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Interpretating Rasch Ability and Difficulty Estimates to Inform Mathematics Learning through an Adaptive Learning System

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Abstract: The Rasch model has been utilized in adaptive learning to provide personalized learning and timely assessment feedback to students in tertiary education. However, much of the work has been focusing on validating the tools, and estimating student ability and item difficulty, with very little research on feedback on learning. This study explored the use of the Rasch model to establish the zone of proximal development (ZPD) to provide feedback on students' mastery of mathematics, potential remediations and future developments. Data containing 251 multiple-choice items and 99 undergraduates reading the *Calculus I* course in a Singapore university were analyzed. The student ability and item difficulty estimates based on Rasch analysis were used to construct the Guttman scalogram and person-kid-map (KIDMAP) diagram. For manageability, students were profiled into red, amber and green groups based upon the Guttman scalogram. These profiles were established based on the ZPD identified via drawing parallel lines on the scalogram. It was evident that the topic on differentials of logarithmic functions remained an area of concern across the red and amber groups of students. For the KIDMAP diagram, the diagnostic information based on students' response patterns resulted in six regions of evidence on students' performance: Mastery, Need-Consolidation, Need-Scaffolding, Mistakes/Special Learning Needs, Non-Mastery Future Goal and Beyond Ability Regions. Through the KIDMAP diagram, the ZPD of each student, areas of mastery and areas needing remediation were identified. Using the Rasch model, purposeful interpretations of student performance and future development areas become accessible when person abilities and item difficulties are calibrated onto a common scale.

1. Introduction

Adaptive learning or personalized learning is a means for educational institutions to tailor learning and instruction to meet the diverse needs of students. The concept of adaptive learning or personalized learning is not new. In fact, educators have been developing personalized learning and differentiated instruction for more than 40 years (e.g., Subban 2006; Tomlinson 1999). Nonetheless, personalized learning has largely been applicable to a certain cohort of students due to the constraints of what a teacher can prepare and implement within limited instructional time. It was not until recently that the advancement in educational technology has made it possible to implement adaptive or personalized learning in tertiary education, as well as to automate and scale the individualization process to tailor the learning experience for each student.

Researchers have been increasingly interested in exploring the use of adaptive or personalized learning systems to address students' individual learning needs. Xie et al. (2019), who studied the trends and development of technology-enhanced adaptive or personalized learning, reported a total of 70 studies from top-tier educational technology journals published between 2007 and 2017, or on average, 6.4 articles related to adaptive or personalized learning. Additionally, Li and Wong (2019)

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noted the upward trend in the number of publications related to personalized learning between 2009 and 2018 in their literature review. They identified a total of 179 relevant articles collected from Web of Science and Scopus databases. The most recent systematic review conducted by Raj and Renumol (2022) also identified 52 publications related to adaptive or personalized learning between 2015 and 2020. The benefits and efficacy of adaptive learning can also be observed through some recent publications (e.g., Wang et al. 2023; Hwang et al. 2020) in which these studies showed evidence of greater gains in learning through the adaptive learning system.

In Singapore, with the growing importance and interest in lifelong learning, the number of courses in the universities that offer working adults the opportunities for continuing education have increased tremendously. However, a high attrition rate was observed, and this has raised concerns in many tertiary education institutions (Guan et al. 2015). Research has shown that adult learners with varied work experiences as well as diverse academic and demographic backgrounds often had multiple roles and commitments, and these could be challenging for them to allocate time for academic study, work and personal lives (Lim and Ho 2022). In order to enhance adult learners' learning experience and provide them with opportunities to achieve their optimal performance, the adaptive learning system has been implemented in the institution in this study. This adaptive learning system aims to enable adult learners who have diverse work experiences, demographic background and academic levels to enhance their prior knowledge so that they could engage with the mathematics content more meaningfully (Lim et al. 2023).

While the study by Lim et al. (2023) showed evidence of some benefit in using adaptive learning to enhance students' achievement, there remains a lack of feedback and diagnostic information to address students' mastery of the target knowledge or skills, the areas needing support and remediation, as well as their future learning goals. Additionally, while instructors could make use of students' raw achievement scores to investigate individual item- and student-responses to better support them in learning, Alagumalai et al. (2005) had warned against traditional methods of using raw scores to interpret assessment data. They argued that raw scores are limited in providing understanding and improving of measures since they are of an ordinal level, sample dependent and require complete datasets.

Bond (2023) and Yan (2023) proposed that researchers and practitioners should move beyond using Rasch analysis for just instrument validation. Instead, they should now apply Rasch analysis for formative assessment or assessment for learning, to provide useful feedback and diagnoses on student learning. Five key areas of applications of the Rasch model as identified by Yan (2023) include: Developing new instruments, creating short-form instruments, developing vertical scales, combining Rasch analysis and path analysis, and applications to classroom testing. Yan (2023) also proffered that Rasch analyses had great potential and important implications to inform learning and teaching by using variable maps and KIDMAPs. Essentially, teaching modifications can be made according to each student's learning needs, and this is the core of formative assessment or assessment for learning (Bond 2023).

In view of the need to provide students with feedback on learning and their future development, this study explored the use of the Rasch Measurement Theory (RMT) to establish students' ZPD, with the aim of providing feedback on learning (i.e., mastery, areas needing remediation and future learning goals).

2. Method

Responses of 99 students reading an undergraduate mathematics course (i.e., *Calculus I*) and having completed the multiple-choice items presented by an in-house adaptive learning system were used for this study. For the purpose of this study, a total of 251 multiple-choice items with four options were developed based on 15 nodes or sub-topics of the course.

The data were analyzed using RUMM2030 (RUMM Laboratory Pty Ltd., Perth, Australia). RMT was employed as it can provide estimates of item parameters by maximum likelihood in the presence of missing responses (Andrich and Marais 2019). In this study, the presence of missing responses was due to the intended nature of the adaptive learning system, where not all students were presented with all the same items. Additionally, to ensure all items are connected, two dummy person responses (i.e., Dummy 1: 101010… and Dummy 2: 010101…) are added to the original datasets (Linacre 2024).

3. Results

The Rasch analysis undertaken in this study was divided into three parts: (1) validation of the mathematics test, (2) Guttman structure and analysis of responses (i.e., Guttman scalogram), and (3) data analysis for diagnostic information (i.e., KIDMAP diagram).

3.1. Validation of Mathematics Test

The Rasch analysis found adequate evidence to suggest that the test was unidimensional, and that the responses to the items fit the Rasch model. Andrich and Marais (2019) stated that responses that fit the Rasch model: (1) provide invariant comparisons of respondents, (2) characterize respondents based on their total scores, and (3) suggest that the items were consistent with one another as expressed by the Rasch model.

The initial Rasch analysis was not viable due to the presence of extreme items (i.e., items that had a very poor fit to the Rasch model, attributable to the lack of student responses). With the deletion of 19 extreme items identified through the item fit residuals that were way beyond the ± 2.5 range for an item to be considered fitting to the Rasch model (Tennant and Conaghan 2007), the Rasch analysis proceeded. This second analysis presented item residual statistics (M = -.00, SD = .77) and person fit residual statistics (M = -.06, SD = 1.01) close to 0 and 1 correspondingly, both of which are theoretically ideal values (Andrich and Marais 2019). The item–trait interaction χ^2 statistic (χ^2 (464, n = 99) = 550.46, p < .05) was significant, suggesting some misfit between the responses and the Rasch model. However, noting the minimum sample of 30 for a Rasch analysis suggested by Linacre (1994), and the sensitivity of the item-trait interaction χ^2 statistic to sample size, the sample size was adjusted algebraically within RUMM2030, for a smaller sample size would have lesser effect on the power of the test of fit (Andrich and Marais 2019). The updated item–trait interaction χ^2 statistic (χ^2 (445, n = 92) = 511.53, p = .05) was non-significant, suggesting that the sample size possibly impacted the fit initially. The Rasch analysis also found the test to be sufficiently targeted, based on the person separation index (0.87) and the person-item distribution (see Figure A).

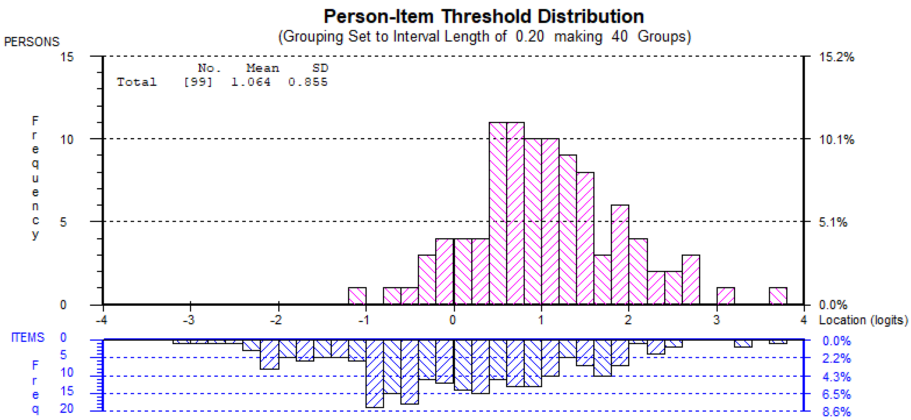

Figure A Person-item distribution.

3.2. Guttman Scalogram and Overall Feedback on Student Learning

The Guttman scalogram is a structure of responses that leads into RMT and is an elaboration of the classical test theory, given that it explains that individuals with higher total scores on a test are expected to have responded to more items correctly, including items that are more difficult (Andrich and Marais 2019). Based on this, the Guttman scalogram provides an illustration of a two-dimensional continuum based on item difficulty and person ability that can be used to determine, for practical significance in this study, content zones instructors should focus on to help students progress, specifically, the ZPD.

Given the adequate psychometric properties of the test as determined through the Rasch analysis, the Guttman scalogram was output via RUMM2030 and then scaled within Microsoft Excel (see Figure B) for the purpose of identifying the ZPD. The scalogram was color coded (i.e., green means correct response; red means incorrect response; white means a non-attempt or the item was not presented). The horizontal axis of the scalogram represented items lined from least difficult (left) to most difficult (right). The vertical axis represented students from low progress (top) to high progress (bottom).

The ZPD is a concept that allows educators to support learning, as purported by the Soviet psychologist and social constructivist Lev Vygotsky (1978). This zone represents the region within which a learner cannot do independently but can do more and operate at a higher level with guidance and assistance (e.g., from a teacher or peer). Identifying the ZPD allows teachers to provide targeted scaffolds and guidance to support learning even within higher education (Wass and Golding 2014). In view of this, it also allows teachers to be aware of what a student is already competent in, and what is beyond the reach of a student at the moment (until he or she crosses the ZPD) The ZPD for this study was identified based on the recommendations by Griffin et al. (2017). To identify the ZPD, the Guttman scalogram was first investigated to locate an area which presented roughly a 50-50 balance between correct (green cells) and incorrect (red cells) responses. A pair of parallel (blue) lines were then drawn to create a boundary for this approximate area, and this area is known as the ZPD. To the left of this pair of parallel lines is the area representing items and skills that each student is relatively more competent in and can interact with independently; to the right is the area that represents items and skills that are beyond the ZPD for each student.

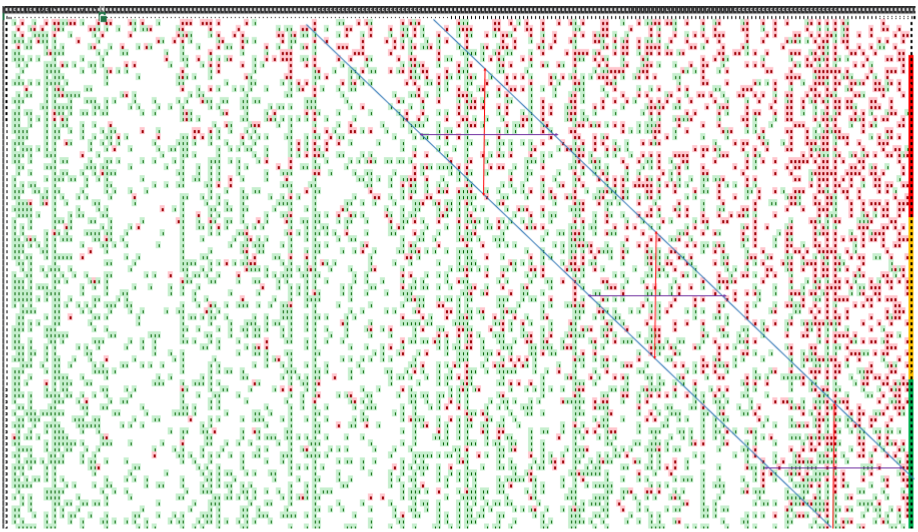

Figure B Guttman scalogram.

While each row in Figure B represents a student, it is not practical for a teacher to investigate the ZPD of each individual student. Hence, Griffin et al. (2017) suggested the use of groups of ZPD. By token of this, three groups of students corresponding to the red, amber and green regions (see rightmost column of Figure B) were identified in this study, by drawing the short red vertical lines between the pair of parallel lines. To identify the items and skills within the ZPDs of these three groups, short horizontal purple lines were drawn to intersect the short red vertical lines at right angles. These horizontal lines referenced to sets of items from the topmost horizontal axis of Figure B; correspondingly, these set of items were mapped to the topics and skills assessed, to identify which were the skills that each group of students would be able to manage, with some guidance. Based on the ZPDs identified for each group of students, it was evident that the topic on differentials of logarithmic functions was an area of concern for all groups, and this information was presented to the course director who then discussed with the course instructors on possible remediations.

3.3. KIDMAP Diagram and Individual Diagnostic Information

While the Guttman scalogram provided an overall picture of areas needing support and remediation for the three groups of students (i.e., red, amber and green), the KIDMAP diagram would offer valuable diagnostic information at the individual student level. The KIDMAP diagram is a graphical presentation of the ZPD and response pattern of each individual student (Chien et al. 2011). An example taken from this study is the KIDMAP for one student, as presented in Figure C. In this KIDMAP, the estimated ability level of the student is represented by “xxx”. Using this estimate, the KIDMAP was divided horizontally into three quadrants and vertically into two quadrants, resulting in six regions, namely Beyond Ability, Non-Mastery Future Goal, Need-Consolidation, Need-Scaffolding, Mastery as well as Mistake/Special Learning Needs Regions (Yao and Mok 2013). Items located in the left quadrants were those that the student answered correctly while the items located in the right quadrants were those that the student answered incorrectly. Items located in the top quadrants presented items that were difficult for the student while items located in the bottom quadrants were items that were easy for the student.

To illustrate, items 2, 3, 12 and 13 in the middle-left quadrant (or Need-Consolidation Region) were items that the student answered correctly. However, consolidation is required as these items were located within the ZPD of the student; there is a 50-50 chance that the student might get these items wrong. Items 14 and 19 in the middle-right quadrant (or Need-Scaffolding Region) were items that the student answered incorrectly. This is also a ZPD region where the student, when given support, will be able to achieve mastery.

Items 5 and 7 in the lower-right quadrant show unexpected failures (or Mistakes/Special Needs Region) and hence, the student should revise these items, as these items were considered easy for the student. Such unexpected failure could alert both the students and instructors, and it would be worthwhile for instructors to investigate the reasons behind these unexpected responses (Yan 2023) to better support the student. Items 1, 6, 8, 9 and 10 were easy for the student who answered them correctly. This is the region of mastery (or Mastery Region).

Figure C also depicts two other regions in the top quadrants. Item 11 shows unexpected success (or Beyond Ability Region) as this item was too difficult for the student but he or she managed to answer it correctly. As noted by Chien et al. (2011), an unexpected response like this is worth noting because it may indicate the student’s strength, luck or cheating. Items 4, 15, 16, 17, 18 and 20 were difficult for the student, as s/he answered them incorrectly. The items within this region (or Non-Mastery Future Goal Region) give direction to the student to plan for his or her future learning goals.

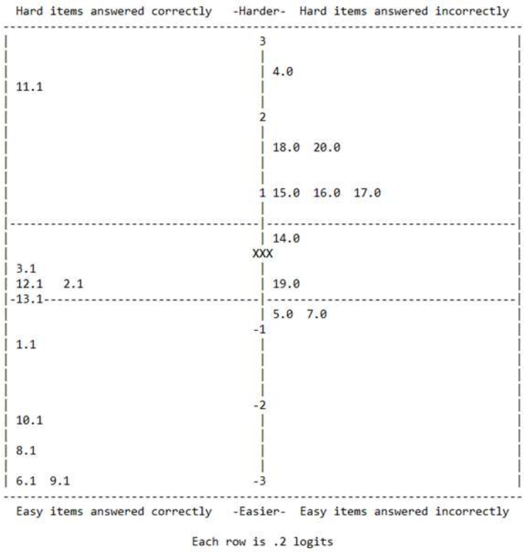

Figure C KIDMAP diagram.

4. Conclusion

This study demonstrated how Rasch analysis could be of practical value to both instructors and students using the data from the adaptive learning system. In this study, the Rasch analysis after removing the misfitting items showed evidence of acceptable data-model fit. This supported the unidimensionality and measurement invariance conditions as required by the Rasch model. Additionally, the Guttman scalogram and the KIDMAP diagram constructed through student ability and item difficulty estimates based on the Rasch model, demonstrated the potential and importance of Rasch analysis in adaptive learning, beyond just the identification of misfitting items or instrument validation. Essentially, the Rasch analysis undertaken in this study benefited both the instructors and students as it: (1) provided them with meaningful insights to determine students' mastery of mathematics skills (2) helped to identify areas needing more support and remediation, and (3) provided information to plan for students' future learning goals. For meaningful applications of adaptive learning, educators and practitioners could consider applying RMT and Rasch analyses to provide feedback on student learning through the Guttman scalogram and KIDMAP diagram.

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