diversification, a common and reasonable investment practice [2]. However, many investors are not good at or do not know how to use diversification to help improve their portfolios.

Therefore, this paper will use three assets, commodities, equity, and bonds, to form a portfolio to demonstrate the practical application of diversification in real investment, which is the content of Markowitz's Portfolio Theory [3]. Gold, the stocks of Ford Motor and Manning&Napier High Yield Bond Series(MNHAX), will be taken as the sample.

This paper will use this theory to analyse the sample portfolio. It uses the expected return and the covariance matrix of assets to compose an optimal portfolio [3]. The matrix will be used to work out the portfolio, which balances risk and return and maximises the returns per unit of risk. The calculation process will be based on Lagrangian, a method for solving constrained optimisation problems.

About the contribution, this paper will show the validity of this theory in actual investment. It is also expected to provide a valuable tool for investors to build their portfolios to reduce risk while meeting their return expectations. Besides, it helps investors evaluate many financial assets simultaneously and allows them to adjust their portfolios if they find a better choice.

The rest of the paper will be structured as follows: Section 2 will introduce the methodology used in the analysis process. Section 3 will apply the methods mentioned in Section 2 to make the analysis and then build the portfolio according to the financial assets selected. Section 4 summarises the study and gives our recommendation for applying this theory.

2 Methodology

2.1 Markowitz Portfolio Theory

The expected return and expected risk are two decisive factors in financial decision-making. For risk-averse investors, they want to have relatively high returns while enduring lower risk. Unfortunately, high returns are always associated with high risks, so investors must make trade-offs.

However, Markowitz's Portfolio Theory, first raised in 1952[4], provides a modern and efficient way to balance the two factors and help investors make better decisions. They assume that investors only care about returns and risk. According to Markowitz[3], investors should invest in a portfolio of many assets instead of any single one in this condition. This changes investors' risk of a single asset into a portfolio risk, which is decided by the proportion of the risky assets chosen. The return also becomes the mathematical weight average of all assets' expected returns. Here is a two-asset example:

$$\mu_P = W_1\mu_1 + W_2\mu_2 \ (W_1 + W_2 = 1) \tag{1}$$

Where μP, $\mu 1$, $\mu 2$ are the expected return of the portfolio, asset 1 and asset 2, respectively. W1 W2 are the fraction of initial wealth allocated to assets 1 and 2, respectively.

To get the portfolio risk, the covariance matrix is needed first. It is shown in Table 1.

Table 1. The covariance matrix of the portfolio variance

	$W_1\mu_1$	$W_2\mu_2$
$W_1\mu_1$	$W_1^2\sigma_1^2$	$W_1W_2\sigma_{12}(W_1W_2\sigma_1\sigma_2\rho_{12})$
$W_2\mu_2$	$W_1W_2\sigma_{12}(W_1W_2\sigma_1\sigma_2\rho_{12})$	$W_2^2\sigma_2^2$

So, the portfolio variance is the sum of all table entries. It can be expressed as:

$$\sigma_P^2 = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2\sigma_{12} = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2\sigma_1\sigma_2\rho_{12} \tag{2}$$

Where $\sigma p,1,2$ represents the portfolio and asset 1,2 standard deviation, respectively. Moreover, $\sigma 12$ is the covariance between assets 1 and 2, while $\rho 12$ denotes the correlation between assets 1 and 2.

Note that the portfolio's risk is represented by its standard deviation, which is the square root of its variance. The mean-variance frontier is now drawn, which minimises the risk with a given expected return [4].

Based on Fig.1, it is clear that the portfolio risk is decreased unless the two assets are perfectly positively correlated. If investors choose the assets with little positive or negative correlation to form their portfolios, the portfolio risk will be even lower. This is the benefit of diversification, which strengthens with the decrease of the correlation or the increase in the number of assets invested. Notice that the segment above the global minimum variance point G has a higher return than below G at any given risk. So, such a frontier with higher returns is called the "efficient frontier".

Next, according to Tobin, a risk-free asset should be introduced [5]. Then, each portfolio can be viewed as a combination of a risky and risk-free asset[6]. For those risky portfolios on the efficient frontier, the Sharpe ratio will give a risk-return trade-off between them [7]. It is given by Eq. (3).

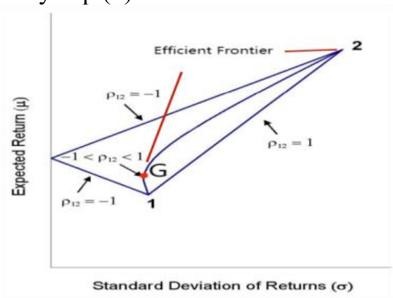

Fig. 1. The mean-variance frontier

Source: Kwan, C. C. (2010). The requirement of a positive definite covariance matrix of security returns for mean-variance portfolio analysis: a pedagogic illustration. Spreadsheets in Education, 4(1).

$$\text{Sharpe ratio} = \frac{\text{rp}-\text{rf}}{\sigma\text{p}} \tag{3}$$

Where rp, rf and σp are the portfolio return, risk-free rate and portfolio risk, respectively.

Notice that in Eq.(3), the Sharpe ratio increases if the portfolio return or risk decreases. Therefore, the portfolio with the highest Sharpe ratio is desirable, and it is also called the tangency portfolio. Because it is the point where the straight line going through rf is tangent to the frontier, so risk-averse investors should choose the combination of the tangency asset and the risk-free asset. This is the separation theorem. It reflects that the optimal choice can be decided separately based on an investor's risk preference [6].

2.2 Lagrangian

Lagrangian should support the calculation to work out the highest Sharpe ratio [7,8].To solve an optimisation problem in the form:

To max f(x), Subject to:

$$h(x) \geq 0 \tag{4}$$

Where f: Rn→ R, h: Rn→ Rm are given functions.

Now develop the Lagrangian:

$$L(x,\lambda) = f(x) + \lambda h(x) \tag{5}$$

Where λ is the Lagrangian multiplier.

According to Kuhn-Tucker, If x* maximises f (x) subject to h(x)≥ 0, there exists such a value λ*≥0 that satisfies the first order condition where x* occurs [9].

$$f'(x^*) + \lambda^* h'(x^*) = 0 \tag{6}$$

Then, the problem can be solved through Eq.(6).

3 Discussion and Analysis

3.1 Basic Description of the Sample Assets

This paper selects gold, Ford Motor stock, and Manning & Napier High Yield Bond Series as three representative risky assets: commodity, equity, and bond.

Gold investing has been popular for a long time. As an investment, gold trading happens physically and in electronic platforms [10]. The gold price surged significantly in the latter years, attracting many speculators and investors [10]. In recent years, investors have preferred gold over other risky assets because of its reduced volatility and consistency in providing returns compared to common stocks [11].

Ford Motor, founded in 1903, is one of the leading automotive manufacturing companies in the US [12]. In 2023, it gained a revenue of $176.191 billion [13]. Besides, Ford Motor mainly provides customers with vehicles like trucks, cars, and SUVs [12].

Manning&Napier High Yield Bond Series(MNHAX) was developed in 2012 [14]. It now becomes an active bond in the bond market with a total return of 6.19% for 5-year bonds, 4.60% for 3-year bonds and 12.72% for one-year bonds [14].

3.2 Analysis of the Data

First, data from 2013 to 2023 was collected from Yahoo Finance (Table 2) to compute the descriptive data of the three risky assets, as shown in Table 3. According to Table 1, the covariance matrix of the three sample assets and the correlations between them will be shown in Table 4 based on the results of Tables 2 and 3.

Table 2. Adjust close prices from 2013 to 2023

Adj close Price ($)			
Date	Gold (per ounce)	Ford Motor (stock)	MNHAX
December 31st, 2013	1201.90	9.29	4.53
December 31st, 2014	1183.90	9.63	4.64
December 31st, 2015	1060.30	9.11	4.50
December 31st, 2016	1150.00	8.39	5.11
December 31st, 2017	1306.30	9.12	5.56
December 31st, 2018	1278.30	5.99	5.50
December 31st, 2019	1519.50	7.76	6.29
December 31st, 2020	1893.10	7.46	6.70
December 31st, 2021	1827.50	17.72	7.39
December 31st, 2022	1819.70	10.25	6.84
December 31st, 2023	2062.40	11.87	7.72
Mean	1482.08	9.69	5.89

Source: Yahoo Finance

Table 3. Description of expected annual returns for sample risky assets

	Expected annual returns (Mean)	Standard deviation
Gold	6.09%	11.32%
Ford Motor	10.16%	49.62%
MNHAX	5.73%	7.64%

Table 4. The covariance matrix with correlations

	Gold	Ford Motor	MNHAX
Gold	0.012805	-0.002539	0.005402
Ford Motor	-0.002539	0.246186	0.019803
MNHAX	0.005402	0.019803	0.005833
Correlation	ρ(Gold,Ford)=-0.045	ρ(Gold,MNHAX)=0.625	ρ(Ford,MNHAX)=0.523

Note that all three assets are not perfectly positively correlated, so it is possible to use them to form a portfolio to obtain the benefit of diversification.

First, deciding the weight of each asset invested is necessary. to form a portfolio In this condition, assume that W1, W2, and W3 are the weights allocated to gold, the Ford Motor stock, and the MNHY bond series, respectively. With Eq.(1), the portfolio return can be expressed as:

$$\mu_P = W_1\mu_G + W_2\mu_F + W_3\mu_B \ (W_1 + W_2 + W_3 = 1) \tag{7}$$

Where $\mu_{G,F,B}$ are expected returns of gold, stock of Ford Motor and MNHAX bond series respectively.

In addition, based on Eq.(2), the portfolio variance will be:

$$\sigma_P^2 = W_1^2\sigma_G^2 + W_2^2\sigma_F^2 + W_3^2\sigma_B^2 + 2W_1W_2\sigma_{GF} + 2W_1W_3\sigma_{GB} + 2W_2W_3\sigma_{FB} \tag{8}$$

Where σG,F,B stands for the standard deviation of gold, stock of Ford Motor and MNHAX bond series respectively.σGF,GB,FB are the covariance between gold and stock, gold and bond series and stock and bond series, respectively.

3.3 Optimise the Portfolio

The next step is to optimise the portfolio. According to the 10-year treasury rate as of February 20th, 2024, the risk-free rate rf should be 4.27%. [15] Recalling Eq.(3)/(7)/(8) the Sharpe ratio of the portfolio is:

$$\frac{W1\mu G + W2\mu F + W3\mu B - rf}{[W1^2\sigma G^2 + W2^2\sigma F^2 + W3^2\sigma B^2 + 2W1W2\sigma GF + 2W1W3\sigma GB + 2W2W3\sigma FB]^{1/2}} \tag{9}$$

Then, the optimisation problem will be developed according to (4) and (9) to max Eq. (9) under the constraints mentioned in Eq. (10).

$$W1 + W2 + W3 - 1 = 0, \ \& \ W1, W2, W3 \geqq 0 \tag{10}$$

Next apply Eq.(5) and Eq.(10) to get the Lagrangian:

$$L(W1,W2,W3,\lambda) = \frac{w1\mu G + W2\mu F + W3\mu B - rf}{[W1^2\sigma G^2 + W2^2\sigma F^2 + W3^2\sigma B^2 + 2W1W2\sigma GF + 2W1W3\sigma GB + 2W2W3\sigma FB]^{1/2}} + \lambda(W_1 + W_2 + W_3 - 1) \tag{11}$$

Applying Eq.(6) and Eq.(11) to get the first order condition:

For W_1,

$$\frac{\mu G\sigma P-(w1\mu G+W2\mu F+W3\mu B-rf)(\sigma G^{2}W1+W2\sigma GF+W3\sigma GB)\sigma P^{-1}}{W1^{2}\sigma G^{2}+W2^{2}\sigma F^{2}+W3^{2}\sigma B^{2}+2W1W2\sigma GF+2W1W3\sigma GB+2W2W3\sigma FB}+\lambda=0 \tag{12}$$

For W_2,

$$\frac{\mu F\sigma P-(w1\mu G+W2\mu F+W3\mu B-rf)(\sigma F^{2}W2+W1\sigma GF+W3\sigma FB)\sigma P^{-1}}{W1^{2}\sigma G^{2}+W2^{2}\sigma F^{2}+W3^{2}\sigma B^{2}+2W1W2\sigma GF+2W1W3\sigma GB+2W2W3\sigma FB}+\lambda=0 \tag{13}$$

For W_3,

$$\frac{\mu B\sigma P-(w1\mu G+W2\mu F+W3\mu B-rf)(\sigma B^{2}W3+W1\sigma GB+W2\sigma FB)\sigma P^{-1}}{W1^{2}\sigma G^{2}+W2^{2}\sigma F^{2}+W3^{2}\sigma B^{2}+2W1W2\sigma GF+2W1W3\sigma GB+2W2W3\sigma FB}+\lambda=0 \tag{14}$$

For λ,

$$W_1+W_2+W_3-1=0 \tag{15}$$

There are four unknowns and four equations, so the unknowns have solutions.

The last step is to write a string of codes to conduct Eq. (10)/(11)/(12)/(13)/(14)/(15) through Python to solve this optimisation problem. First, the number of portfolios to simulate is set at 10000. Then, generate random weights for each asset with a sum of 1. Next, plot the portfolios on the mean-variance frontier and use the risk-free rate to get the optimal portfolio with the data shown in Table 3 and Table 4. At last, we get the mean-variance frontier and the optimal portfolio shown in Fig.2:

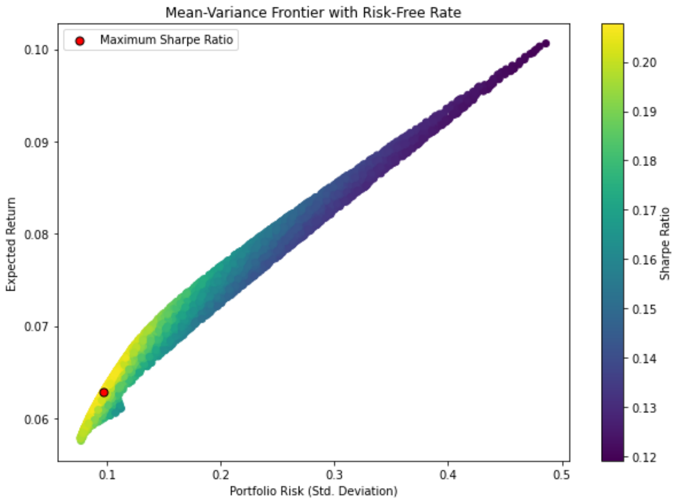

Fig. 2. The mean-variance Frontier with the risk-free rate

According to the results computed by Python, the optimal portfolio consists of 49.81% gold, 8.23% of the stock of Ford Motor and 41.96% of the Manning&Napier High Yield bond series(MNHAX). The portfolio return is about 6.27%, while the risk is about 9.64%. Moreover, the highest Sharpe ratio equals to 0.2079.

4 Conclusion

The discussion and analysis of the sample assets show a possible way of reducing risk while gaining the highest returns per unit risk. This meets the requirements of investors very well based on the present situation, where there are increasingly risky assets, and investment decisions are becoming increasingly frequent. Recalling Markowitz's Portfolio Theory, investors only care about the risk and returns in the condition that they are all assumed to be risk-averse. So, the sample application of this theory shown in this paper satisfies the demand for a balance between risk and returns quite well. The purpose of this paper is precisely this. It aims to provide reference guidance for investors to help them make better decisions.

This paper employs the Markowitz Portfolio Theory and Lagrangian methodology to help form the optimal portfolio. This theory fully applied the benefits of diversification. Investors' portfolio returns can be expressed as a weighted sum of the assets chosen, while the risk is the sum of all entries in the covariance matrix. Moreover, it reflects investors' trade-offs between risk and returns through a sharp ratio. Lagrangian is an efficient and convenient way of solving optimisation problems. It uses the Lagrangian multiplier λ to help find the optimal portfolio with the highest Sharpe ratio. Moreover, the results are determined by solving the first-order condition.

Gold, the stock of Ford Motor and the Manning&Napier High Yield Bond Series(MNHAX) are selected as samples to form the portfolio. All three are active and representative risky assets in their commodity, equity and bond categories. These three categories cover the asset selection of most investments, so the sample portfolio formed consists of them is more practical.

According to the outcome computed by Python, the optimal portfolio is composed of 49.81% gold, 8.23% of stock of Ford Motor and 41.96% of Manning&Napier High Yield bond series(MNHAX), with the highest Sharpe ratio equaling 0.2079. This means that for risk-averse investors, the focus of investment is mainly on gold and bonds, but the stock of Ford Motor is still needed to ensure a relatively good return.

Through reading this paper, investors can better understand and learn how to apply Markowitz's Portfolio Theory in real-life investment to evaluate different assets and improve their portfolios based on their risk aversion.

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