Simulation of tsunami wave propagation due to volcanic and tectonic earthquakes

Ahmad Zakaria[1*], Subuh Tugiono[1], A. Ashruri[1]

[1] Department of Civil Engineering, Lampung University, Lampung 35145, Indonesia
*ahmad.zakaria@eng.unila.ac.id

Abstract. Generally, long wave propagation numerical models are used to model the propagation of tsunami waves, which both are caused by volcanic earthquakes and tectonic earthquakes that occur in the middle of the sea. In this research, the time and height of tsunami wave propagation are presented and simulated. Tsunami waves are simulated with the assumption that if in the future at the location of Mount Anak Krakatau an eruption of Mount Anak Krakatau occurs, or from around the fault zone on the west coast of Sumatra a tectonic earthquake occurs. It is assumed that the earthquake that occurred will cause a tsunami wave. The waves will propagate to the coastal areas of Lampung and the west coast of Sumatra. To model the propagation of tsunami waves, nonlinear wave equations are used. The explicit finite difference method is used to approach this equation. Aim of this research presented here is to show that the propagation of tsunami waves caused by tectonic earthquakes is more dangerous than tsunami waves caused by volcanic earthquakes if it is assumed that the energy or height of the tsunami waves at the source location is the same.

Keywords: Tsunami, volcanic and Tectonic Earthquakes, Anak Krakatau

1 Introduction

Generally, numerical models are used by researchers to model natural events such as wave propagation. Wave propagation theory is used by researchers, especially in developing models for the propagation of water surface wave motion. Researchers usually use long wave theory in developing numerical models of surface wave propagation moving towards the coast. This modelling is very important in the field of civil engineering. In this paper, the results of the development of a numerical model/program called SIGERD (Simulation of Diffraction Refraction Waves) are presented. This program uses the Fortran programming language and the explicit finite difference method, for a numerical approach to the two-dimensional nonlinear long wave equation, this equation is used to model the propagation of tsunami waves.

Many studies on surface wave propagation have been carried out, including those carried out by [1]. In their research, [1] carried out physical and numerical modelling. Where, the wave breaker was physically modelled to study wave run-up, reflection, dissipation, and dispersion effects. Wave propagation in breakwaters has also been

A. Zakaria et al. (eds.), *Proceedings of the 1st International Conference on Industry Science Technology and Sustainability (IConISTS 2023)*, Advances in Engineering Research 235,
https://doi.org/10.2991/978-94-6463-475-4_26

modelled to investigate transmission effects. Apart from that, a researcher has also tested wave propagation models at offshore breakwaters to model the shoreline response [2].

Numerical simulations of 2-D non-linear surface wave propagation have been introduced by many researchers, including [3], where the simulations carried out were used to study the case of one-dimensional tsunami wave run-up. Apart from that, a discussion of 2-D non-linear wave propagation has also been presented by [4], where in their research they studied the problem of dispersion of two-dimensional waves. In a study conducted by [5], he reviewed the achievements and findings of studies related to numerical modelling of tsunami wave observations since the 2011 Tohoku earthquake. Wave propagation simulations using 2-dimensional hyperbolic equations have also been used by [6] to study dispersion problems at physical boundaries (boundaries) and the influence of changes in gird width on the level of model accuracy, from low accuracy to high accuracy. Based on research that has been carried out, and by using 2-dimensional non-linear wave equations [3], a 2-dimensional surface wave propagation model was developed [7, 8] to simulate the propagation of tsunami waves due to volcanic eruptions. children of Krakatoa. With this simulation, it is hoped that the risk due to tsunami waves can be estimated.

To be able to estimate the risk of a tsunami if Mount Anak Krakatau erupts, researchers can do this by simulating tsunami waves numerically. Many researchers have carried out tsunami wave simulation modeling,

both due to volcanic and tectonic earthquakes. To be able to model tsunamis [9], [10], [3], [11], [4], [12–14], [15], and [16] have carried out tsunami modelling. Modelling of tsunamis resulting from volcanic earthquakes has also been carried out, including by [17] and [18]. Here, [18] conducted a review of the tsunami event caused by Krakatoa in 1883. In tsunami simulation modelling, hydrodynamic equations with an explicit difference approach up to order 2 accuracy were used.

Furthermore, the aim of this research is to simulate the propagation of tsunami waves which are assumed to occur either due to tectonic earthquakes (faults) or volcanic earthquakes (due to the eruption of Mount Anak Krakatau) in the waters of the west coast of Sumatra and its surroundings. And are tsunamis caused by tectonic earthquakes more dangerous than tsunamis caused by volcanic earthquakes?

2 Numerical Modelling

The research approach employed in this study exclusively utilizes quantitative methods. By using numerical methods, the research costs incurred will be cheaper compared to physical modelling and modelling research in the field.

In this research, the wave equation used to model two-dimensional nonlinear wave propagation was used to model tsunami wave propagation. For modelling 2-D nonlinear wave propagation [3], a series of momentum and continuity equations are usually used as follows,

2.1 Momentum Equation,

x-axis direction

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-g\frac{\partial \eta}{\partial x}-r.u.f \tag{1}$$

y-axis direction

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial y}+u\frac{\partial v}{\partial x}=-g\frac{\partial \eta}{\partial y}-r.v.f \tag{2}$$

2.2 Continuity Equation,

$$\frac{\partial \eta}{\partial t}=-\frac{\partial (Du)}{\partial x}-\frac{\partial (Dv)}{\partial y} \tag{3}$$

Where:
$f = \frac{\sqrt{u^2+v^2}}{D}$
η = water surface elevation
r = friction coefficient = 0.025
u = wave propagation speed in the x direction
v = wave propagation speed in the y direction
g = acceleration due to gravity
D = total depth = $h + \eta$
h = water depth (m)
Δt = time step = 1 sec
$\Delta x = \Delta y$ = grid width = 850 m

The solution to the 2-D nonlinear surface wave equation used in this research is an explicit finite difference method. Using this method, the equation can be approached using the first derivative approach to time as follows,

$$\frac{\partial u}{\partial t}=\frac{u_{i,j}^{k+1}-u_{i,j}^{k}}{\Delta t} \tag{4}$$

$$u\frac{\partial u}{\partial x} = u_{i,j}^{k}\left(\frac{u_{i+1,j}^{k}-u_{i,j}^{k}}{\Delta x}\right) \tag{5}$$

$$v\frac{\partial u}{\partial y} = v_{i,j}^{k}\left(\frac{u_{i,j+1}^{k}-u_{i,j}^{k}}{\Delta y}\right) \tag{6}$$

$$\frac{\partial \eta}{\partial x} = g\left(\frac{\eta_{i+1,j}^{k}-\eta_{i,j}^{k}}{\Delta x}\right) \tag{7}$$

$$r.u.f \;=\; r.u_{i,j}^{k}.\frac{\sqrt{\left(u_{i,j}^{k}\right)^{2}+\left(v_{i,j}^{k}\right)^{2}}}{D_{i,j}^{k}} \tag{8}$$

$$\frac{\partial v}{\partial t} = \frac{v_{i,j}^{k+1}-v_{i,j}^{k}}{\Delta t} \tag{9}$$

$$v\frac{\partial v}{\partial y} = v_{i,j}^{k}\left(\frac{v_{i,j+1}^{k}-v_{i,j}^{k}}{\Delta y}\right) \tag{10}$$

$$u\frac{\partial v}{\partial x} = u_{i\,,j}^{k}\left(\frac{v_{i+1,j}^{k}-v_{i,j}^{k}}{\Delta x}\right) \tag{11}$$

$$g\frac{\partial \eta}{\partial y} = g\left(\frac{\eta_{i,j+1}^{k}-\eta_{i,j}^{k}}{\Delta y}\right) \tag{12}$$

$$r.v.f \;=\; r.v_{i,j}^{k}.\frac{\sqrt{\left(u_{i,j}^{k}\right)^{2}+\left(v_{i,j}^{k}\right)^{2}}}{D_{i,j}^{k}} \tag{13}$$

$$D_{i,j}^{k} = h_{i,j}+\eta_{i,j}^{k} \tag{14}$$

$$\frac{\partial (Du)}{\partial x} = D_{i,j}^{k}.\left(\frac{u_{i,j}^{k}-u_{i-1,j}^{k}}{\Delta x}\right)+u_{i,j}^{k}.\left(\frac{D_{i,j}^{k}-D_{i-1,j}^{k}}{\Delta x}\right) \tag{15}$$

$$\frac{\partial \eta}{\partial t} = \frac{\eta_{i,j}^{k+1}-\eta_{i,j}^{k}}{\Delta t} \tag{16}$$

The propagation of modelled surface waves is limited by boundaries because they are not physically real. Boundaries are usually referred to as nonphysical boundaries or open boundaries. To simulate waves that can propagate across nonphysical boundaries, mathematical equations must be applied to these boundaries. This mathematical equation is used to minimize or reduce nonphysical reflections from the computational boundary. Many methods have been developed to reduce non-physical reflections, of course each method has advantages and disadvantages.

In this research, the boundary condition method that is usually used in wave propagation modelling is the transparent boundary condition method. Boundary condition methods are needed to reduce waves propagating beyond nonphysical boundaries. From the boundary, wave reflection is not permitted. The equation used as an open limit is as introduced by [19] as follows,

$$\frac{\partial \eta}{\partial t} + c\frac{\partial \eta}{\partial x} = 0 \tag{17}$$

Using Equation (17) above, it is possible to reduce nonphysical reflections from the boundaries. Using non-linear surface wave equations and the Reynolds transparent boundary condition method, surface wave propagation can be modeled. For surface wave propagation, the above boundary conditions can be used by changing the wave velocity (c) to the gravitational acceleration (g). The same can be done for speed (u and v). Using different values of wave velocity, tsunami wave propagation can be modelled to see to what extent the effects of wave propagation and propagation can be simulated.

For wave sources, use the Ricker wavelet function [6] which can be presented as follows,

$$f(t) = \left(t - \frac{2}{\pi . f_o}\right) . e^{-\frac{1}{2}\omega^2\left(t-\frac{2}{\pi . f_o}\right)^2} \tag{18}$$

Where:

$\omega = 2.\pi.\ f_o$ and

f_o = wavelet fundamental frequency

Equation (18) is used to simulate the source of tsunami waves. This equation is better used as a wave source function compared to the sine function.

3 Wave propagation Scenarios

In this numerical modelling, the surface wave sources used are wave sources in the form of lines to simulate wave sources resulting from tectonic earthquakes and wave sources in the form of points to simulate wave sources resulting from volcanic earthquakes. The wave source in the form of a line is assumed to be a source of many point waves and forms lines/faults. To model the wave source, the Ricker wavelet equation is applied as used by [6]. In this research, the partial wave propagation model uses scenario assumptions as presented in Fig. 1. Grid scheme was used to simulate the propagation of tsunami waves caused by the eruption of Anak Krakatau as presented in research conducted by [8] and [7]. The bathymetric data used in the numerical simulations were obtained from the General Bathymetric Chart of the Oceans (GEBCO), where the bathymetric data uses an accuracy of 30 seconds. Volcanic earthquakes are assumed to originate from Mount Anak Krakatau, where the coordinate position of Anak Krakatau is assumed to be a point source, where the tsunami wave height at the source location is assumed to be equal to 200 meters [18]. This level is based on published literature regarding the height of the tsunami waves due to the eruption of Mount Krakatoa in 1883. Apart from being based on the literature above, tectonic earthquakes are also assumed to occur at a location from several locations that are predicted to be most likely to occur. It is also based on information from literature released by the USGS.

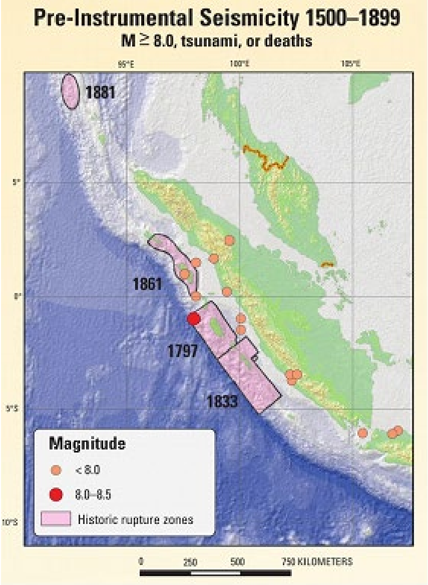

Fig. 1. Map of the location of the earthquake with the largest tsunami in Sumatra [20].

4 Results and Discussion

From the results of 2-dimensional tsunami wave propagation modelling for various scenarios of source types and wave locations, the following results are obtained:

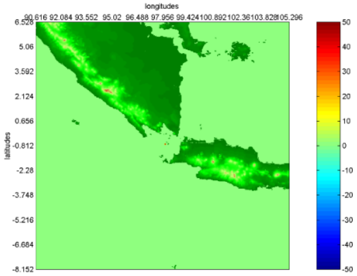

Fig. 2. Snapshot of tsunami wave propagation at t = 50 seconds due to a volcanic earthquake (scen 1)

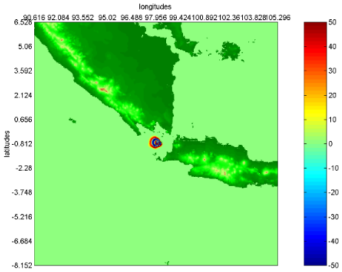

Fig. 3. Snapshot of tsunami wave propagation at t = 500 seconds due to a volcanic earthquake (scen 1)

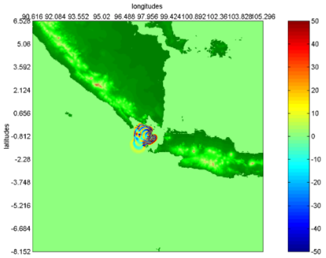

Fig. 4. Snapshot of tsunami wave propagation at t = 1000 seconds due to a volcanic earthquake (scen 1)

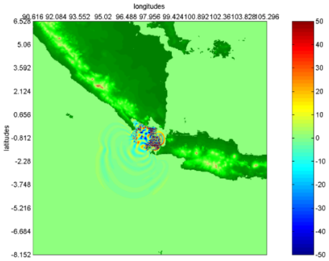

Fig. 5. Snapshot of tsunami wave propagation at t = 2000 seconds due to a volcanic earthquake (scen 1)

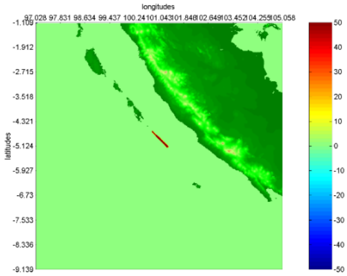

Fig. 6. Snapshot of tsunami wave propagation at t = 50 seconds due to a tectonic earthquake (scen 2)

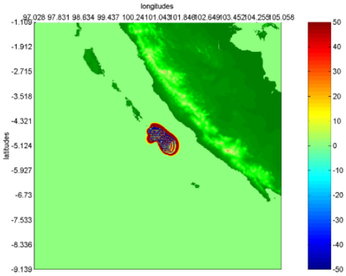

Fig. 7. Snapshot of tsunami wave propagation at t = 500 seconds due to a tectonic earthquake (scen 2)

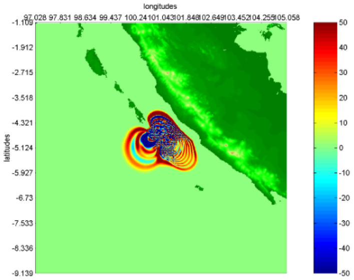

Fig. 8. Snapshot of tsunami wave propagation at t = 1000 seconds due to a volcanic earthquake (scen 2)

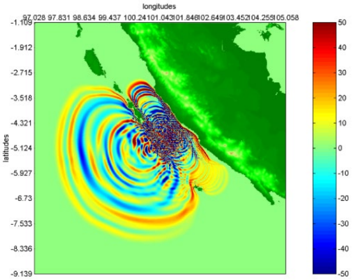

Fig. 9. Snapshot of tsunami wave propagation at t = 2000 seconds due to a volcanic earthquake (scen 2)

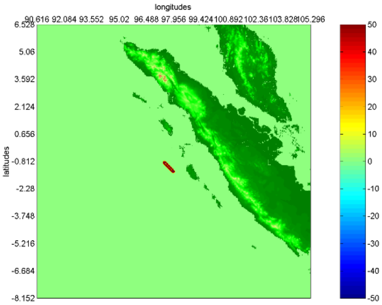

Fig. 10. Snapshot of tsunami wave propagation at t = 50 seconds due to a tectonic earthquake (scen 3)

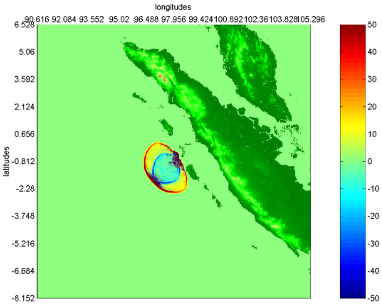

Fig. 11. Snapshot of tsunami wave propagation at t = 500 seconds due to a tectonic earthquake (scen 3)

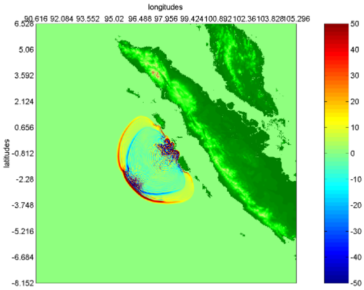

Fig. 12. Snapshot of tsunami wave propagation at t = 1000 seconds due to tectonic earthquake (scen 3)

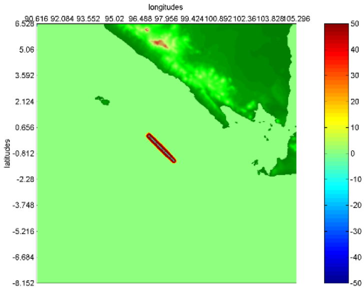

Fig. 13. Snapshot of tsunami wave propagation at t = 50 seconds due to a tectonic earthquake (scen 4)

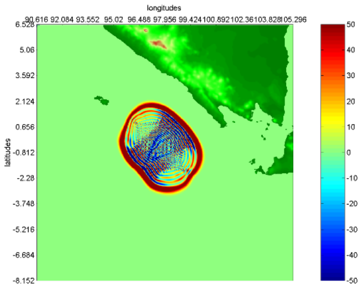

Fig. 14. Snapshot of tsunami wave propagation at t = 500 seconds due to a tectonic earthquake (scen 4)

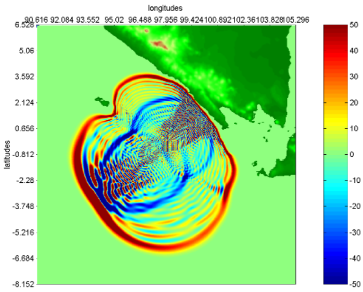

Fig. 15. Snapshot of tsunami wave propagation at t = 1000 seconds due to a tectonic earthquake (scen 4)

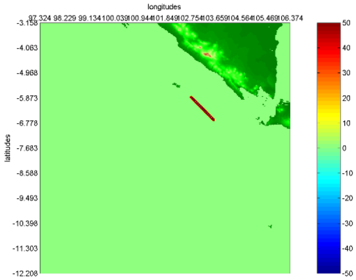

Fig. 16. Snapshot of tsunami wave propagation at t = 50 sec due to a tectonic earthquake (scen 5)

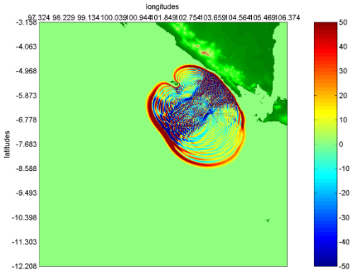

Fig. 17. Snapshot of tsunami wave propagation at t = 1000 sec due to a tectonic earthquake (scen 5)

Fig. 2. to Fig. 5. shows a snapshot of the simulation of tsunami wave propagation due to a volcanic and tectonic earthquake or the consequences if the Anak Krakatau volcano erupts (scen 1) and a snapshot of the consequences if a tectonic earthquake occurs (scen 2 to scen 5). These results also show that changes in tsunami wave height due to volcanic earthquakes will decrease over time as they propagate, but changes in tsunami wave height due to tectonic earthquakes do not decrease or experience significant changes. This means that tsunamis caused by tectonic earthquakes are more dangerous or hazardeous than tsunamis caused by volcanic earthquakes.

From scen 1 it also shows that the tsunami wave height caused by the volcanic earthquake (Mount Anak Krakatau) on the west coast is much smaller than the tsunami wave height in the Tanggamus and Teluk Betung coastal areas. Meanwhile, the tsunami wave height in the Tanggamus and Teluk Betung areas was estimated to be below 5% of the wave height at the source (Child of Krakatoa). The propagation time of tsunami waves due to volcanic earthquakes in the west coast area is much longer than the propagation time of tsunami waves in the Tanggamus coastal area and the Teluk Betung coastal area, which is more than 60 minutes. Research conducted by [18] also shows that the propagation time for tsunami waves caused by volcanic earthquakes can reach Teluk Betung in more than 60 minutes.

From the results of scen 2, it is estimated that the tsunami can reach the coast of Bengkulu after 35 minutes with a height that can reach 25% of the wave height at the source of the tectonic earthquake. Meanwhile, the tsunami could reach Krui Beach after 44 minutes with a tsunami height of more than 20%. After 52 minutes the tsunami could reach the coast of Padang with a height of more than 30%. For scen 3, the tsunami danger to the west coast of Sumatra is estimated to be much smaller compared to scen 2. For scen 4 and scen 5, the tsunami can reach the coast of the west coast of Lampung in more than 17 minutes with a height that can reach 40% of the wave height at earthquake source. The results of this study show that the propagation time of tsunami waves due to tectonic earthquakes to the west coast of Sumatra in general and the west coast of Lampung Province is much faster than the propagation of tsunami waves due to tectonic earthquakes to the coasts of the Tanggamus coast and the Teluk Betung coast. This study also shows that the high percentage of tsunami waves reaching the coast from their source due to volcanic earthquakes is smaller than the high percentage of tsunami waves resulting from tectonic earthquakes.

5 Conclusions and Suggestions

5.1 Conclusion

Based on the results of research that has been carried out, it can be concluded that tsunamis caused by tectonic earthquakes are more hazardous than tsunamis caused by volcanic earthquakes. The tsunami that will occur as a result of a tectonic earthquake in the western region of Sumatra could be more dangerous for the western coastal region of Lampung Province because the tsunami wave propagation time is less than 20 minutes.

5.2 Suggestion

To obtain more accurate results, the simulations carried out should also use various other long wave equations. To be able to estimate the risk more accurately, the simulation should be carried out using an equation that can calculate inundation by applying the 2-dimensional moving boundary equation. However, unfortunately this is not easy to do because it must use much smaller and denser bathymetric and topographic data. So, to carry out the simulation a supercomputer is needed.

Acknowledgement. I would also like to express my gratitude to Lampung University for their extraordinary encouragement. The study was performed with financial support by Institute for Research & Community Service, Lampung University.

References

1. Neelamani, S., Sumalatha, B.V.: Wave reflection, run-up, run-down and pressures on seawalls defended by an offshore breakwater. J Indian Inst Sci. 86, 15–31 (2006)
2. Ty Wamsley, Hanson, H., Kraus, N.C.: Wave Transmission at Detached Breakwaters for Shoreline Response Modeling. , Vicksburg (2002)
3. Kowalik, Z., Murty, T.S.: Numerical simulation of two-dimensional tsunami runup. Marine Geodesy. 16, 87–100 (1993). https://doi.org/10.1080/ 15210609309379681
4. Horrillo, J., Kowalik, Z., Shigihara, Y.: Wave dispersion study in the Indian Ocean-tsunami of December 26, 2004. Marine Geodesy. 29, 149–166 (2006). https://doi.org/10.1080/01490410600939140

5. Sugawara, D.: Numerical modeling of tsunami: advances and future challenges after the 2011 Tohoku earthquake and tsunami. Earth Sci Rev. 214, 103498 (2021). https://doi.org/https://doi.org/10.1016/j.earscirev.2020.103498
6. Zakaria, A.: Numerical modelling of wave propagation using higher order finite-difference formulas, (2003)
7. Zakaria, A.: Simulasi perambatan gelombang tsunami akibat meletusnya gunung anak krakatau. In: Seminar Sains MIPA Aplikasinya. pp. 235–246. FMIPA UNILA, Bandarlampung (2009)
8. Zakaria, A., Susilowati, K.: Simulasi waktu perambatan dan tinggi gelombang tsunami akibat meletusnya gunung anak Krakatau. In: Prosiding Seminar Nasional Magister Teknik Sipil Universitas Lampung. Magister Teknik Sipil Universitas Lampung, Bandarlampung (2012)
9. Goto, C., Shuto, N.: Numerical simulatiom of tsunami propagation and runup. In: K. Iida and T. Iwasaki (ed.) Tsunami - Their Science and Engineering. pp. 439–451. Terrapub, Tokyo (1983)
10. Goto, C., Ogawa, Y., Shuto, N., Imamura, F.: Numerical Method of Tsunami Simulation with Leap-Frog Scheme. UNESCO, Paris (1997)
11. Marchuk, A.J., Anisimov, A.: A method for numerical modeling run-up on the coast of an arbitrary profile. In: ITS 2001 Proceedings (2001)
12. Grilli, S.T., Vogelmann, S., Watts, P.: Development of a 3D numerical wave tank for modeling tsunami generation by underwater landslides. Eng Anal Bound Elem. 26, 301–313 (2002). https://doi.org/10.1016/S0955-7997 (01) 00113-8
13. Watts, P., Ioualalen, M., Grill, S.T., Shi, F., Kirby, J.T.: Numerical simulation of December 26, 2004 Indian ocean tsunami using higher order boussinesq model, Ocean waves measurement and analysis. In: Fifth International Symposium WAVES 2005. , Madrid, Spain (2005)
14. Watts, P., Grill, S.T., Kirby, J.T., Fryer, G.H., Tappin, D.R.: Landslide., tsunami case studies using a boussinesq model and a fully nonlinier tsunami generation model. Natural Hazards and Earth System Sciences. 3, 391–402 (2023)
15. Shigihara, Y., Fujima, K., Homma, M., Saito, K.: Numerical methods of linier dispersive wave equation for the practical problems. In: Asian and Pacific Coasts. p. 14. , Jeju, South Korea (2005)
16. Kowalik, Z., Proshutinsky, T., Proshutinsky, A.: Tide-tsunami interactions. Science of Tsunami Hazards. 24, 242–256 (2006)
17. Kawamata, S., Imamura, F., Shuto, N.: Numerical simulation of the 1883 Krakatau tsunami. In: XXV Congress, Int. Assoc. Hydroenviron. Eng., Tokyo (1993)
18. Hantoro, W.S., Latief, H., Susilohadi, Airlangga, A.Y.: Volcanic tsunami of krakatau: chronology model and its mitigation in sunda strait. In: Proceedings of International Symposium on Geotechnical Hazards: Prevetion, Mitigation and Engineering Response. pp. 331–354 (2007)
19. Reynolds, A.C.: Boundary conditions for the numerical solution of wave propagation problems. Geophysics. 43, 1099–1110 (1978). https://doi.org/10.1190/ 1.1440881
20. Hayes, G.P., Bernardino, M., Dannemann, F., Smoczyk, G., Briggs, R., Benz, H.M., Furlong, K.P., Villaseñor, A.: Seismicity of the Earth 1900–2012 Sumatra and vicinity. , Denver (2013)