Visual teaching of Higher Mathematics based on GeoGebra

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Abstract. GeoGebra is a dynamic software that combines geometry and algebra, which is a means to achieve visualization, using GeoGebra to realize the visualization of the teaching content of Higher Mathematics by taking the limits of the number series, Taylor's formula, and rotating surfaces as examples, turning abstraction into concrete, static into dynamic, combination of numbers and shapes, and quantitative research problems.

Keywords: GeoGebra; Higher Mathematics; visualization.

1 Introduction

With the development of science and technology and the progress of The Times, especially the development of information technology represented by the Internet, the deep integration of education and information technology has attracted people's attention. Improving the teaching level of Higher Mathematics and developing the core quality of Higher mathematics are inseparable from the organic integration of Higher Mathematics and information technology. It is necessary to use information technology to design various learning activities to improve students' learning ability of Higher Mathematics, encourage students to solve some learning problems of Higher Mathematics through information technology, and make contributions to the modernization of education in the country[7]. Visualization methods are becoming more and more perfect with the help of information technology. GeoGebra is a visualization tool that does not require programming, is simple and easy to use, and is powerful[8]. This paper imparts GeoGebra perfectly into the teaching process of Higher Mathematics, and finds its convergence and focus in the teaching process of Higher Mathematics. For some difficult to understand mathematical concepts, GeoGebra can make visual pictures and dynamic courseware for students to understand. Through the mathematical tools in GeoGebra, combined with relevant mathematical theories, students can exercise scientific thinking and modeling ability.

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2 Research Background

Markus Hohenwarter, a professor at Atlanta University in Florida, developed GeoGebra, a free mathematical software program, in 2002. GeoGebra is a combination of the English words "Geometry" and "Algebra", which has both "geometric" and "algebraic" functions. GeoGebra can draw static graphics, but also can use sliders to display graphics dynamically, and also can perform a variety of algebraic calculations, can be a good visualization of abstract mathematical concepts[4].

The combination of algebraic window and geometric window of GeoGebra software can make the highly abstract knowledge of Higher Mathematics visualized, concrete, dynamic and diversified. Using GeoGebra software to assist the teaching of Higher Mathematics can make the abstract, boring and boring mathematics class vivid, rich and concrete, so that students have a fresh and fresh feeling, and can stimulate students' interest in learning, improve their enthusiasm, understanding and exploration, and finally improve the teaching effect of Higher Mathematics class[5].

3 Experimental Methods

Higher Mathematics studies the concepts and theoretical knowledge of differential calculus, integral calculus and geometry of space analysis. Because the theoretical knowledge of Higher Mathematics is abstract and difficult for students to understand in the process of learning, in order to enable students to better understand Higher Mathematics, visualization teaching of Higher Mathematics is carried out based on GeoGebra software[3]. Taking the limit of number series, Taylor formula, surface and its equation as examples, the visual teaching experiment is carried out to stimulate students' interest in learning, improve students' spatial imagination ability, and promote students' understanding of mathematical concepts.

3.1 Limit Visualization of Series

Limit is one of the most important and fundamental concepts in Higher Mathematics, and it is an essential tool for studying calculus. The important concepts in calculus such as continuity, derivative and integral are defined in terms of limits.

Limit Thought: Liu Hui's circle cutting technique.

Find the area of a circle of radius 1, that is π, first use the inner positive 3-sided area to approximate, and then bisect each side to get the inner positive 6-sided shape, obviously the approximate effect is better, continue flat, and so on... Liu Hui said, "If the cut is fine, the loss is small, and if the cut is cut again, so that it cannot be cut, it is in the circle, and nothing is lost!" This is our ancient limit thought.

In the course of teaching, GeoGebra software can be used to realize the idea of limit through the method of combination of number and form. In the command bar type: $A=(0,0)$, create a point A; Enter: c : circumference $(A,1)$ to create a circle with the center c of the circle A and 1 as the radius; Create a slider n with a minimum value of 3, a maximum value of 30, and an increment of 1; Enter: sequence $((1;2\pi/ni),i,0,n-1)$ in the command bar to create the inner regular polygon of the circle.

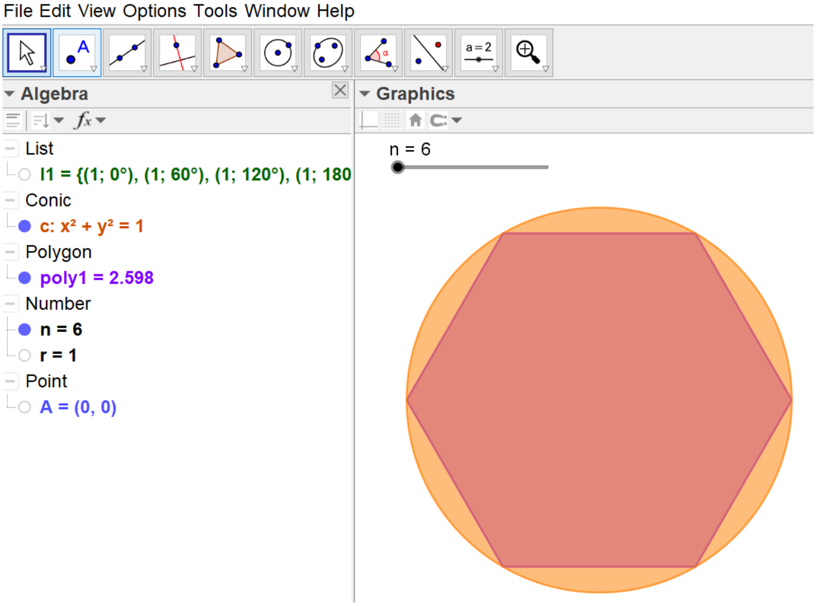

Fig. 1. Inscribed regular hexagon

Figure 1 Through GeoGebra dynamic demonstration, it can be intuitively seen that the inner regular polygon of the circle is getting closer and closer to the circle.

Series limit definition[1]: given series $\{a_n\}$ and the real number a, towards $\forall\varepsilon>0$, $\exists N=N(\varepsilon)$, when $n>N$, there is $|a_n-a|<\varepsilon$, then a is called the limit of series $\{a_n\}$, or $\{a_n\}$ converges to a. Write it as $\lim_{n\to\infty}a_n=a$ or $a_n\to a\,(n\to\infty)$. A sequence of numbers is said to converge if its limit exists, otherwise it is said to diverge.

The arbitrariness of the positive number ε reflects an infinite process of series approximation. Not the only one. N does not have to look for the lowest positive integer a that fits the definition N.

Geometric interpretation of series limit: Given $\varepsilon>0$, a ribbon-shaped region with a as the center and ε as the radius can be determined. If the limit of series $\{a_n\}$ is a, it can be seen from the definition of series limit that there must be N so that when $n>N$, all points fall into this ribbon-shaped region $a-\varepsilon<a_n<a+\varepsilon$. The former N item may be outside or within the neighborhood.

The geometric meaning of the series limit states that when $n > N$, all points a_n fall within the open interval $(a-\varepsilon, a+\varepsilon)$, and only a finite number fall outside the interval.

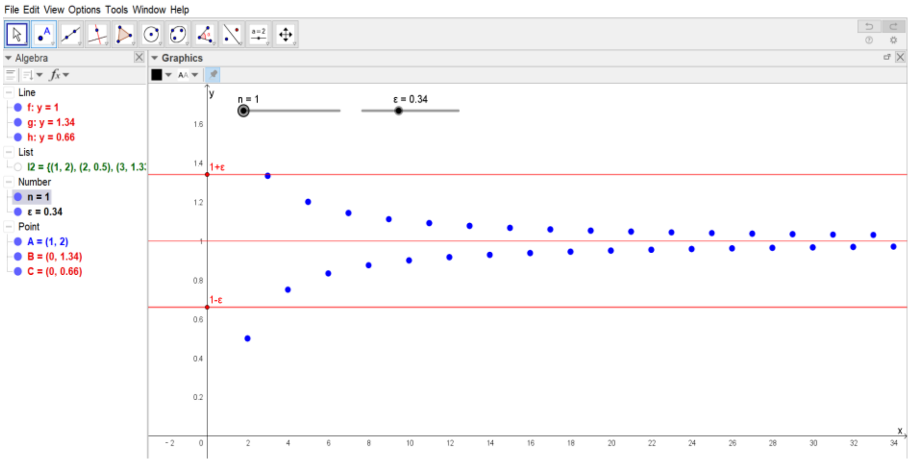

Fig. 2. Geometric significance of series limits

Figure 2 shows the geometric meaning of the series through GeoGebra, which is conducive to helping students understand the relationship between ε and N in combination with graphics. Through the organic integration of number and shape, the definition and geometric meaning of abstract sequence limit are visualized by GeoGebra software, and the mystery of sequence limit is revealed.

3.2 Taylor Formula Visualization

Taylor formula is an important content in Higher Mathematics. It is widely used in theoretical research and numerical calculation. It can be applied to approximate calculation, limit calculation, inequality proof and so on. It is a common method used to solve practical problems. Therefore, it is very important to master Taylor's formula and its thought.

Taylor's formula[1]: If the function $f(x)$ has a derivative of n at x_0, then there is a neighborhood of x_0, and for any x in that neighborhood, there is:

$$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \cdots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n + R_n(x),$$

$$R_n(x) = o((x-x_0)^n).$$

When $x_0 = 0$ Taylor's formula is called Maclaurin's formula:

$$f(x)=f(0)+f'(0)(x)+\frac{f''(0)}{2!}(x)^2+\cdots+\frac{f^{(n)}(0)}{n!}(x)^n+o(x^n).$$

The larger n is, the more closely[2] $f(0)+f'(0)(x)+\dfrac{f''(0)}{2!}(x)^2+\cdots+\dfrac{f^{(n)}(0)}{n!}(x)^n$ approximates $f(x)$. To visualize the abstract Taylor formula, Figure 3 uses GeoGebra software to show the Taylor formula for the concrete function $y=\sin x$.

Enter $f: y=\sin(x)$ in the GeoGebra command bar to create a sine function; Input : $O=(0,0)$, establish point O; Create a slider: a, the minimum value is 1, the maximum value is 21, the increment is 2; Input: $g(x)=$ Taylor formula $(\sin(x),0,a)$; The solid line is the sinusoidal function $y=\sin x$, and the dashed line is Taylor's formula.

When $a=1$, the approximate formula $\sin x\approx x$ is obtained.

When $a=3$, the approximate formula $\sin x\approx x-\dfrac{1}{3!}x^3$ is obtained.

When $a=5$, the approximate formula $\sin x\approx x-\dfrac{1}{3!}x^3+\dfrac{1}{5!}x^5$ is obtained.

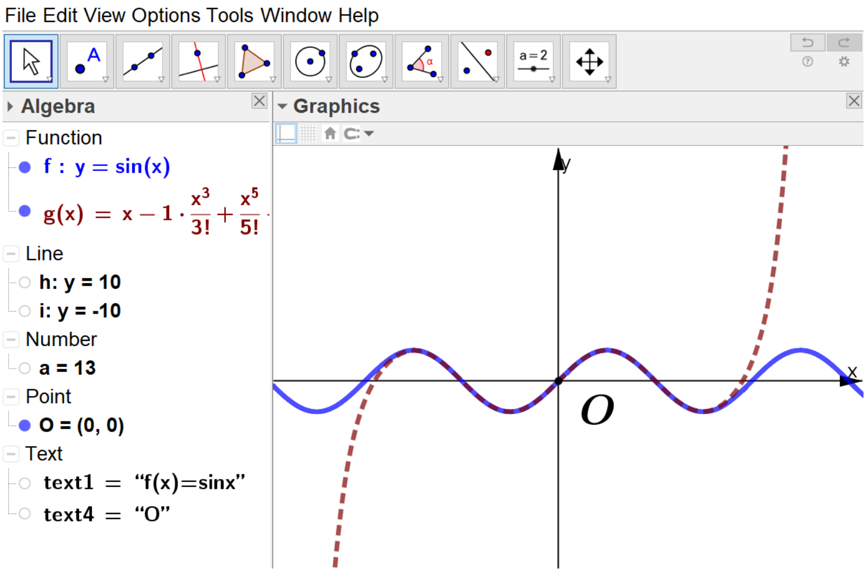

Fig. 3. Taylor expansion of order $n=13$

Through the dynamic display of GeoGebra software, the process of polynomial function $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots+(-1)^{m-1}\frac{x^{2m-1}}{(2m-1)!}$ approximating sine function $y=\sin x$ is intuitively demonstrated. It can be seen that the larger n is, the higher the approximate precision of polynomial function approximating sine function is, and the smaller the error is.

In the same way, the Taylor expansion of any function at a certain point can be obtained by GeoGebra software.

3.3 Visualization of Rotary Surface

The surface of rotation is a common special surface. It is the basis for the calculation of multiple integrals and surface integrals[6].

Definition of a surface of rotation[1]: A surface formed by rotating a plane curve around a certain straight line on the plane is called a surface of rotation. This fixed line is called the axis of the surface of rotation.

How do we determine the equation of a surface of rotation?

Problem: Given a curve $C: f(y,z)=0$ on the surface of yOz, find the equation of the surface S formed by C rotating around the axis of z.

Analysis: $\forall M(x,y,z)\in S$, C rotates around z axis and intersects C with $M_0(0,y_0,z_0)$. M, M_0 satisfy:

$$\begin{cases} f(y_0,z_0)=0, \\ x^2+y^2=y_0^2, \\ z=z_0 \end{cases}$$

By eliminating y_0, z_0 there is $f(\pm\sqrt{x^2+y^2},z)=0$. The coordinates of points not on S do not satisfy the above formula. So $f(\pm\sqrt{x^2+y^2},z)=0$ equation of S.

The axis around which the surface of rotation rotates, the variable does not change, and the other variable is replaced by the missing variable in the form of the complete square root of both positive and negative.

By the same method, the equation of the surface of rotation $f(y,\pm\sqrt{x^2+z^2})=0$ generated by the rotation of the curve C around the axis y can be obtained.

The definition of the equation of rotary surface is relatively abstract. Through GeoGebra dynamic display, the abstract definition can be visualized and the generation process of rotary surface can be experienced.

In the GeoGebra command bar input : $c=$ curve $(0,t^2/4+1,t,t,-2,2)$, create the curve on the yOz plane; Create a slider u, the minimum value is 0°, the maximum value is 360°, and the increment is 2°; Input: $d=$ surface $(c, u, z\text{axis})$, create a rotary surface. When the slider u moves, the surface of rotation pattern is obtained.

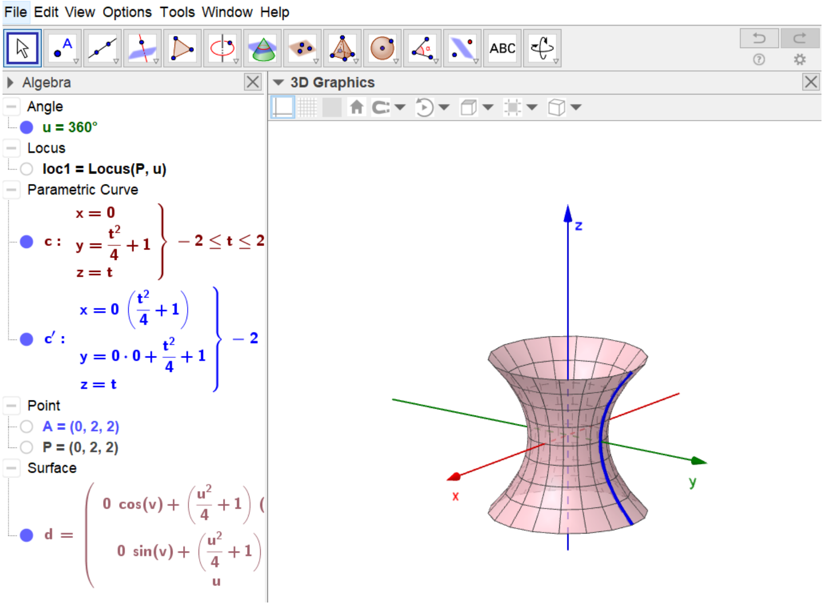

Fig. 4. Surface of rotation

As shown in Figure 4, GeoGebra shows the equation of the surface of rotation obtained by rotating the same curve around different coordinate axes, so as to analyze the characteristics of the equation of the surface of rotation.

In the GeoGebra command bar input : $a=$ curve $(t, t^2, 0, t, -3/2, 3/2)$, create the curve on the xOy plane; Create slider: α, minimum value is 0°, maximum value is 360°, and increment is 2°; Input: $b=$ Surface $(a, \alpha, x\text{axis})$. The surface of rotation about x axis is obtained.

Input: $c=$ surface $(a, \alpha, y\text{axis})$ to obtain the surface of rotation rotated about the axis of y.

GeoGebra gifs show that the equation of a surface of rotation has at least two square terms with the same coefficient, and the other variable corresponds to the axis of rotation, which axis the curve rotates around, which variable will not change.

4 Analysis of Data

In the teaching process, two district teams were used for comparative teaching, one district team used GeoGebra software for teaching, and the second district team used traditional methods for teaching. The average scores of the three tests and the final tests of the first district team were 80,83,84,82, and the average scores of the three tests and the final tests of the second district team were 76,75,80,78. The average score of the Division 2 team was significantly lower than the average score of the Division 1 team.

5 Conclusion

The application of GeoGebra in the teaching process of Higher Mathematics not only attracts students' attention, but also makes the original boring and abstract mathematics class lively and interesting. Students can ask questions about difficult math problems in the courseware, and teachers can immediately use GeoGebra software to explain, which can quickly and conveniently answer the doubts in students' minds, which is conducive to breaking the "teacher-centered" teaching method, and creating a broad space for students to ask questions and solve problems. Under the guidance of teachers, students can use GeoGebra to investigate the corresponding mathematical problems, cultivate students' awareness of independent learning, and improve students' ability to analyze problems.

For mathematical problems that were not easy to explain and demonstrate before, GeoGebra can now create appropriate teaching situations, concretize abstract problems, and transform static problems into dynamic ones, breaking through difficulties in the teaching process, allowing students to form some intuitive mathematical changes in their minds, and making it easier for students to understand the corresponding mathematical knowledge. Greater efficiency to improve the classroom teaching effect.

Exploring the perfect combination of traditional classroom teaching of Higher Mathematics and mathematics software teaching, not only reflects the rigor and meticulousness of traditional classroom, but also shows the vividness of modern technical means, so that teachers and students can better invest in the teaching and learning of Higher Mathematics.

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