



Review on the Application of the SIR Model in Predicting Urban Traffic Congestion: Successes and Future Directions

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Abstract. Simulating the propagation of congestion in urban rail transit systems is a complex and multifaceted task, especially when dealing with periods of excessive overcrowding. This study aims to address this challenge by presenting a predictive model for traffic congestion based on the SIR (Susceptible, Infected, Recovered) model commonly used in epidemiology. By formalizing the phenomenon of congestion as a process of susceptibility recovery, we hope to provide a more comprehensive understanding of how it spreads within urban rail networks.

In developing our predictive model, we have identified six key contributing factors that influence the rate at which congestion spreads. These factors include passenger flow, train intervals, ease of passenger transfers between lines or stations, timing of congestion events throughout the day, initial station affected by congestion, and overall station capacity. By considering these factors in our model, we aim to provide transit authorities and planners with valuable insights into how they can effectively manage and mitigate congestion within their systems.

To illustrate the potential impact of our SIR-based model on managing urban rail transit congestion, we offer an illustrative example that demonstrates its practical application. Through this example scenario, we hope to showcase how our approach can enhance existing strategies for addressing overcrowding and improving overall system efficiency.

Ultimately, our goal is to contribute to the development of more effective solutions for managing congestion in urban rail transit systems through data-driven modeling and analysis. We believe that by leveraging advanced predictive models such as the SIR framework, transit authorities can make informed decisions that lead to better service reliability and improved passenger experiences.

Keywords: infectious disease control, SIR epidemic model, transportation networks.

1 Introduction

Nowadays, the world is facing an unprecedented challenge posed by the rapid spread of a contagious virus called Coronavirus or COVID-19 [1]. In response to this epidemic's swift transmission, the Chinese government accomplished several measures in late January to impede its outbreak such as imposing a lockdown on Wuhan city and closing all access routes leading to it [2]. As of now, the pandemic has been recognized in nearly 213 countries and worldwide. On January 13th, 2020, Thailand became the first country outside of China to report an outbreak of COVID-19 [3]. By May 28th, 2020, there were approximately 5.76 million cases globally with around 2.39 million recoveries and sadly resulting in about 358 thousand deaths [4].

The declaration of COVID-19 as an epidemic and Global Public Health Emergency was made by the World Health Organization (WHO) on January 30th, 2020 [5], coinciding with a significant date that highlighted its worldwide threat on March 11th, 2020 [6]. As a result, COVID-19 was classified as a global pandemic. Governments across the globe have implemented measures to contain the spread of this new coronavirus by imposing restrictions on movement and social interactions. The implementation of social distancing has been widely acknowledged as an effective strategy in managing transmission. The rapid increase in global cases has presented significant challenges in various sectors such as public health, politics, economics, education, and social behavior. Additionally, poverty rates and unemployment levels have surged globally due to the consequences of this outbreak. Managing COVID-19 has proven exceptionally difficult due to its highly contagious nature combined with prolonged incubation periods and limited understanding of how it spreads. Countries worldwide are actively collaborating in extensive efforts aimed at reducing or preventing further outbreaks [7]. Numerous healthcare organizations and pharmaceutical companies are racing against time to develop vaccines or treatments for COVID-19, however, none have achieved success thus far. The impact on the global economy has been severe with many nations facing substantial economic crises caused by the devastating effects of this virus [8].

Given the current global uncertainty, it is crucial for policymakers to possess accurate assessments of the impact COVID-19 has had so far and its potential future consequences [9]. Precisely predicting the spread of this pandemic can aid officials in implementing effective preventive measures and preparing for healthcare interventions. While obtaining precise estimations may present challenges, researchers can employ established methodologies to generate approximate forecasts. This valuable information equips authorities with vital data for making well-informed decisions regarding strategies aimed at minimizing the repercussions of COVID-19 [10].

The SIR model, a widely used mathematical framework for predicting pandemics, is based on the acronym Susceptible-Infected-Recovered. It divides individuals into three categories: those susceptible to the disease, those confirmed as infected and capable of spreading it, and those who have either recovered or died from the illness [11]. These classifications represent different stages of a contagious disease. The SIR model is particularly useful in estimating healthcare needs during an epidemic by assuming that individuals who recover from a disease gain lifelong immunity and

cannot be reinfected. Despite their simplicity, SIR models have shown significant accuracy in forecasting pandemics.

Apart from the process of transmitting, the expansion of urban traffic congestion is acknowledged as a complicated phenomenon that requires the utilization of computationally intensive microscopic models for analysis. This research introduces a framework that utilizes SIR models, similar to those employed in modeling disease transmission among populations, to demonstrate how traffic congestion spreads and dissipates within cities. By conducting empirical analyses across different cities, we validate the proposed dynamics based on contagion. This framework can be utilized for monitoring, predicting, and controlling the proportion of congested links in urban networks over time.

2 Basic Definition of Sir Model

Lorenzo Pellis and his team have made a noteworthy discovery indicating that practical scenarios can be incorporated into stochastic epidemic models [12], even without considering the temporal dynamics of the outbreak. Despite ignoring the temporal dynamics, this model still preserves the distribution of final sizes. In this research, the SIR model is utilized to examine traffic congestion on highways by considering three states: susceptible to congestion (S), experiencing congestion (I), and relieved from congestion (R). These states are represented as time-dependent functions denoted as $S(t)$, $I(t)$, and $R(t)$ respectively [13].

In areas with heavy traffic, the presence of vehicles has a stochastic impact on nearby uncongested vehicles, characterized by an average transmission rate denoted as λ . At the same time, vehicles gradually move away from the congested area at an average recovery rate of μ (transitioning from infected to recovered state). Moreover, vehicles affected by congestion can also contribute to further congestion through the average transmission rate λ (transitioning from susceptible to infected state) [14].

In the SIR propagation model, a constant total number of vehicles, denoted as N , is utilized throughout the process. The variable $S(t)$ represents vehicles experiencing free-flowing traffic conditions, while $I(t)$ signifies congested vehicles and $R(t)$ denotes departing vehicles. Consequently, there exists a differential equation that governs the interrelationships among these variables:

$$\frac{dS}{dt} = -\beta S I \quad (1)$$

$$\frac{dI}{dt} = \beta S I - v I \quad (2)$$

$$\frac{dR}{dt} = v I \quad (3)$$

3 Modified Sir Model

Given that the initial SIR model is a dynamic mathematical framework at a micro level utilizing ordinary differential equations, there exists potential for enhancement in characterizing vehicles operating in congested scenarios, particularly those engaged in continuous movement. Additionally, while addressing computationally demanding differential equations within the SIR infectious disease model tends to oversimplify intricate random behaviors encountered, adopting a cellular automata approach simplifies both the verification and resolution of complexities associated with this matter [15]. Furthermore, this alternative method enables a more precise depiction of complex stochastic behaviors by offering an improved configuration setup. As a result, simulations incorporating both temporal and spatial congestion propagation attributes become viable.

Cellular automata is a discrete model that operates on time-space and state, comprising of four essential components: cell, cell space L , cell neighbor K , and cell rule F . Specifically, $CA = (L, Sd, K, F)$, where Sd represents the set of states for each individual cell. By utilizing the CA-SIR model in conjunction with these definitions [16], it becomes feasible to effectively tackle the problem of traffic congestion propagation. A network is formed by N cells, wherein each cell corresponds to a vehicle. The future condition of a cell relies on its present state and the conditions of adjacent cells. The ensemble of cell states is denoted as $Sd_{i,j}^t$, representing the state of the cell in row i and column j at time t . We define $Sd_{i,j}^t = \{0, 1, 2\}$, where 0 indicates susceptible vehicles to congestion (S), 1 represents congested vehicles (I), and 2 denotes unaffected vehicles by congestion (R). Neighborhood rules are based on Moore-type neighbors. Evolution rule for cells: When $Sd_{i,j}^t = 0$, if there are congested vehicles nearby, each congested vehicle has a probability λ of being influenced. If successful, $Sd_{i,j}^t$ becomes 1; otherwise, $Sd_{i,j}^{t+1}$ remains as 0. When $Sd_{i,j}^t = 1$, with a probability b during each unit time step, a congested vehicle may transition into an uncongested impact vehicle. If successful, $Sd_{i,j}^t$ becomes 2; otherwise, $Sd_{i,j}^{t+1}$ remains as 1.

4 Case Study and Contributing Factors

In addition to the SIR model and propagation rate, we will also consider factors such as passenger flow patterns, train schedules, and station capacity in our quantitative analysis of urban railway networks. By taking a comprehensive approach, we aim to provide a more thorough understanding of the operational conditions and challenges faced by these networks.

The case study conducted at Beijing Xizhimen Station serves as an important example of how our approach can be applied in real-world scenarios. The station's role as an interchange for multiple lines makes it a particularly relevant location for studying oversaturation and congestion issues within urban railway systems.

Figure 1 provides a visual representation of the rail transit network at Xizhimen Station, allowing us to better understand the layout and connections between different lines. This information is crucial for identifying potential bottlenecks or areas where improvements can be made.

Our calibration process involved collecting data on various aspects of station operations, including passenger counts, walking speeds, departure intervals, travel times between stations, and total journey durations. These data points were essential for validating the accuracy of our calculations using the SIR model.

Furthermore, by comparing our findings with existing research (as presented in Tables 1, 2 and 3), we are able to situate our analysis within the broader context of urban railway network studies. This comparative approach allows us to identify any discrepancies or similarities with previous work and contributes to the overall robustness of our methodology.

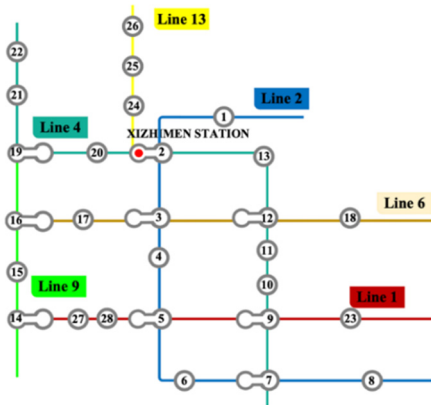


Fig. 1. A part of Beijing metro network [17]

The difference in transfer rate between line 4 and line 2 becomes apparent when comparing other research findings [18], while the remaining transfer rates show similarity. In contrast to our collected data, the project evaluation report from China Metro Engineering Consulting Company [19] provides more precise information. Therefore, we have chosen to use our collected data as a reference point for parameter calibration. Table 2 presents time-related parameter values for three lines at Xizhimen Station. Additionally, Table 3 compares walking speeds during transfers at Beijing metro line 5 and Haidian Huangzhuang Station, which serves as an interchange station for both line 10 and line 4, with the data obtained at Xizhimen Station.

Table 1. The number of the rate at which passengers are transferred [17]

Subway routes	Transfer rate during peak hours (%)	Transfer rate during normal hours (%)	Transfer rate (%) in 2015
Line 4 to line 2	13	9	75
Line 13 to line 2	36	27	27

Line 2 to line 4	27	28	51
Line 13 to line 4	31	2	8
Line 2 to line 13	40	39	43
<i>Line 4 to line 13</i>	<i>6</i>	<i>5</i>	<i>7</i>

Table 2. The number of the parameters related to time [17]

Metro lines	Duration of walking during rush hours (min)	Duration of walking during regular hours (min)	Duration of waiting during busy periods (min)	Duration of waiting during regular operating hours (min)	Duration of travel between a pair of stations (min)	Overall duration of operation (min)
Line 4 to 2	3.72	2.69	2.09	4.51	2.02	39.12
Line 13 to 2	6.64	3.64	2.01	4.52	2.03	39.07
Line 2 to 4	3.82	2.70	2.03	4.01	2.02	79.02
Line 13 to 4	5.25	3.03	2.53	4.03	2.11	79.03
Line 2 to 13	6.79	3.56	2.62	5.49	3.09	50.11
<i>Line 4 to 13</i>	<i>8.11</i>	<i>5.11</i>	<i>2.63</i>	<i>5.50</i>	<i>3.04</i>	<i>50.09</i>

Table 3. Peak-hour pedestrian velocity within the transfer corridor for passengers [17]

Metro lines	4-2	13-2	2-4	2-13	2-13	4-13
<i>The pace of walking (m/s)</i>	<i>0.828</i>	<i>0.732</i>	<i>0.793</i>	<i>0.720</i>	<i>0.617</i>	<i>0.644</i>

Table 4. The data input for the calculation of calibration values [17]

Metro lines	Data sequence of peak-hour features X_0	Data sequence of correlation factor during peak hours X_A	Data sequence of correlation factor during peak hours X_B	Data sequence of regular-hour features X_0	Data sequence of correlation factor during regular hours X_A	Data sequence of correlation factor during regular hours X_B
Line 4 to 2	3.73	0.12	0.16	0.34	0.08	0.22
Line 13 to 2	6.63	0.33	0.29	0.51	0.26	0.27
Line 2 to 4	3.83	0.24	0.14	0.32	0.27	0.14
Line 13 to 4	5.27	0.35	0.15	0.15	0.04	0.13
Line 2 to 13	6.78	0.40	0.23	0.61	0.38	0.21
<i>Line 4 to 13</i>	<i>8.12</i>	<i>0.08</i>	<i>0.26</i>	<i>0.33</i>	<i>0.06</i>	<i>0.27</i>

The calculation results for calibration model is shown in Table 4. Based on the data presented in Tables 1 and 2, Table 5 displays the calculated value of k_{peak} , which represents the propagation rate. For simulation purposes under oversaturated conditions, line 4's propagation rate ($k_{line\ 4} = 28.7\%$) is selected as the peak-hour k_{peak} .

Table 5. Variations in the congestion propagation rate k_{peak} across various lines [17]

Subway routes	Rate of dissemination (%)
Line 4 to 2	16.5
Line 13 to 2	30.2
Line 2 to 4	17.3
Line 13 to 4	19.7
Line 2 to 13	30.4
Line 4 to 13	15.1
Line 2	30.1
Line 4	27.8
<i>Line 13</i>	<i>19.9</i>

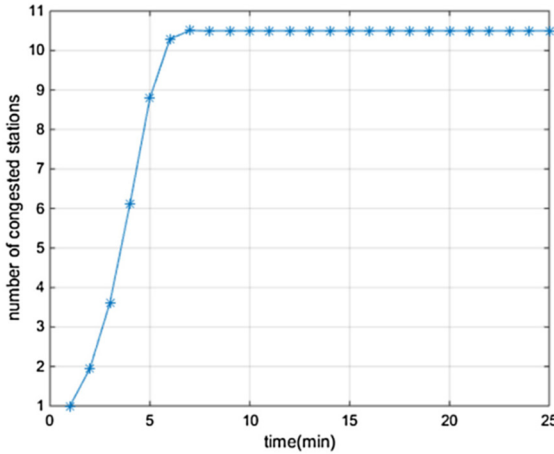


Fig. 2. Congestion propagation procedure [17]

The congestion propagation process depicted in Figure 2 demonstrates a rapid increase in the number of congested stations within the first 5 minutes. Subsequently, there was a decrease observed at time step 8, followed by a minor reduction between time step 8 and 25. In order to compare the simulation results with real-life circumstances, we made adjustments to parameter N_i as shown in Figure 3a. The alternatives for measures taken by operators are illustrated in Figure 2, which include bypassing stations 1 and 3 without stopping, and reducing the time step from 5 to 3 for neighboring stations to the initial station experiencing congestion (N_1). As a result of these adjustments, within a span of 25 minutes, the number of congested stations

decreases to only six, with stabilization occurring between time step 10 and 25. By varying its value (e.g., using values such as 5, 4, 3 and 2), we can better illustrate how parameter N_i impacts the system. This is demonstrated through simulations presented in Figure 3a. Additionally, Figure 3b displays different evolutions of propagation rates: 0.23, 0.20, 0.16 and 0.10, respectively. From this figure, it can be concluded that if we reduce the propagation rate, the total number of congested stations will also decrease. However, a comparison between (b) and (a) clearly indicates that reducing adjacent stations significantly mitigates an increase in total congestion while improving efficiency accordingly.

Studies have indicated that the expansion of congested stations is directly impacted by an increase in the rate of transmission. Therefore, a more effective strategy would involve bypassing certain stations without stopping to decrease the number of neighboring stations N_i . Furthermore, there are several measures available to alleviate overcrowding by reducing the transmission rate k . These tactics involve managing the flow of passengers at entry and exit points, adjusting train schedules, improving transfer convenience, and expanding station accessibility.

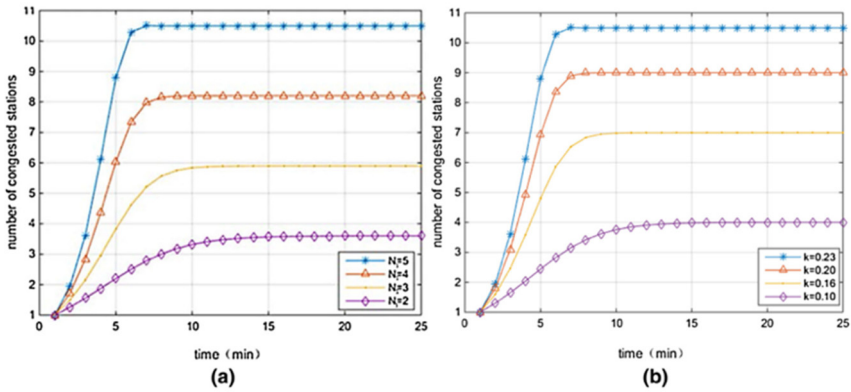


Fig. 3. Reproduction of congestion spread simulation with varying parameter values. [17]

5 Conclusions

The SIR model, which stands for Susceptible, Infected, and Recovered, is a widely used mathematical model in epidemiology to understand the spread of infectious diseases. In recent years, this model has also been applied to study congestion in transportation networks. By simulating the transmission of congestion in an overloaded network, the SIR model provides valuable insights into how congestion spreads and how it can be resolved.

One novel method that has been introduced to determine the speed at which congestion spreads is through analyzing various factors that influence its spread. These factors include passenger movement characteristics, time intervals between train departures, ease of passenger transfers, timing of congestion occurrences, initial station

where congestion starts, and station capacity. By taking these factors into account, strategies can be generated for resolving oversaturation and improving congestion management.

However, there are still additional significant factors that require further investigation to enhance the capabilities of the SIR model. For example, understanding how different types of transportation modes (such as buses or subways) interact with each other during peak travel times could provide valuable insights into managing overall network congestion more effectively.

Overall, the SIR model provides a comprehensive analysis of processes related to congestion and offers insights into predicting trends so that traffic controllers can quantitatively observe these processes. As technology continues to advance and data collection becomes more sophisticated in transportation systems around the world, the application of mathematical models like SIR will continue to play a crucial role in improving our understanding and management of network congestions [20]. By differentiating between various measures aimed at enhancing congestion conditions, this model enables traffic controllers to evaluate outcomes and select optimal solutions for effectively resolving oversaturated conditions. This includes the ability to analyze the impact of infrastructure improvements, such as adding new lanes or implementing public transportation options, on reducing congestion.

As the need for daily passenger transportation grows, it becomes crucial to include an assessment of congestion propagation and efficiency differentiation in urban transit networks' expansion plans. This will involve considering factors such as population growth, land use patterns, and economic development when planning for future transit system expansions.

In addition, effective recovery strategies can be developed by employing a comprehensive quantitative model like SIR. This model can replicate current conditions within urban transit systems and accurately predict congestion spread trends based on real-time data analysis. By utilizing this predictive tool, traffic controllers can proactively implement measures to alleviate potential congestion hotspots before they become problematic for commuters.

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