# A Rasch-based Analysis of Malaysian Students' Hierarchical Understanding of Rational Numbers

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**Abstract.** Rational numberis an important topic in mathematics since the knowledge and skills in this topic are very useful in more advanced topics. However, many students are unable to master this topic due to various misconceptions that exist because of properties of Rational Numbers. As such, in this study, we take the opportunity to investigate Malaysian students hierarchical understanding of Rational Numbers. A total of 1135 students aged 13 years old were randomly selected for this study. We used eight different forms of mathematics test to gauge their response on items related to Rational Numbers. The tests were administered separately but were linked together by several common items using the common item non-equivalent group design. In general, most of the items were able to be placed at the common scale, and students' understanding of this topic is as expected. However, there were cases where the difficulty of the items was rather unexpected. As such, we proposed that the findings of the present study to be discussed further with the teachers to provide better understanding of the hierarchical understanding in this topic.

**Keywords:** Rational Number, Hierarchical Understanding, Common Item Non-equivalent Group Design, Rasch Model, Mathematics

## 1 First Section

### 1.1 A Subsection Sample

Rational numbers can be defined as those real numbers that can be represented as the quotient a/b of two integers, with b unequal to zero [1]. Integers, whole numbers, natural numbers, fractions, and decimals are all examples of rational numbers. Note that 0 is also a rational number because it can be expressed in terms of 0/1,0/(2,),0/24, etc. However, pi (□ = 3·14159265… ), and Euler's number (e = 2·718281…..), were not rational numbers. Rather, they are called irrational numbers which include non-terminating decimals. It is also interesting to see while square root of 4 a rational number, square root of 2 was not. Note that even though 1/3=0.333… seems to include non-terminating decimals, the number is still called rational number since it can be represented as a/b. Understanding of rational numbers is important not only for the daily life (such as counting money or making measurement) but also for further mathematical development such as probabilities, trigonometry and advanced statistics [2, 3, 4].


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Knowledge about rational number is considered as one of the prime factors that contribute to competency in mathematics.

Rational numbers share many properties with natural numbers, such as 1, 2, 3, 15, 235, etc., which student are familiar with. Firstly, all natural numbers are rational numbers. For example, the natural number 7 can be written as 7/1 which is a rational number. Secondly, rational and natural numbers can both be ordered according to their sizes, i.e. 3 is bigger than 2, 0.28 is bigger than 0.19. Nevertheless, there is also important difference between rational numbers and natural numbers which can become potential obstacles for learners [5]. Pouta et al. [6] quotes that one of the main reasons for difficulties in learning rational number was because of the increasing complexity of operations especially with fractions and decimals. For example, Gabriel et al. [7] explained the need to process local and global values when dealing with fractions – something that they don't have to deal with in natural numbers. Meanwhile, to add or subtract two decimal numbers, we need to line up the decimal places, so that we know that we are adding values with corresponding place values. The procedure was somewhat different from natural numbers where we line up the right-hand side. It should be noted that the procedure was the same. The fact that the decimal point is invisible, it doesn't look the same.

Researchers such as McMullen et al. [8] and Van Dooren et al. [9] observed that generally, students tend to overgeneralize their natural number knowledge to rational numbers. The so-called natural number bias refers to the inappropriate application of natural number rules [10]. For example, since 9 is bigger than 8, then 1/9 should be bigger than 1/8. Nevertheless, it should be noted that the conceptual change from natural to rational numbers was actually a long process, and natural number bias can result in misconceptions during many phases of the process [11].

There are many studies conducted on fractions and decimals. For example, a study by Resnick et al. [12] found that many students failed to understand fractions magnitude. Students were found to have tendency to apply their whole-number understandings when solving fractions problems [13]. Meanwhile, studies in decimals were mainly focussing on the misconceptions such as if the number is longer, it is usually the larger the number Karp et al. [14] or the misconception of smaller number tends to have longer decimals longer the decimal, the smaller the number [15]. Meanwhile, study by Liu et al. [16] found that application of decimals was largely limited by their learning experiences. Nevertheless, studies beyond fraction and decimals such as combination of fraction and decimals and their operations, or problem-solving involving rational numbers are rather scarce even though they are related to real-live experience.

As such, the purpose of study was to examine hierarchical understanding of rational numbers among school students. We believe that the hierarchy of understanding in rational number exist based on the fact that fraction-related knowledge can be ordered accordingly [17, 18]. Understanding the hierarchy is important because it means that the knowledge at one stage is pre-requisite for achieving tasks at a higher stage. In another words, students unable to complete items designed to test lower stages will be unlikely to successfully complete items designed to test higher stages. Therefore, information from the present study can help teachers to identify learning standards that require reinforcement and enrichment before moving to the next more difficult learning standards.

# 2 Methodology

## 2.1 Participants

Participants in this study consist of 1,135 secondary school students with an average age of 13 years old. The gender distribution is 518 males (45.6%) and 617 females (54.4%) from schools in the states of Kedah, Penang, and Perak in the northern parts of Malaysia. The selection of the schools was based on purposive sampling, in which the researchers' identified schools with various degrees of achievement in mathematics.

## 2.2 Instruments

This study employed eight mathematics tests that were administered to eight schools. The tests were conducted separately but were linked together by eight sets of common items using the common item non-equivalent group design (CINEG) [19]. CINEG design was employed in this study since it was relatively easy and in line with how tests were administered in Malaysia. The common items serve as statistical means for equating test forms so that scaled scores are directly comparable. We employed concurrent calibration for parameter estimations. Altogether, we link the eight tests using a total of 45 common items measuring 13 topics specified in the curriculum specifications [20]. Nevertheless, only results involving the topic of Rational Numbers will be presented in this article. Content validity of the test was observed by the head of the mathematics panel of each school. The tests included both multiple-choice and partial credit items. In the multiple-choice format, participants chose one correct answer from a list of four possible choices. One mark was given to the correct answer and no mark for the incorrect answer. In the partial credit format, the scoring was based on the completion of the steps in solving the problem. The marks for each item ranged from 1 to 4 marks, and the total marks for each test were 100. Examples of a multiple-choice item and a 2-marks partial credit item are given in Table 1.

**Table 1.** Test items and scoring

| Format | Item | Scoring |
|---|---|---|
| Multiple-choice | (16s – 5) + (4s + 2 – 3s) =<br>**A** 17s – 7 **B** 17s – 3<br>**C** 23s – 3 **D** 23s + 7 | **B** …………1 mark<br>**A, C, D** …. 0 mark |
| Partial Credit | Tulis tiga gandaan sepunya yang pertama bagi nombor 4 dan 6.<br>*Write the first three common multiples of the numbers 4 and 6* | 12 ……….. 1 mark<br>24, 36 …… 1 mark |

### 2.3 Instruments

The quality of each item in the item pool was examined by using Rasch model software WINSTEPS 3.74. The plan of analysis started with assessing the assumptions of the Rasch model, specifically, (1) the model-data fit and (2) the unidimensionality assumptions. The first assumption was that the data must fit the model's expectation. Model-data fit refers to the extent to which the data collected matches expectations from the model. This assumption was examined using the infit and outfit mean-square (MNSQ) values generated from WINSTEPS 3.74. While both statistics are sensitive towards unexpected responses, the infit MNSQ deals with responses by the respondents that are targeted towards them while the outfit MNSQ explains far from the targeted respondents [21]. According to Bond and Fox [22], the assumption is met when the values of the infit and outfit MNSQs were in the range from 0.6 to 1.4. Meanwhile, the unidimensionality assumes that items in a test measure a single construct [23]. The assumption was examined from the principal component analysis of the residuals procedure in the software. The assumption is met when the variance explained by the measurement dimension from the procedure is more than 40% [24].

In this study, apart from examining the assumptions, we also reported statistics such as the item reliability and item separation indices and the point measure correlation (PTMEA Corr.). Item reliability statistics refer to the ratio between true to observed item variance [24]. This provides information on the consistency of the ordering of item difficulty if an instrument is administered to a comparable sample of participants. High item reliability statistic indicates the consistent ordering of the items' difficulty and vice versa. Meanwhile, the item separation index is an indication of the adequacy of the measurement to distinguish between participants. For example, if the separation index is 2, then it is possible to distinguish the participant into two ability groups. It should be noted that a proper measurement should be able to distinguish clearly the ability of the participants. For a proper measurement, the item reliability index should be more than .94 [25], while the separation index should not be less than 2.0 [22]. With regards to the PTMEA Corr., the positive values of this statistic indicate that the particular item is working together with other items in the same direction to measure the intended construct [22].

## 3 Findings

Table 2 showed statistics for all 66 items measuring 17 learning standards. Learning standard 1.1.1 was measured using two MCQ items, namely, EA1 and DA4 with difficulty measures of -3.18 and -1.29 logits respectively. The negative sign showed that respondents have more than a 50% chance of getting both items correctly, with EA1 was considered as easier based on its smaller measure value. The SE indicated the standard error of the estimation – in which higher SE indicates more notable irregularities in the measurement of learning standard 1.1.1. Meanwhile, the infit and outfit MNSQ values of 1.03 and 1.15 for EA1 signify that there were only 3% and 15% variation from the model's expectations for the on-target and off-target participants. Finally, the positive value of .30 of the PTMEA yielded evidence that Item EA1 was working together with other items in measuring the participants' ability in Rational Numbers

Overall, the assumption of model-data fit is met based on the range of infit and outfit MNSQ, which is between 0.62 - 1.38 logits, well within the range of 0.6 - 1.4 logits suggested by Bond and Fox (2015). Conversely, results from the PCA of residuals showed that raw variance explained by both the students and the items measures was 54%, which was more than the intended value of 40% [24]. As such, we provided ample evidence that the unidimensionality assumption was also fulfilled. Both item reliability (.97) and item separation index (6.10) exceed the intended value of 2.0 – providing indication that the consistency of the ordering of item difficulty measures if the tests were administered to comparable group of respondents. Finally, positive values of the PT MEAS. Corr. Give evidence that all items were suitable and working together in defining the construct of ability in Rational Number.

**Table 2**. Item statistics (in logits)

| Learning Standards[@] | Item Label | Format* | Diff. Meas. | SE | Infit MNSQ | Outfit MNSQ | PT MEA. Corr |
|---|---|---|---|---|---|---|---|
| 1.1.1 | EA1 | MCQ | -3.18 | 0.59 | 1.03 | 1.15 | .30 |
| 1.1.1 | DA4 | MCQ | -1.29 | 0.22 | 1.04 | 0.99 | .34 |
| 1.1.2 | DB1 | PC | -1.48 | 0.11 | 1.13 | 1.27 | .56 |
| 1.1.2 | LB4 | PC | -1.24 | 0.13 | 1.35 | 1.30 | .48 |
| 1.1.2 | L11R82# | MCQ | -0.69 | 0.17 | 0.98 | 0.95 | .24 |
| 1.1.2 | WA2 | PC | -0.42 | 0.25 | 1.04 | 1.12 | .17 |
| 1.1.2 | L1R7 | PC | -0.39 | 0.10 | 1.02 | 0.89 | .36 |
| 1.1.2 | GB1 | PC | -0.20 | 0.09 | 1.29 | 1.32 | .26 |
| 1.1.2 | WB3 | PC | 1.63 | 0.18 | 1.03 | 1.01 | .33 |
| 1.1.3 | LB7 | PC | -1.15 | 0.18 | 0.68 | 0.62 | .72 |
| 1.1.3 | LC14 | PC | -1.06 | 0.13 | 0.83 | 0.73 | .70 |
| 1.1.3 | L9R76 | PC | -0.90 | 0.15 | 1.04 | 0.90 | .21 |
| 1.1.3 | LA2 | MCQ | 0.45 | 0.31 | 1.08 | 1.08 | .22 |
| 1.1.4 | EA2 | MCQ | -2.46 | 0.42 | 1.01 | 0.83 | .13 |
| 1.1.4 | FA1 | MCQ | -1.88 | 0.20 | 0.90 | 0.83 | .41 |
| 1.1.4 | L4R30 | PC | -1.75 | 0.19 | 0.97 | 0.94 | .37 |
| 1.1.4 | RA2 | MCQ | -1.60 | 0.46 | 0.98 | 0.75 | .20 |
| 1.1.4 | LA1 | MCQ | -1.41 | 0.28 | 0.98 | 0.96 | .38 |
| 1.1.4 | LA7 | MCQ | -1.03 | 0.27 | 1.00 | 1.08 | .35 |
| 1.1.4 | FB1 | PC | 0.07 | 0.16 | 0.87 | 0.85 | .48 |
| 1.2.1 | VA2 | MCQ | -1.93 | 0.18 | 1.04 | 1.03 | .39 |
| 1.2.1 | LC2 | PC | -0.56 | 0.17 | 1.22 | 1.19 | .38 |
| 1.2.1 | EC5 | PC | 0.37 | 0.07 | 1.00 | 1.05 | .42 |
| 1.2.2 | FC1 | PC | -1.19 | 0.09 | 0.72 | 0.68 | 0.65 |
| 1.2.3 | VA3 | MCQ | -3.67 | 0.26 | 1.00 | 0.73 | 0.34 |
| 1.2.3 | DA2 | MCQ | -2.84 | 0.33 | 1.02 | 1.05 | 0.22 |
| 1.2.3 | JA3 | MCQ | -1.22 | 0.26 | 1.02 | 1.01 | 0.12 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1.2.3 | L1R1 | MCQ | -0.90 | 0.17 | 1.00 | 1.06 | 0.16 |
| 1.2.3 | EB1 | PC | -0.57 | 0.08 | 0.91 | 0.91 | 0.46 |
| 1.2.3 | GC17 | PC | 0.12 | 0.12 | 1.06 | 1.03 | 0.28 |
| 1.2.3 | DC2 | PC | 0.46 | 0.14 | 1.34 | 1.27 | 0.38 |
| 1.2.3 | LB8 | PC | 0.53 | 0.21 | 0.95 | 1.00 | 0.40 |
| 1.2.3 | FC2 | PC | 1.10 | 0.09 | 0.90 | 0.87 | 0.55 |
| 1.2.6 | GA2 | MCQ | -1.57 | 0.28 | 1.04 | 1.22 | .05 |
| 1.2.6 | EA4 | MCQ | -1.54 | 0.28 | 1.01 | 0.97 | .16 |
| 1.2.6 | LA4 | MCQ | -1.41 | 0.28 | 0.95 | 1.00 | .40 |
| 1.2.6 | FC4 | PC | -0.46 | 0.08 | 1.22 | 1.37 | .43 |
| 1.2.6 | LA3 | MCQ | -0.44 | 0.27 | 0.86 | 0.81 | .52 |
| 1.2.6 | DC4 | PC | 0.67 | 0.11 | 0.89 | 0.85 | .70 |
| 1.2.6 | WC1 | PC | 0.87 | 0.09 | 1.12 | 1.12 | .46 |
| 1.2.6 | JC6 | PC | 1.37 | 0.08 | 1.13 | 1.20 | .36 |
| 1.3.2 | WA19 | MCQ | -1.74 | 0.40 | 1.00 | 0.80 | 0.19 |
| 1.3.2 | LC5 | PC | -0.72 | 0.15 | 0.90 | 0.87 | 0.61 |
| 1.3.2 | L11R81# | MCQ | 0.83 | 0.13 | 0.96 | 0.93 | 0.34 |
| 1.3.3 | WA1 | MCQ | -1.88 | 0.43 | 1.00 | 0.88 | 0.17 |
| 1.3.3 | VC2 | PC | -1.25 | 0.09 | 1.15 | 1.38 | 0.68 |
| 1.3.3 | EC2 | PC | 0.54 | 0.10 | 0.98 | 0.95 | 0.38 |
| 1.3.3 | FC3 | PC | 1.36 | 0.10 | 1.03 | 1.00 | 0.44 |
| 1.3.4 | L7R64# | PC | 0.94 | 0.06 | 1.02 | 1.11 | 0.52 |
| 1.4.1 | L1R6# | PC | -1.20 | 0.11 | 1.02 | 0.91 | .31 |
| 1.4.1 | L7R63# | PC | -0.06 | 0.10 | 1.07 | 1.09 | .27 |
| 1.4.3 | VA4 | MCQ | -2.00 | 0.18 | 0.99 | 0.97 | .44 |
| 1.4.3 | JA1 | MCQ | -0.65 | 0.22 | 1.03 | 1.07 | .11 |
| 1.4.3 | L3R20# | MCQ | -0.63 | 0.16 | 0.94 | 0.86 | .49 |
| 1.4.3 | DC3 | PC | 0.10 | 0.13 | 1.14 | 1.03 | .51 |
| 1.4.3 | L11R87# | PC | 0.44 | 0.10 | 0.99 | 0.95 | .37 |
| 1.4.4 | JA4 | MCQ | -0.61 | 0.21 | 1.02 | 0.90 | .30 |
| 1.4.4 | DC23 | PC | -0.08 | 0.10 | 1.07 | 1.33 | .70 |
| 1.5.1 | VB1 | PC | -1.81 | 0.08 | 0.80 | 0.64 | .80 |
| 1.5.1 | JC1 | PC | 0.23 | 0.09 | 0.95 | 0.99 | .46 |
| 1.5.2 | LC15 | PC | -0.13 | 0.11 | 1.11 | 0.81 | .55 |
| 1.5.2 | JC2 | PC | 0.22 | 0.08 | 1.18 | 1.13 | .37 |
| 1.5.2 | EB5 | PC | 0.74 | 0.06 | 0.98 | 0.91 | .53 |
| 1.5.2 | DC12 | PC | 0.92 | 0.12 | 1.23 | 1.23 | .51 |
| 1.5.3 | L11R86# | PC | -0.57 | 0.13 | 1.10 | 1.15 | .28 |
| 1.5.3 | VC3 | PC | -0.23 | 0.11 | 1.08 | 0.86 | .62 |

[@]Learning Standards:

1.1.1 Recognize positive and negative numbers based on real-life situations.

1.1.2 Recognize and describe integers.

1.1.3 Represent integers on number lines and make connections between the values and positions of the integers with respect to other integers on the number line

1.1.4 Compare and arrange integers in order.
1.2.1 Add and subtract integers using number lines or other appropriate methods. Hence, make a generalization about the addition and subtraction of integers.
1.2.2 Multiply and divide integers using various methods. Hence make a generalization about the multiplication and division of integers.
1.2.3 Perform computations involving combined basic arithmetic operations of integers by following the order of operations.
1.2.6 Solve problems involving integers.
1.3.1 Represent positive and negative fractions on number lines.
1.3.2 Compare and arrange positive and negative fractions in order.
1.3.3 Perform computations involving combined basic arithmetic operations of positive and negative fractions by following the order of operations.
1.3.4 Solve problems involving positive and negative fractions.
1.4.1 Represent positive and negative decimals on number lines.
1.4.2 Compare and arrange positive and negative decimals in order.
1.4.3 Perform computations involving combined basic arithmetic operations of positive and negative decimals by following the order of operation.
1.4.4 Solve problems involving positive and negative decimals.
1.5.1 Recognize and describe rational numbers.
1.5.2 Perform computations involving combined basic arithmetic operations of rational numbers by following the order of operations.
1.5.3 Solve problems involving rational numbers.
*Format: MCQ = multiple choice question, PC = partial credit
#Linking items

In general, it seems that the teachers developed relatively easy items for this topic since the respondents have more than 50% of getting correct answer for 43 (66.1%) items. The following item VA3 was the easiest item (measure = -3.67 logits), followed by item EA1 (measure = -3.18 logits) and item DA2 (measure = - 2.84 logits). In contrast, item WB3 was endorsed as the most difficult item (measure = 1.63 logits), followed by JC6 (measure = 1.37 logits), and FC3 (measure = 1.36 logits). It is also evident that PC items were more difficult compared to the MCQs. As expected, in general, MCQ items were less difficult than the PC items. Also, participants were more able to solve items related to arithmetic compared to those related to number theory and geometry. In addition, results also showed that the participants have less difficulty in solving items related to procedural knowledge compared to conceptual understanding and problem solving.

**Table 3.** Example of easy items

| | | |
|---|---|---|
| Item Label: VA3<br>Learning standards: 1.2.3<br>Measure = -3.67 logits | Item Label: EA1<br>Learning standards: 1.1.1<br>Measure = -3.18 logits | Item Label: DA2<br>Learning standards: 1.2.3<br>Measure = -2.84 logits |

| | | |
|---|---|---|
| $468 \div (6 \div 3) \times 2 =$<br><br>**A** –76<br>**B** –61<br>**C** 61<br>**D** 76 | Sarah berada 4 m di bawah paras laut<br><br>Apakah integer yang sesuai untuk mewakili kedudukan Sarah?<br>*What is an appropriate integer to represent Sarah's position?*<br><br>**A** –4<br>**B** +4<br>**C** ×4<br>**D** ÷4 | $5 - (6 - 20) - (-9) =$<br><br>**A** –76<br>**B** –61<br>**C** 61<br>**D** 76 |

**Table 4.** Example of difficult items

| Item Label: WB3<br>Learning standards: 1.1.2<br>Measure = 1.63 logits | Item Label: JC6<br>Learning standards: 1.2.6<br>Measure = 1.37 logits | Item Label: FC3<br>Learning standards: 1.3.3<br>Measure = 1.36 logits |
|---|---|---|
| Berikut menunjukkan beberapa nombor. Nyatakan nombor perdana.<br>*The following are some numbers. State the prime number.*<br>[2 markah/*marks*]<br>(1) (2) (9) (17) (33) | Puan Sofia menyediakan beberapa buah hamper untuk suatu majlis di sekolahnya. Setiap hamper mengandungi sebotol air mineral, sebungkus coklat dan sebiji epal. Sebuah kedai runcit menjual 12 botol air mineral sekotak, 30 bungkus coklat sekotak dan 36 biji epal sekotak.<br>Hitung bilangan minimum kotak epal yang perlu dibeli olehnya bagi menyediakan hamper itu tanpa baki.<br>*Puan Sofia prepared some hampers for an event at her school.* | Selesaikan setiap yang berikut:<br>*Solve the following*:<br>[2 markah/*marks*]<br><br>$3\frac{1}{3} \times \left(\frac{2}{5} - \frac{3}{4}\right)$ |

| | | |
|---|---|---|
| | *Each hamper contains a bottle of mineral water, a packet of chocolates and an apple. A grocery store sells 12 bottles of mineral water per box, 30 packs of chocolate per box and 36 apples per box.* *Calculate the minimum number of boxes of apples he needs to buy to provide the hamper with no remainders* [3 markah/*marks*] | |

## 4 Discussions

Several important observations can be made from the results. Firstly, results for the easiest items were rather expected. Previous studies in Malaysia [26, 27, 28] have shown that students have a high mastery level when answering items that measure procedural understanding, such as items VA3 and DA2. One possible explanation was that the ability to perform a series of computational tasks has always been exposed to the students since primary school [29]. Therefore, the students were quite familiar with the types of items. Item EA1 was endorsed as one of the easiest-to-score since the item was very similar to the examples in the textbook as explained by van den Ham and Heinze [30]. It is plausible that the teachers had gone through similar items with the students in the classroom. As such, when asked again in these tests, students just need to recollect the solution steps taught in class instead of engaging in high-level cognitive tasks like interpreting or evaluating.

The fact that item WB3 was endorsed as the most difficult items is quite interesting. According to the teachers, the item only requires students to identify the prime numbers which were 2 and 17. Nevertheless, looking at the students' answers, most of them only choose one answer, which is 17. At least two observations can be made based on this finding. Firstly, students may feel that there is only one answer for each item for one test, as in the MCQ format. Secondly, the item is actually not an easy item since it measures students conceptual understanding of prime numbers which is considered more difficult from procedural understanding [31]. Meanwhile, item JC6 involves problem-solving that requires students to have some prerequisite skills such as number fact, arithmetic, as well as information skills [32, 33]. Meanwhile, item FC3 involves combination of operations where, according to Khalid and Embong [34], Malaysian students have difficulties in the following skills: (1) Parenthesis apprehension, (2) knowledge of the basic concept, (3) solving problems without using calculator, and (4) deep understanding of concepts.

Apart from the six items discussed here, 60 more items in the Table 2 that were calibrated based on learning standards and their measures. These items can be discussed further with education experts and schoolteachers to provide information regarding students' understanding of the topic of Directed Numbers. The discussion results can be used to improve teaching practices and enable students to solve problems related to this topic better.

## 5 Conclusion

The study's purpose was to examine the hierarchical understanding of rational numbers among school students. The results show that the Rasch model linking procedure has the potential to be used to obtain more information about a single topic and input to improve teaching practices for that topic.

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