Research on the Stability Analysis of Car Following Model Influenced by Multiple Factors in the Internet of Vehicles Environment

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Abstract. In the context of Internet of Vehicles (IoV) environment, although drivers can timely obtain information that affects their driving behavior through the IoV technology, but manual driving will still be necessary for a long time in the future under the current trend of technological development. Therefore, in the actual driving process, the driver not only retains memory of the status information of the front and rear vehicles themselves, but also can obtain more vehicle information through the IoV. Therefore, based on existing car following models, this paper establishes a car following model that considers the dual memory effect of the following vehicle and the information of multiple preceding vehicles, and conducts stability analysis on the model. The results show that the model can effectively improve the stability of traffic flow.

Keywords: Car following model, Multi front vehicle information, Traffic flow, IoV

1 Introduction

In recent years, with the surge in the number of vehicles, various traffic problems have arisen. The IoV, as an important application of the Internet of Things in the transportation field and the development direction of the next generation of Intelligent Transportation Systems (ITS) [1], can effectively alleviate the basic contradictions in road traffic. In the context of the IoV, real-time road traffic status information is obtained between vehicles, providing drivers with the ability to exceed line of sight, providing important information support for vehicle safety, and enabling collaborative driving between vehicles within a certain range. Modeling traffic flow in the context of IoV has become a current research hotspot. The car following model is an important micro traffic flow model, which has a wide range of applications in micro traffic flow mechanics analysis and simulation.

Numerous scholars have achieved numerous research results in modeling car following models based on the IoV. One research direction is to further expand research

Z. Ahmad et al. (eds.), *Proceedings of the 2024 5th International Conference on Urban Construction and Management Engineering (ICUCME 2024)*, Advances in Engineering Research 242,
https://doi.org/10.2991/978-94-6463-516-4_13

based on optimal velocity (OV) model and their extended models, the OV model can further simulate phenomena such as stop and go, traffic congestion, etc. As the most classic car following model, the optimal velocity model has attracted widespread attention from many scholars since its proposal [2]. On this basis, Jiang et al. [3] established a full velocity difference (FVD) model, to improve the problem of following vehicles without slowing down. In addition, with the development of IoV technology and its impact on traffic flow, scholars have begun to pay attention to car following behavior in IoV environments, such as the impact of multi vehicle information on the car following behavior. Kuang et al. [4] considered multiple expected average speed effects (including the average speed of the preceding vehicle group and the average expected speed field effect) in the Vehicle to Vehicle (V2V) environment, proposed an extended vehicle following model, and demonstrated that obtaining more vehicle information through the V2V environment has a stabilizing effect on traffic flow. Ma et al. [5] proposed an extended car following model based on ITS environment by considering dual speed difference and rear-view effect, this model can improve the stability of traffic flow for autonomous vehicles.

Based on the current research status, it can be seen that different scholars have established models that are both interrelated and complementary, considering different factors. But with the popularization of IoV, the car following in the future will also change accordingly, but for a long time, manual driving will still be the main mode. Therefore, it is very necessary to establish an extended car following model that considers both the characteristics of manual drivers and the impact of the IoV environment. Therefore, this paper constructs a car following model that considers the characteristics of drivers and the influence of multiple preceding vehicles in the context of IoV. And stability analysis and discussion of influencing factors were conducted on the proposed extended model, indicating that the proposed extended model has a positive significance for stabilizing traffic flow.

2 The Extended Model

In 2015, Cao [6] introduced the driving memory effect and constructed an average memory car following model. The equation is show as equation (1).

In equation (1), $\Delta x_n(u)$ represents the interval difference between the preceding vehicle $n+1$ and the current vehicle n at time u, τ_0 represents the driver's memory time, $v_n(t)$ represents the velocity of the current vehicle n, α represents the sensitivity parameter, and V represents the optimal velocity. Theoretical analysis and numerical simulations have shown that reducing driver memory time can alleviate traffic congestion.

$$\frac{dv_n(t)}{dt} = \alpha\left[V\left(\frac{1}{\tau_0}\int_{t-\tau_0}^{t} \Delta x_n(u)du\right) - v_n(t)\right] \tag{1}$$

In 2023, Wang et al. [5] proposed a new extended car following model considering the impact of front memory and rear-view effects in the context of ITS, with the following expression:

$$\frac{dv_n(t)}{dt}=\alpha\left[pV_F\left(\frac{1}{\tau_0}\int_{t-\tau_0}^{t}\Delta x_n(u)du\right)+(1-p)V_B\left(\frac{1}{\tau_1}\int_{t-\tau_1}^{t}\Delta x_{n-1}(r)dr\right)-v_n(t)\right]+\lambda\left[q\Delta v_n(t)+(1-q)\Delta v_{n+1}(t)\right] \tag{2}$$

In equation (2), τ_0 and τ_1 respectively represent the memory time for the state message of the front and rear vehicles, $\Delta x_{n-1}(r)$ represent the interval difference between the vehicle n and the rear vehicle $n-1$, and α represents the Sensitivity parameters for the front vehicle. Analysis results have proven that considering the dual memory effect of the rear vehicle can effectively improve traffic flow stability in ITS environment.

It can be seen that driver memory effect is an important part of studying traffic flow performance. Recent years have also seen developments in IoV technology that are actually impacting actual driving behavior, IoV can enable drivers to obtain more vehicle information through various devices. Therefore, Kuang et al. [6] considered multiple expected average velocity effects (including the average velocity of the preceding vehicle group and the average expected velocity field effect) in the IoV environment, proposed an extended vehicle following model, and demonstrated that obtaining more vehicle information through the V2V environment has a stabilizing effect on traffic flow. The equation is shown in equation (3):

$$\frac{dv_n(t)}{dt}=a\left[(1-p)V\left(\Delta x_n(t)\right)+pV_{mf}\left(\Delta x_n(t)\right)-v_n(t)\right]+\lambda\left[\overline{v}_n(t)-v_n(t)\right] \tag{3}$$

In equation (3), $V_{mf}\left(\Delta x_n(t)\right)=\frac{1}{r}\sum_{l=1}^{r}V\left(\Delta x_{n+l}(t)\right)$ is called the average expected velocity field. The parameter p has a value range of $p\in(0,0.5)$, aiming to ensure that the optimal velocity $V\left(\Delta x_n(t)\right)$ plays a Plays a dominant role in the model. In addition, as the value of parameter p gradually increases, the effect of the average expected velocity field on traffic flow operations becomes more significant. The above model also proves that with the introduction of the IoV environment, more vehicle information is considered to have a stabilizing effect on traffic flow.

In summary, currently there are no scholars who have comprehensively considered the information of rear vehicles, dual memory characteristics, and multiple front vehicles to construct a car following model in IoV environment. Therefore, this paper will establish an extended car following model for the traffic behavior of manually driven vehicles in the connected vehicle environment. The equations of the model are as follows:

$$\frac{dv_n(t)}{dt}=\alpha\left[pV_F\left(\frac{1}{\tau_0}\int_{t-\tau_0}^{t}\Delta x_n(u)du\right)+(1-p)V_B\left(\frac{1}{\tau_1}\int_{t-\tau_1}^{t}\Delta x_{n-1}(r)dr\right)-v_n(t)\right]+\lambda\left[\frac{1}{m}\sum_{l=1}^{m}v_{n+l}(t)-v_n(t)\right] \tag{4}$$

In equation (4), τ_0 and τ_1 respectively represent the memory time for the state information of the front and rear vehicles, $\Delta x_{n-1}(r)$ represent the interval difference between the current vehicle n and the rear vehicle $n-1$, and α represent the sensitivity coefficient for the front vehicle, m is the number of vehicles ahead that need to be considered in the model. According to the integral related theorem, it can be inferred that:

$$\frac{1}{\tau_0}\int_{t-\tau_0}^{t} \Delta x_n(u)du = \Delta x_n(t-\tau_2) \tag{5}$$

$$\frac{1}{\tau_1}\int_{t-\tau_1}^{t} \Delta x_{n-1}(r)dr = \Delta x_{n-1}(t-\tau_3) \tag{6}$$

In the equation (5-6), $\tau_2 \epsilon [0,\tau_0]$, $\tau_3 \epsilon [0,\tau_1]$. $\Delta x_n(t-\tau_2)$ is the average memorized distance between vehicles in time period $[t-\tau_0, t]$, and $\Delta x_{n-1}(t-\tau_3)$ is the average memorized interval between vehicles in time period $[t-\tau_1, t]$. τ_2 and τ_3 respectively represent the memory intensity of the state information of the front and rear vehicles. Furthermore, according to Taylor's formula expand $\Delta x_n(t-\tau_2)$ and $\Delta x_{n-1}(t-\tau_3)$ as flows:

$$\Delta x_n(t-\tau_2) = \Delta x_n(t) - \tau_2\frac{d\Delta x_n(t)}{dt} = \Delta x_n(t) - \tau_2\Delta v_n(t) \tag{7}$$

$$\Delta x_{n-1}(t-\tau_3) = \Delta x_{n-1}(t) - \tau_3\frac{d\Delta x_{n-1}(t)}{dt} = \Delta x_{n-1}(t) - \tau_3\Delta v_{n-1}(t) \tag{8}$$

Next, expand $V_F\left(\Delta x_n(t-\tau_2)\right)$ and $V_B\left(\Delta x_{n-1}(t-\tau_3)\right)$, the following equation can be obtained:

$$V_F\left(\Delta x_n(t-\tau_2)\right) = V_F\left(\Delta x_n(t)\right) - \tau_2\Delta v_n(t)V_F^{'}\left(\Delta x_n(t)\right) \tag{9}$$

$$V_B\left(\Delta x_{n-1}(t-\tau_3)\right) = V_B\left(\Delta x_{n-1}(t)\right) - \tau_3\Delta v_{n-1}(t)V_B^{'}\left(\Delta x_{n-1}(t)\right) \tag{10}$$

In equation (9-10), $V_F^{'}(\Delta x_n(t))$ and $V_B^{'}(\Delta x_{n-1}(t))$ are the derivatives of the optimal velocity function relative to the distance between the vehicle's headway

Substitute equations (7-10) into equation (4), which can be rewritten as:

$$\begin{aligned}\frac{dv_n(t)}{dt} &= \alpha[p(V_F(\Delta x_n(t)) - \tau_2\Delta v_n(t)V_F^{'}(\Delta x_n(t))) + (1-p)(V_B(\Delta x_{n-1}(t)) - \tau_3\Delta v_{n-1}(t)V_B^{'}(\Delta x_{n-1}(t))) - v_n(t)] \\ &+\lambda\left[\frac{1}{m}\sum_{l=1}^{m} v_{n+l}(t) - v_n(t)\right]\end{aligned} \tag{11}$$

The optimization velocity function is:

$$V_F\left(\Delta x_n(t)\right) = \alpha'\left[\tanh\left(\Delta x_n(t) - h_c\right) + \tanh(h_c)\right] \tag{12}$$

$$V_B\left(\Delta x_{n-1}(t)\right) = -\alpha''\left[\tanh\left(\Delta x_{n-1}(t) - h_c\right) + \tanh(h_c)\right] \tag{13}$$

In the optimization velocity function equation, α' and α'' are normal numbers, and h_c is the safe distance.

3 Stability Analysis

The study of traffic flow stability is essentially an important theoretical analysis method in micro traffic flow theory, which mainly focuses on the evolution of traffic flow when vehicles driving in a stable queue face random disturbance. By analyzing the stability of traffic flow, the stability conditions of traffic flow can be obtained, and then the characteristics of the traffic flow and the stability area can be analyzed. At the same time, traffic flow stability analysis has certain guiding significance for analyzing traffic congestion, traffic accidents, and guiding vehicle traffic and improving traffic efficiency [7-8]. Assuming a fleet of N vehicles traveling on a road of length L, the given fleet starts in a stable state. At this point, all vehicles maintain the same front distance h and velocity $pV_F(h) + (1-p)V_B(h)$ while driving forward. Therefore, the position solution for stable traffic flow is:

$$x_n^0\left(t\right) = hn + \left(pV_F(h) + (1-p)V_B(h)\right)t \qquad h = L/N \tag{14}$$

Assuming $y_n(t) = e^{(ikn+zt)}$ is a small disturbance applied to the steady-state position, it can be obtained that:

$$x_n(t) = x_n^0(t) + y_n(t) \tag{15}$$

Substituting equations (14-15) into equation (11) and linearizing equation (11) yields:

$$\begin{aligned}\frac{d^2 y_n(t)}{dt^2} &= \alpha[p(V_F'(\Delta y_n(t)) - \tau_2 \Delta y_n'(t) V_F'(\Delta y_n(t))) + (1-p)(V_B'(\Delta y_{n-1}(t)) - \tau_3 \Delta y_{n-1}'(t) V_B{}'(\Delta y_{n-1}(t))) - y_n'(t)] \\ &+\lambda\left[\frac{1}{m}\sum_{l=1}^{m} y_{n+l}'(t) - y_n'(t)\right]\end{aligned} \tag{16}$$

Based on the Fourier equation, expand $y_n(t)$ in equation (16), and let $\tau_2 = l_1\tau$, $\tau_3 = l_2\tau$ (l_1 and l_2 are parameters proportional to the memory strength of the running information of the front and rear vehicles, respectively), to obtain:

$$z^2 = \alpha[p(V_F'(h)(e^{ik} - 1) - z l_1 \tau V_F'(h)(e^{ik} - 1)) + (1-p)(V_B'(h)(1 - e^{-ik}) - l_2 \tau z V_B'(h)(1 - e^{-ik})) - z] + \lambda z\left[\frac{1}{m}\sum_{l=1}^{m} e^{ikl} - 1\right] \tag{17}$$

Expanding z and e^{ik} in equations (17) according to $z = z_1(ik) + z_2(ik)^2 + \cdots$ and $e^{ik} = 1 + ik + \frac{1}{2}(ik)^2 + \cdots$ yields the first and second terms of ik, respectively.

$$z_1 = pV_F'(h) + (1-p)V_B'(h) \tag{18}$$

$$z_2 = \frac{\lambda}{\alpha} z_1 \sum_{l=1}^{m} l - \frac{z_1^2}{\alpha} - z_1\left[pl_1\tau V_F'(h) + (1-p)l_2\tau V_B'(h)\right] + \frac{pV_F'(h) + (1-p)V_B'(h)}{2} \tag{19}$$

According to linear stability theory, if $z_2 < 0$, the traffic flow will become unstable, otherwise it will still be stable. So, the conditions for stabilizing traffic flow are:

$$a > 2\left[pV_F'(h) + (1-p)V_B'(h)\right] + 2\left[pl_1V_F'(h) + (1-p)l_2V_B'(h)\right] - \lambda(m+1) \tag{20}$$

4 Sensitivity Analysis of Parameters on Model Stability

Based on the above theoretical analysis, it can be seen that the changes in different parameters of the car following model will have an impact on the stability of the model. Different parameters lead to different stable regions of the car following model. Therefore, this section focuses on the influence coefficient of rear vehicles, the number of front vehicles considered, and the sensitivity coefficient of driver memory effect on the stability of the car following model, and analyzes their sensitivity to model stability.

4.1 The Influence Coefficient of Rear Vehicle

In traditional driving environments, when the front distance is small, in order to prevent dangerous driving such as rear end collision caused by the front car braking too quickly, the driver observes the running status of the following car through the rearview mirror. In the IoV environment, the current vehicle will have a more comprehensive and detailed understanding of the relevant information of the following vehicles through the communication technology. Therefore, this chapter will conduct a stability analysis of the influence coefficient of rear vehicle information on the extended model proposed in this paper.

With other parameters set to $m = 2$, $\lambda = 0.2$, $l_1 = l_2 = l = 0.1$ and kept constant, set the rear vehicle influence coefficient parameter p to 0.95, 0.9, 0.85, 0.8 and plot the distance between the front of the vehicle h and sensitivity coefficient α. The phase diagram is shown in Figure 1, where different curves represent the critical stability curves of the following model under different values of the influence coefficient of the following vehicle. According to the linear stability theory, it can be inferred that the stable zone is above the curve in the figure, and the unstable zone is below the curve. From Fig. 1, it can be seen that as the influence coefficient p of the rear vehicle decreases, the stable zone of the following model gradually expands. This indicates that the introduction of following vehicle information can maintain stable operation of the fleet within a larger range of values α , which is conducive to further enhancing the smoothness of traffic flow.

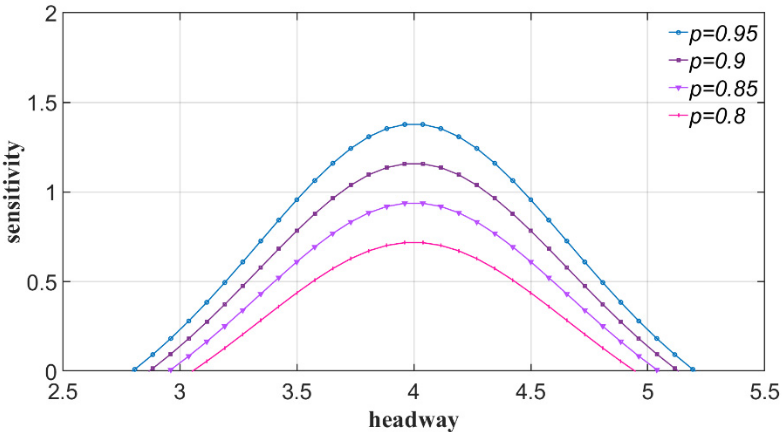

Fig. 1. Schematic diagram of the influence coefficient of the rear vehicle on the stability of the extended model

4.2 The Number of Vehicles Ahead

In actual traffic flow operation, especially in the IoV environment, only considering the optimal velocity difference term of the current vehicle will make the obtained optimal velocity information not comprehensive enough. Adequate multi vehicle state information is beneficial for following vehicles to obtain more accurate acceleration decision information. Therefore, this section will analyze the effectiveness of introducing multiple front vehicle information in the extended model and its sensitivity to model stability.

When the other influencing parameters are set to $\lambda = 0.2$, $l_1 = l_2 = l = 0.1$, $p = 0.95$ and kept constant, the number of vehicles in front of the vehicle under consideration m is set to 1, 2, 3, and 4, respectively. The phase diagram of the front distance h and the sensitivity coefficient α is plotted as shown in Fig. 2. The different curves in the Fig. 2 represent the critical stability curves of the car following model with different values of the number of vehicles ahead. According to the linear stability condition theory, the upper part of the curve in the figure is the stable zone, while the opposite part is the unstable zone. From Fig. 2, it can be seen that the stable region of the car following model is larger when the number of front cars considered is $m > 1$ than when $m = 1$, and as the number of front cars considered increases, its stable region gradually expands. This indicates that introducing information from multiple preceding vehicles is beneficial for increasing the linear stability range and can to some extent promote traffic flow stability.

4.3 The Sensitivity Coefficient of Driver Memory Effect

Although this paper is a car following model established against the backdrop of the IoV environment, it is essentially a model of manual driving. Therefore, it is necessary to take into account the inherent characteristics of the driver and conduct stability analysis. In the process of establishing the model in this article, the dual memory characteristics of the driver on the front and rear vehicles were considered. Therefore, this section analyzes the impact of the driver's memory characteristics on the stability of the model based on this.

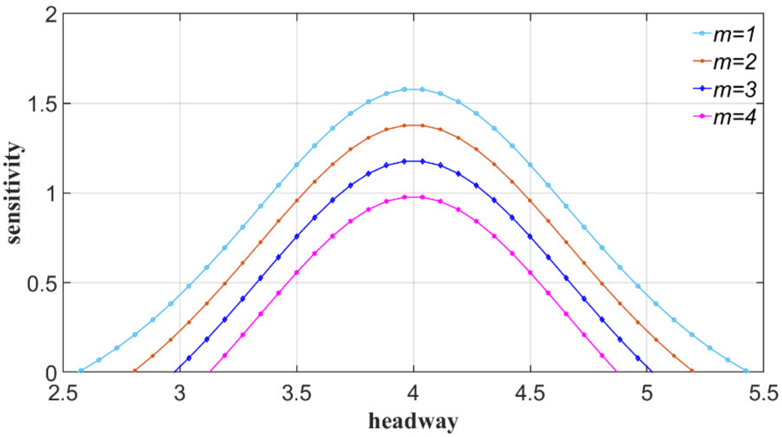

Fig. 2. Schematic diagram of the impact of the number of front vehicles considered on the stability of the extended model

When the other influencing parameters are set to $\lambda = 0.2$, $p = 0.95$, $m = 2$ and kept constant, the driver's memory characteristic parameters are $l_1 = l_2 = l$ set to 0.1, 0.2, 0.3, and 0.4 respectively. Draw the phase diagram of the front distance h and the sensitivity coefficient α as shown in Fig. 3. Different curves in the figure represent the critical stability curves of the following model with different values of the number of preceding vehicles considered. According to the theory of linear stability conditions, the upper part of the curve in the figure is the stable zone, while the opposite is the unstable zone. This indicates that if the driver minimizes memory time as much as possible, the critical traffic density and capacity will increase respectively. The above analysis indicates that if the driver reduces the memory time, the stability of traffic flow will be improved. Therefore, when $l_1 = l_2 = l = 0.1$, it means that the memory effect is relatively short, which means that the stable region is the largest. This result is consistent with real traffic scenarios.

5 Conclusion

In the IoV environment, an extended car following model is proposed to describe the traffic behavior of manually driven vehicles in the IoV environment, taking into account the dual memory characteristics of rear vehicles and multiple front vehicle information. And the critical stability conditions of the model were derived through linear stability analysis. In addition, this paper analyzes the impact of the coefficient of influence of rear vehicles, the number of front vehicles considered, and the sensitivity coefficient of driver memory effect on the stability of the extended model. The stability analysis results show that the above parameters have an important impact on the stability of the model, which strongly proves the effectiveness of the components in the model and lays a foundation for further research in the future. This article is based on research on manually driven vehicles, but in the future, there will be a long period of time for manual and autonomous vehicles to share road resources together. Therefore, the penetration rate of autonomous vehicles and other heterogeneous traffic flow situations will become a new focus of attention. In the subsequent research process, further research will be strengthened to establish a vehicle following model suitable for the aforementioned heterogeneous traffic flow.

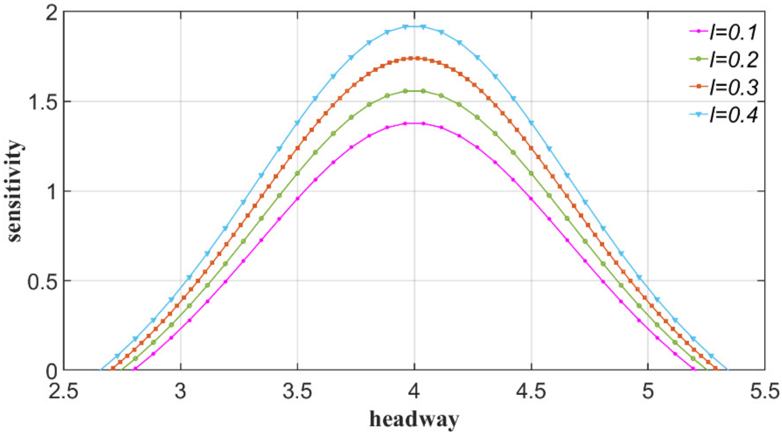

Fig. 3. Schematic diagram of the impact of driver memory sensitivity coefficient on the stability of the extended model

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