A Multiple Allocation Hub-and-Spoke Network Considering Capacity Selection in a Competitive Environment

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Abstract. The hub-and-spoke network is a crucial network structure in the modern logistics industry, significantly enhancing transportation efficiency. However, issues such as hub congestion and failure cannot be overlooked, as they have a direct impact on the operational effectiveness of this network model. Furthermore, due to substantial market potential attracting continuous capital influx, there is excessive competition within the industry leading to inefficient resource utilization. In this context, we investigate the design problem of a multiple allocation hub-and-spoke network considering capacity selection under competitive environments. To address this issue, we construct a two-layer programming model: the upper layer represents the leader's mixed integer programming model concerning hub layout schemes while the lower layer depicts followers' mixed integer programming models based on their observations of leader behavior and its influence on their own hub layout schemes. Given the complexity involved in solving this two-layer programming model and aiming to reduce computational complexity while ensuring solution quality, we have designed a variable neighborhood search algorithm for solving it. The results indicate that factors such as number of hubs, decision-making order, consideration of competitors' behavior will affect hub location schemes, capacity decisions and market share among participants in market competition.

Keywords: Competition, Capacity Selection, Multiple Allocation

1 Introduction

The rapid growth of e-commerce in recent years has significantly boosted the robustness of the logistics industry. According to data from the China Federation of Logistics and Purchasing, as a critical sector supporting national economic development, the total value of social logistics in 2022 reached 347.6 trillion yuan, marking a year-on-year increase of 3.4%, signifying that the scale of logistics demand has attained a new level with stable growth. In December 2018, the National Development and Reform Commission and Ministry of Transport jointly released the National Logistics Hub Layout and Construction Plan, underscoring that logistics hubs serve as core infrastructure within the logistics network system and play an essential role in regional transportation

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networks. Currently, most logistics enterprises employ O'Kelly's hub-and-spoke network model for their transportation network structure. Compared to direct-through networks, this model achieves economies of scale through goods aggregation at hub nodes, thereby reducing overall network operational costs.

While the utilization of economies of scale presents a significant advantage within hub networks, it may also result in potential commodity overload at select hubs. Consequently, the matter of hub capacity has been subject to more comprehensive discussion within the realm of hub-and-spoke network design. Chen respectively proposed hybrid integer linear programming models with and without capacity constraints to optimize transportation services in hybrid hub-and-spoke networks [1]. Zhao proposed a three-stage optimization model with a two-level hub-and-spoke structure that was a maximum-coverage location model that accounts for capacity constraints [2]. Huang constructs a robust optimization model of container liner routes in feeder line network, which based on the hub-and-spoke maritime network under the capacity and time constraints[3].

Additionally, hub facilities are susceptible to unforeseeable disruptions stemming from inclement weather, natural calamities, labor conflicts, and deliberate sabotage, thus the reliability of hub-and-spoke networks is often studied. Rostami proposed a two-stage formula for the single-distribution hub location problem, which involves redistributing the affected traffic to an alternate hub in the event of a hub failure [4]. Zhao proposed a new mathematical model for the design of reliable hub-spoke networks, which allows multiple distribution at non-hub nodes and involves multiple distribution modes under multiple hub failures [5]. Shen studied a reliable hub location model, taking into account not only the fixed cost of constructing the hub, but also the fixed cost generated by the junction arc that exhibits economies of scale in transportation [6].

The capacity and reliability of hub nodes also are evaluated in conjunction when examining the design of hub-and-spoke networks. Azizi studied the single assignment reliable hub location problem with multiple capacity levels and traffic-related discount factors [7]. Huang studied a design problem of hub spoke network for container shipping hubs that considered both hub failure and hub congestion, and defined congestion cost as a power function of container flow exceeding port service capacity [8].

In the above research, the competition factor in the design of hub-and-spoke network is missing. The first study on the location of competitive hubs was first proposed by Marianov [9]. From the perspective of followers, the study assumed that in a hub network where the leader has made optimal layout, followers participate in market competition by building their own hub nodes, with the goal of maximizing the market share of followers. Tiwari studies the hub location problem of hub-and-spoke network established by an airline in order to maximize its market share in the highly competitive aviation market [10]. Yang addresses a competitive airline hub-and-spoke network design problem in a duopolistic market with sequential airline entry [11].

To sum up, the current hub location model design still has two aspects of less attention:(1) The combination of competition and failure; (2) The combination of competition and capacity. Therefore, this study focuses on multiple allocation hub-spoke network design considering capacity selection under competitive environment.

2 Problem Description and Model

2.1 Problem Description

In the logistics industry market, the timing of logistics enterprises is not equal to the competition. Logistics enterprises make decisions according to the order of entering the industry market, and the different order of decision will make them in different positions. In a market divided by geographical region, there are two enterprises providing homogeneous logistics and distribution services, among which the first one to enter the market is called the leader, and the one who enters the market after the leader is the follower. Both companies provide logistics and transportation services to all consumers in the region through their own hubs, and their goal is to gain the most market share. The problem consists of three stages: the first stage is the leader chooses the location and capacity level of the hub node, and decides the allocation scheme of non-hub nodes under normal conditions and failure conditions. In the second stage, followers choose the location and capacity level of hub nodes according to the leader's hub layout, and decide the distribution plan of non-hub nodes under normal conditions and failure conditions. In the third stage, customers assign probabilities according to the combined service cost provided by the leaders and followers.

2.2 Mathematical Notations

Table 1. Mathematical notations and definitions of various parameters and variables.

Var.	definition
N	Set of nodes in hub-and-spoke network
H^p	Set of the potential hub nodes for leader
H^r	Set of the potential hub nodes for follower
L	Set of the hub capacity levels
w_{ij}	The flow between node i and node j
c_{ij}	The transportation cost between node i and node j
F_k^l	The cost for installing a hub with capacity level l at node k
Q_k^l	The capacity level l at node k
a	Discount factor between hubs
p	The number of hub nodes for leader
r	The number of hub nodes for follower
β_{ij}	The service cost between node i and node j provided by the leader
γ_{ij}	The service cost between node i and node j provided by the follower
S_{ij}	The share that the leader gets between node i and node j
s_{ij}	The share that the leader gets between node i and node j
h_k^l	1 if capacity level l is decided for hub k and 0 otherwise
X_{ikmj}	1 if flow from node i to node j routed via hubs located at nodes

	k and m of the leader and 0 otherwise
x_{ikmj}	1 if flow from node i to node j routed via hubs located at nodes k and m of the follower and 0 otherwise
Z_k	1 if node k is selected as the hub of leader and 0 otherwise
z_k	1 if node k is selected as the hub of follower and 0 otherwise
U_{kn}	1 if n is the backup hub of leader when hub k is disrupted and 0 otherwise
u_{kn}	1 if n is the backup hub of leader when hub k is disrupted and 0 otherwise

2.3 Multiple Allocation under Competition Model

The service cost between node i and node j provided by the leader (including transportation costs and additional service charges depending on the level of capacity) can be expressed as:

$$\beta_{ij} = (c_{ik} + ac_{km} + c_{mj})X_{ikmj} + \frac{\sum_{k\in H^p}\sum_{l\in L}F_k^l h_k^l}{\sum_{i\in N}\sum_{j\in N} w_{ij}} \tag{1}$$

The same can be obtained:

$$\gamma_{ij} = (c_{ik} + ac_{km} + c_{mj})X_{ikmj} + \frac{\sum_{k\in H^r}\sum_{l\in L}F_k^l h_k^l}{\sum_{i\in N}\sum_{j\in N} w_{ij}} \tag{2}$$

After the leaders and followers have make their respective hub location decision, the share obtained by the leaders for the flow between node i and node j can be calculated by the following formula:

$$S_{ij} = \frac{\frac{1}{\beta_{ij}}}{\frac{1}{\beta_{ij}}+\frac{1}{\gamma_{ij}}} = \frac{\beta_{ij}\gamma_{ij}}{\beta_{ij}(\beta_{ij}+\gamma_{ij})} \tag{3}$$

The same can be obtained:

$$s_{ij} = \frac{\frac{1}{\gamma_{ij}}}{\frac{1}{\beta_{ij}}+\frac{1}{\gamma_{ij}}} = \frac{\beta_{ij}\gamma_{ij}}{\gamma_{ij}(\beta_{ij}+\gamma_{ij})} \tag{4}$$

Lower-level Follower Hub Location Model.

$$max\sum_i\sum_j s_{ij}w_{ij} \tag{5}$$

$$s.t$$

$$\sum_k z_k = r \tag{6}$$

$$\sum_m x_{ikmj} \le z_k \ \forall i, j, k \tag{7}$$

$$\sum_k x_{ikmj} \le z_m \ \forall i, j, m \tag{8}$$

$$\sum_i \sum_j \sum_m s_{ij} w_{ij} x_{ikmj} \leq \sum_{l \in L} Q_k^l h_k^l \ \forall k \tag{9}$$

$$\sum_{l \in L} h_k^l = z_k \ \forall k \tag{10}$$

$$\sum_{n \neq k} u_{kn} = h_n^l \ \forall i \neq k \tag{11}$$

$$u_{kn} \leq z_n \ \forall k, n, i \neq k \tag{12}$$

$$\sum_i \sum_j \sum_m s_{ij} w_{ij} x_{inmj} + \sum_{i \neq k} \sum_{j \neq k} s_{ij} w_{ij} x_{ikmj} u_{kn} \leq \sum_{l \in L} Q_k^l h_k^l \ \forall k, n \tag{13}$$

$$x_{ikmj}, z_k, h_k^l, u_{kn} \in \{0,1\} \tag{14}$$

The symbols of the model formula are shown in Table 1. The objective function (5) represents that followers seek to maximize their market share. Constraints (6) mean the number of hubs to be built. Constraints (7) and (8) ensure that all traffic transportation paths between all nodes pass through the hub. Constraints (9) indicate that the traffic entering the hub does not exceed the capacity limit of the hub. Constraints (10) indicate that a hub can only select one capacity level. Constraints (11) ensure that there is only one backup facility for non-hub nodes. Constraints (12) ensure that the backup facilities of non-hub nodes are hub nodes; Constraints (13) ensure that the traffic entering the hub does not exceed the capacity limit of the hub when the hub fails; Constraints (14) represent binary variable constraints.

Upper-level Leader Hub Location Model.

$$max \sum_i \sum_j S_{ij} w_{ij} \tag{15}$$

$$s.t$$

$$\sum_k Z_k = p \tag{16}$$

$$\sum_m X_{ikmj} = z_k \ \forall i, j, k \tag{17}$$

$$\sum_k X_{ikmj} = z_m \ \forall i, j, m \tag{18}$$

$$\sum_i \sum_j \sum_m S_{ij} w_{ij} X_{ikmj} \leq \sum_{l \in L} Q_k^l h_k^l \ \forall k \tag{19}$$

$$\sum_{l \in L} h_k^l = Z_k \ \forall k \tag{20}$$

$$\sum_{n \neq k} U_{kn} = h_n^l \ \forall i \neq k \tag{21}$$

$$U_{kn} \leq z_n \ \forall k, n, i \neq k \tag{22}$$

$$\sum_i \sum_j \sum_m S_{ij} w_{ij} X_{inmj} + \sum_{i \neq k} \sum_{j \neq k} S_{ij} w_{ij} X_{ikmj} U_{kn} \leq \sum_{l \in L} Q_k^l h_k^l \ \forall k, n \tag{23}$$

$$x_{ikmj}, z_{ik}, h_k^l, \ u_{ikn} \in \{0,1\} \tag{24}$$

The objective function (15) represents that leaders seek to maximize their market share. Constraints (16) mean the number of hubs to be built. Constraints (17) and (18) ensure that all traffic transportation paths between all nodes pass through the hub. Constraints

(19) indicate that the traffic entering the hub does not exceed the capacity limit of the hub. Constraints (20) indicate that a hub can only select one capacity level. Constraints (21) ensure that there is only one backup facility for non-hub nodes. Constraints (22) ensure that the backup facilities of non-hub nodes are hub nodes; Constraints (23) ensure that the traffic entering the hub does not exceed the capacity limit of the hub when the hub fails; Constraints (24) represent binary variable constraints.

3 Solution Algorithm

The established model is a two-element dual-layer mixed integer programming framework. The upper layer addresses the leader's decision on hub location, considering potential follower competition, while the lower layer focuses on hub location after followers observe the leader's choice. The upper constraint closely relates to the lower programming solution, rendering this non-convex and non-smooth model a complex optimization problem falling within NP-hard category. However, for NP-hard problems with exponentially increasing difficulty as scale expands, precise solutions may lead to slow processing and high costs. Hence, this paper employs the variable neighborhood search algorithm for resolution.

The algorithm proceeds as follows: initially generating a random scheme for the leader's hub location; subsequently selecting by followers the hub location scheme with maximum market share based on the leader's layout; finally reselecting by leaders their hub location scheme targeting maximum market share according to followers' choices until no further increase in leader's market share occurs. This paper proposes a variable neighborhood search algorithm encompassing four types of neighborhood structures: (1) exchanging identities between a hub node and a non-hub node assigned to that pivot point generates new solutions in first structure; (2) exchanging distribution of two non-hub nodes creates new solutions in second structure; (3) randomly changing assignment of a non-hub node constitutes third structure; (4) randomly selecting two hub nodes and exchanging all assigned non-hub nodes forms fourth structure.

4 Case Study

In this section, CAB standard data set is used, and the first 15 nodes are selected for this experiment. By calculating the total traffic to be transmitted in the entire transportation network, six capacity levels are generated, which are 15%, 30%, 45%, 60%, 75% and 99% of the total flow, and the cost for installing a hub are set at 25 million, 50 million, 75 million, 100 million, 125 million and 150 million yuan respectively. The discount factor is set to 0.6 for both leaders and followers.

4.1 The Experimental Result

The focus of this section is on the decision-making process of leaders in selecting their hub layout plan, taking into account the choices made by followers. It is important to note that the followers' decisions are contingent upon those made by the leaders, with the latter aiming to maximize their own market share through their hub layout plan. The objectives of this study are as follows: (1) To compare and observe the additional market share gained by leaders when not considering follower behavior. (2) To assess and compare the extent of improvement in leader hub capacity utilization when follower behavior is taken into consideration, as opposed to when it is not considered. (3) To analyze and observe how a leader's consideration of follower behavior impacts both sides' hub allocation schemes.

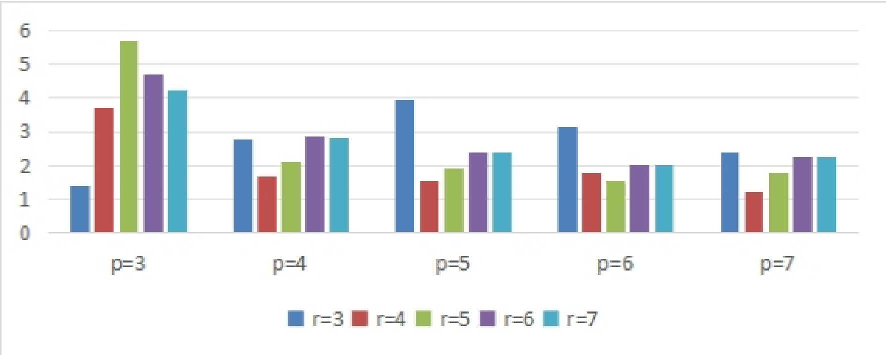

Fig. 1. Market share gains of competitive strategy under different number of hubs for the leader

The advantages of a competitive strategy for leaders in a multi-distributed hub-and-spoke network are depicted in Figure 1 under various scenarios. In this network, irrespective of the number of hubs, leaders who take into account follower competition behavior can attain an additional market share ranging from 1.77% to 6.38%, compared to those who do not consider follower competition behavior. It is noteworthy that the overall market share gains achieved through competitive strategies decrease as the number of hubs increases, indicating that an escalation in the number of hubs may mitigate the impact of followers on their market share. This outcome is attributed to the fact that the decision-making scope for leaders is influenced by the quantity of hubs, thus alterations in a leader's strategy may have relatively constrained effects.

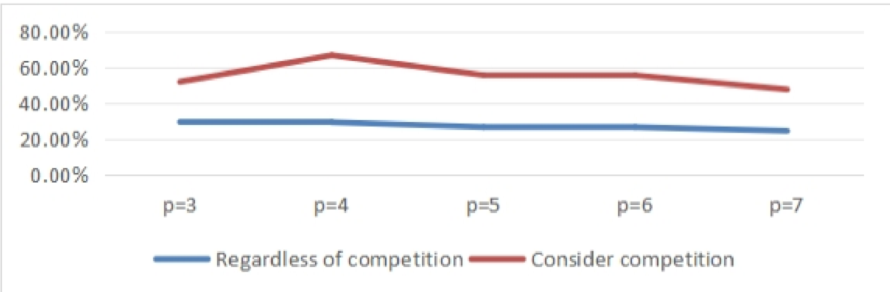

Fig. 2. Comparison of average capacity utilization under different number of hubs for the leader.

In a multi-allocation hub-and-spoke network, taking into account the behavior of followers can result in a substantial enhancement of the leader's hub capacity utilization rate. Specifically, as shown in Figure 2, this enhancement ranges from a minimum of 25.37% to as high as 41.15%, compared to scenarios where the leader does not consider follower behavior. The observed increase in capacity utilization can be attributed to both reduced construction costs for the leader's hub and an expansion of market share.

Table 2. Hub allocation scheme for Leader and Follower.

		Regardless of competition		Competitive considerations	
p	r	Leader location scheme	Follower location scheme	Leader location scheme	Follower location scheme
3	3	4,12,13	4,6,7	4,7,12	4,7,12
4	4	4,7,12,14	4,6,7,12	4,7,9,12	4,7,9,12
5	5	4,7,9,12,14	1,4,7,9,12	1,4,7,9,12	1,4,7,9,12
6	6	3,4,6,7,12,14	1,3,4,7,9,12	1,3,4,7,9,12	1,3,4,7,9,12
7	7	1,3,4,7,9,12,14	1,3,4,7,9,12,14	1,3,4,7,9,12,14	1,3,4,7,9,12,14

Table 2 records the hub location results in a multiple allocation hub-and-spoke network with the leader considering and not considering the competitive behavior of the followers. By comparing the observations, whether the leader considers the impending competition will affect his or her own decision and, in turn, the decision of the followers. Combining Figure 3 and observing the number of times each node is selected in Table 2, it can be found that in all cases, nodes 4 and 9 are chosen by the leader and the followers the most. This is because, relative to the other nodes, nodes 4 and 9 have a relative advantage in traffic and location. On the contrary, nodes 10, 12, and 14 have never been chosen, as they are too far away from other nodes and have poor traffic flow, leading the leader and the followers to be un-willing to choose them as hub sites.

Fig. 3. The flow of each node and its distance from the rest of the nodes.

5 Conclusions

Building upon existing literature, this study extends the classical multiple allocation hub-and-spoke network model to the hub competition location model that incorporates capacity decisions and reliability. Subsequently, a variable neighborhood search algorithm is designed for solving the problem. Computational experiments using the CAB dataset demonstrate the necessity of researching hub location problems considering competition.

By observing and summarizing the experimental data, this study ultimately concludes: (1) The number of hubs affects both leader's hub location plan and overall service cost in the multiple allocation hub-and-spoke networks. An increase in the number of hubs does not completely change the original hub location plan. Generally, it adds other hubs to the original plan. However, an increase in the number of hubs significantly reduces overall service costs, thereby facilitating market share acquisition. (2) Competitive strategies significantly impact leader's hub location plans, capacity decisions, and market share in the multiple allocation hub-and-spoke networks. Hub location plans considering competitive strategies differ greatly from those without consideration of competitive strategies; furthermore, considering competitive strategy can greatly improve capacity utilization rates and increase market share. (3) Node selection as a hub point exhibits significant correlation with its flow level as well as geographical position. Nodes with higher flow levels and more central geographical positions are associated with higher priority for selection as hubs points.

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