



Scenario-Based Teaching Enhances Mathematical Core Competencies—A Study on Cultivating Mathematical Thinking Ability of Middle School Students under the Background of the New Curriculum Standards

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Abstract. The study analyzed factors influencing students' mathematical thinking abilities and developed a survey questionnaire. Based on the survey results, data analysis was conducted using SPSS software to establish a test model for assessing students' levels of mathematical thinking abilities. The current status of mathematical thinking abilities among secondary school students was examined. Finally, in accordance with the requirements of the new curriculum standards, strategies for cultivating students' mathematical thinking abilities were proposed.

Keywords: Mathematical thinking ability, The new curriculum standards, Mathematical literacy, Training strategy

1 Introduction

Scenario-based teaching is a teaching method that places students in real-life situations, providing a more in-depth and practical learning experience. It involves simulating authentic situations to help students connect theoretical knowledge with practical problem-solving. The approach encourages students to apply what they've learned to real-world scenarios, emphasizing the practical application of theoretical knowledge. This teaching method aligns with the 2022 version of the compulsory education mathematics curriculum standards, promoting active learning, practical skill development, and the cultivation of interdisciplinary abilities.

Regarding the understanding of the connotation of mathematical thinking, Liu Xiaomei believes that mathematical thinking ability is the capability to approach and solve problems from a mathematical perspective. Common types of mathematical thinking include analytical thinking, logical thinking, creative thinking, model thinking, algebraic and geometric thinking, spatial thinking, and problem-solving thinking [1]. Lian Yuankun states that mathematical thinking serves the construction of students' knowledge. In the era of pursuing the value of mathematical ideas and methods, mathematical thinking plays a supporting role in students' understanding of mathematical

ideas and methods. Under the current background of junior high school mathematics teaching and the cultivation of core literacy, it is necessary to consider how to cultivate students' mathematical problem-solving thinking [2]. With the progress of society and the development of education, the awareness of cultivating students' mathematical thinking ability has gradually become the main theme of education and teaching [3].

Looking at the research of foreign scholars on the mathematical thinking ability, considerable achievements have been made, but these studies have a relatively broad scope, strong theoretical content, a short time since the promulgation of the new curriculum standards, and less connection with the 2022 version of the new curriculum standards. There is not much in-depth connection with the national situation, so applying the research of predecessors to teaching practice is the focus of this paper's research.

The study employs a questionnaire survey method, designing a survey questionnaire under the guidance of curriculum standards. Students from selected classes are surveyed to collect, analyze, and organize data. The paper intends to construct a model for testing students' mathematical thinking levels using a questionnaire.

2 Data Analysis

2.1 Construction of a Mathematical Thinking Ability Test Model for Junior High School Students

Firstly, the construction of primary indicators is considered. From a psychological perspective, mathematical thinking ability is related to the cognitive development stage[4]. During the compulsory education stage, students have different cognitive abilities at different ages. The existing mathematical cognition of students is the foundation for learning mathematical knowledge and enhancing mathematical thinking ability[5]. It is also a fundamental attitude towards the mathematical discipline. Students' evaluation of their current learning is also an important factor influencing their mathematical learning. The evaluation of learning outcomes is the self-monitoring of students' own learning situations. The existing mathematical literacy of students is the foundation for cultivating their mathematical thinking ability[6]. The most important aspect of improving students' quality is to cultivate their mathematical thinking ability. Students' perception of teachers' teaching is a crucial aspect of the teaching process. The teacher's classroom teaching methods and content directly impact the cultivation of students' mathematical thinking ability. All these aspects are also requirements of the "Compulsory Education Mathematics Curriculum Standards (2022 Edition)."

Based on international curriculum objectives, metacognition and mathematical cognition theory, evaluation and learning motivation theory, the 2022 edition of compulsory education curriculum standards, PISA 2021 mathematics framework, and the analysis of domestic and foreign education experts on factors influencing mathematical thinking ability, this study identifies four aspects influencing students' mathematical thinking ability at the middle school stage from the perspective of the mathematics discipline: students' existing cognition[7], students' evaluation of current learning, students' existing mathematical literacy, and students' perception of teachers[8].

In order to analyze each indicator more deeply and scientifically, this study extracts the connotations of each indicator separately, combines various aspects of teaching and students' psychological development stages, and analyzes to form secondary indicators of the testing model. The framework for constructing the indicators of the testing model at each level is presented in Table 1.

Table 1. Middle School Stage Student Mathematical Thinking Ability Indicator System

Primary Indicators		Secondary Indicators			
A. Students' Existing Cognition	A1 Emotional and Mood (Emotional Attitudes and Values)	A2 Knowledge Processing	A3 Knowledge Organization (Observing the Real World)	A4 Knowledge Organization (Reflecting on the Real World)	A5 Learning Feedback
	B. Students' Evaluation of Current Learning	B1 Learning Status	B2 Learning Process	B3 Learning Outcomes	B4 Problem-Solving Results
C. Students' Existing Mathematical Literacy	C1 Mathematical Thinking	C2 Mathematical Language	C3 Mathematical Perspective		
D. Students' Perception of Teachers' Teaching	D1 Teaching Methods	D2 Teaching Skills	D3 Mathematical Culture	D4 Teaching Content	

Each secondary indicator is independent under its relevant primary indicator. Mathematical thinking activities are comprehensive cognitive activities influenced by "antecedents", "consequences" and "external factors". In different mathematical contexts and situations, the various factors of secondary indicators are interconnected and permeate, playing different roles. This constructs mathematical thinking ability into an organic, multidimensional whole.

2.2 Survey Results of Mathematical Thinking Ability Among Middle School Students

The survey distributed a total of 218 test papers, with 216 valid responses, resulting in an effective response rate of 99.1%. After collecting the test papers, descriptive statistical analysis was performed on the gathered data using SPSS Statistics R26.0.0. According to the analysis results, the mean values of these 16 indicators ranged from 3.33 to 4.30, indicating a relatively concentrated distribution of mean values. Therefore, the approval ratings for these 16 indicators are relatively high. Subsequent reliability and validity analyses were conducted on the data, with the results shown in Tables 2 and 3. The Cronbach's Alpha coefficient for this survey questionnaire was 0.955, exceeding

0.7, indicating strong reliability due to the high coefficient. The questionnaire results indicate that collecting data on various indicators at different levels is reliable and scientifically sound. Table 3 reveals that the Kaiser-Meyer-Olkin (KMO) measure is 0.964, exceeding 0.7, indicating a certain level of correlation between the independent variables in the questionnaire design. The questionnaire demonstrates high validity, and the significance is less than 0.001, rejecting the hypothesis of independence among variables and indicating a strong correlation between variables.

Table 2. Reliability Statistical Analysis of the Survey Questionnaire

Cronbach's Alpha	Number of Items
.955	16

Table 3. KMO and Bartlett's Test

KMO Measure of Sampling Adequacy	.964	
	Approximate Chi-Square	2592.190967
Bartlett's Sphericity Test	Degrees of Freedom	120
	Significance	.000

2.3 Analysis of the Current Situation of Mathematical Thinking Ability among Middle School Students

This section mainly analyzes the differences in current mathematical thinking ability levels among junior high school students from the perspectives of gender and grade. In order to visually observe the impact of these factors on students' mathematical thinking ability, independent samples t-tests and one-way analysis of variance (ANOVA) were conducted using SPSS based on students' gender and grade.

(1) Independent Samples Test on Gender Differences and Main Factors Affecting Learning

Table 4. Group Statistics of Main Factors Influencing Learning and Gender

Gender	Number of cases	Mean	Standard Deviation	Standard Error of Mean
Male	127	31.235	9.8530	.8743
Female	89	32.497	10.2619	1.0878

Table 5. Independent Samples Test

Levene's test for homogeneity of variances	t-test for equality of means
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							stand- ard er- rors	differ- ence		
A 1	assuming equal variance	.283	.595	.135	214	.893	.015	.111	-.203	.233
	not as- suming equal variance			.134	186.1 34	.893	.015	.111	-.205	.235
A 2	assuming equal variance	.066	.939	1.07 6	214	.283	.129	.120	-.107	.365
	not as- suming equal variance			1.08 2	193.5 71	.280	.129	.119	-.106	.364
A 3	assuming equal variance	.036	.850	2.24 9	214	.026	.260	.116	.032	.488
	not as- suming equal variance			2.25 1	190.2 66	.026	.260	.116	.032	.488
A 4	assuming equal variance	.280	.597	1.32 8	214	.185	.181	.137	-.088	.451
	not as- suming equal variance			1.33 0	190.3 95	.185	.181	.136	-.088	.451
A 5	assuming equal variance	.067	.796	.857	214	.393	.110	.128	-.143	.362
	not as- suming equal variance			.859	191.3 58	.391	.110	.128	-.142	.361

B 1	assuming equal variance	.5 8 6	.445	.555	214	.579	.064	.115	-.163	.291
	not as- suming equal variance			.567	202.9 17	.571	.064	.113	-.158	.286
B 2	assuming equal variance	.2 9 8	.586	1.47 6	214	.141	.174	.118	-.058	.406
	not as- suming equal variance			1.48 9	195.1 84	.138	.174	.117	-.056	.404
B 3	assuming equal variance	3. 5 1	.062	.977	214	.330	.124	.127	-.127	.375
	not as- suming equal variance			1.00 3	205.0 79	.317	.124	.124	-.120	.368
B 4	assuming equal variance	.4 9 3	.483	1.41 5	214	.158	.186	.132	-.073	.446
	not as- suming equal variance			1.43 1	196.6 56	.154	.186	.130	-.070	.443
C 1	assuming equal variance	1. 3 9 7	.239	1.15 9	214	.248	.140	.121	-.098	.379
	not as- suming equal variance			1.17 9	200.2 07	.240	.140	.119	-.094	.375
C 2	assuming equal variance	.9 8 6	.322	1.54 1	214	.125	.180	.117	-.050	.411

As shown in Table 6, at the predetermined significance level of 0.05 (5%), the results of the independent samples t-tests revealed significant differences between genders in three dimensions of students' mathematical thinking abilities: knowledge organization ability in observing the real world (A3), mathematical perspective (C3), and opinions on teacher teaching methods (D1) (Sig. two-tailed P-value < 0.05). In summary, gender differences in students' mathematical thinking abilities are primarily evident in the dimensions of observing the real world, mathematical perspective, and opinions on teaching methods, while no significant differences were observed in the other thirteen dimensions.

The above conclusion indicates that there is a certain degree of similarity in the dimensions of students' current cognition, evaluations of their current learning, existing mathematical literacy, and their perceptions of teachers' teaching.

(3) ANOVA Test on Students' Understanding of Mathematics Curriculum Standards and Mathematical Thinking Abilities

According to Table 7, with a predetermined significance level of 0.05 (5%), the significance (P) values for each dimension are all less than 0.05, indicating significant differences.

Table 7. Differences in Students' Understanding of Mathematics Curriculum Standards and Single-Factor ANOVA Test on Various Mathematical Thinking Abilities

		Sum of Squares	Degrees of Freedom	Mean Square	F	Significance
A1	Between Groups	7.623	3	2.541	4.149	.007
	Within Groups	129.817	212	.612		
	Total	137.440	215			
A2	Between Groups	37.084	3	12.361	21.036	.000
	Within Groups	124.578	212	.588		
	Total	161.662	215			
A3	Between Groups	30.751	3	10.250	17.698	.000
	Within Groups	122.786	212	.579		
	Total	153.537	215			
A4	Between Groups	43.787	3	14.596	18.542	.000
	Within Groups	166.875	212	.787		
	Total	210.662	215			
A5	Between Groups	34.628	3	11.543	16.387	.000
	Within Groups	149.330	212	.704		
	Total	183.958	215			
B1	Between Groups	28.554	3	9.518	16.759	.000
	Within Groups	120.404	212	.568		
	Total	148.958	215			

B2	Between Groups	21.978	3	7.326	11.518	.000
	Within Groups	134.837	212	.636		
	Total	156.815	215			
B3	Between Groups	37.753	3	12.584	18.495	.000
	Within Groups	144.247	212	.680		
	Total	182.000	215			
B4	Between Groups	27.745	3	9.248	11.676	.000
	Within Groups	167.917	212	.792		
	Total	195.662	215			
C1	Between Groups	31.875	3	10.625	16.921	.000
	Within Groups	133.120	212	.628		
	Total	164.995	215			
C2	Between Groups	29.044	3	9.681	16.252	.000
	Within Groups	126.290	212	.596		
	Total	155.333	215			
C3	Between Groups	49.370	3	16.457	22.677	.000
	Within Groups	153.848	212	.726		
	Total	203.218	215			
D1	Between Groups	26.964	3	8.988	12.335	.000
	Within Groups	154.476	212	.729		
	Total	181.440	215			
D2	Between Groups	23.933	3	7.978	12.162	.000
	Within Groups	139.063	212	.656		
	Total	162.995	215			
D3	Between Groups	17.107	3	5.702	7.172	.000
	Within Groups	168.555	212	.795		
	Total	185.662	215			
D4	Between Groups	33.745	3	11.248	17.550	.000
	Within Groups	135.880	212	.641		
	Total	169.625	215			

The above conclusion suggests that students' understanding of mathematics curriculum standards is significantly related to various abilities under students' existing cognition, evaluations of current learning, existing mathematical literacy, and opinions on teacher teaching methods.

2.4 Analysis of the Comprehensive Ability Level of Mathematical Thinking among Middle School Students

In the previous section, a mathematical expression for the model assessing students' mathematical thinking ability levels was developed through the organization of relevant theoretical studies and survey questionnaire data, represented as $Y = 0.056A + 0.27B + 0.42C + 0.25D$. Multiplying the effective percentages of each primary indicator by 100, the scores for the respective sections are obtained. Regression analysis was conducted using SPSS software, and the results are presented in Table 8.

Table 8. Linear Regression Analysis

	Unstandardized	Standardized Coeffi-		Collinearity Statis-			
	Coefficient	cient				tics	
	B	Standard Error	Beta	t	Signifi-	Tolerance	VIF
					cance		
A1	2.569	1.135	.205	2.264	.025	.525	1.904
A2	-.028	1.455	-.002	-.019	.985	.272	3.682
A3	-.305	1.548	-.026	-.197	.844	.253	3.959
A4	1.164	1.150	.115	1.012	.313	.334	2.998
A5	1.953	1.209	.180	1.615	.108	.345	2.897
B1	-.773	.987	-.064	-.783	.434	.640	1.562
B2	-1.564	1.233	-.133	- 1.269	.206	.390	2.566
B3	-1.621	1.105	-.149	- 1.468	.144	.418	2.390
B4	.539	1.124	.051	.480	.632	.376	2.661
C1	-1.405	1.421	-.123	-.989	.324	.279	3.588
C2	-4.432	1.467	-.376	- 3.020	.003	.278	3.600
C3	3.207	1.290	.311	2.485	.014	.275	3.642
D1	-.914	1.201	-.084	-.761	.448	.355	2.816
D2	1.207	1.160	.105	1.041	.299	.423	2.361
D3	-2.131	.936	-.198	- 2.276	.024	.571	1.751
D4	1.689	1.085	.150	1.557	.121	.465	2.149

a. Dependent Variable : Overall Status of Mathematical Thinking Abilities

The data was processed, imported into SPSS, and histograms for the total scores at each stage were plotted. By observing the histogram of overall mathematical thinking ability, it is found that the total scores of the student questionnaires roughly follow a normal distribution. Additionally, Beta values had both positive and negative values, indicating that these factors had both positive and negative impacts on thinking abilities. All VIF values were below 5, indicating the absence of multicollinearity.

Using SPSS, the data was further tested to more accurately examine whether the total test scores exhibited a normal distribution. By observing the normal P-P plot for total scores, the scatterplot appeared to approximate an upward-sloping diagonal line. The

actual cumulative probability of the data closely matched the expected cumulative probability, indicating that the data collected through the questionnaire exhibited normality.

Based on the distribution of individual raw total scores from the survey questionnaire, the current status of students' mathematical thinking abilities was categorized into four levels: the first level ranged from 0 to 40 points, the second level from 41 to 50 points, the third level from 51 to 70 points, and the fourth level from 71 to 80 points. 4% of students are at the first level, 27% are at the second level, 52% are at the third level, and 17% are at the fourth level. It illustrates that the overall mathematical thinking ability of first-year middle school students in the school is relatively high, with the majority falling into the second and third levels.

On the basis of the entire dataset, the categorization of students' mathematical thinking ability levels is defined as follows: the lowest level is categorized as the first level, and the highest level is categorized as the fourth level.

First Level: Low Mathematical Thinking Ability. Students have limited knowledge of mathematics, low motivation for learning, lack confidence and patience in problem-solving, weak foundational knowledge, inability to construct a logical system, low interest and curiosity in mathematics, excessive reliance on teachers, and a lack of independent thinking and divergent thinking abilities. They have minimal understanding of mathematical culture.

Second Level: Moderate Mathematical Thinking Ability. Students have fragmented knowledge of mathematics, can solve basic problems, possess some motivation but tend to give up easily, can complete homework with a low accuracy rate, have limited self-awareness of their learning situation, lack accurate understanding and profound comprehension of knowledge, weak mathematical language skills, relatively simplistic thinking, high dependence on teachers, and view mathematics primarily as a problem-solving tool.

Third Level: High Mathematical Thinking Ability. Students exhibit a strong interest and motivation in mathematics, can solve basic mathematical problems, construct a logical thinking system, apply mathematical thinking to observe and solve real-world problems, have strong mathematical language skills, actively engage in pre-class preparation and reflection, evaluate their learning through exams and assignments, show moderate dependence on teachers, possess awareness of mathematical culture, and view mathematics as a skill.

Fourth Level: Very High Mathematical Thinking Ability. Students have a profound understanding of mathematics, strong logical thinking, confidence and patience in solving complex problems, enjoy the problem-solving process, can apply mathematical thinking to other disciplines, demonstrate divergent thinking, accurately assess their learning situation, autonomously explore and delve into mathematical problems, flexibly apply knowledge to solve new problems, have a moderate dependence on teachers, a high interest in mathematical culture, and immerse themselves in the role of mathematicians during thinking and reasoning.

3 Conclusion and Suggestion

3.1 Research Conclusion Summary

The first study indicates that, following an independent sample test on gender differences regarding the main factors influencing mathematics learning, the statistical results reveal no significant association between students' gender and their perception of the main reasons affecting mathematics learning. The statistical outcomes demonstrate no significant differences, whether in tests for homogeneity of variance or in mean comparisons. In summary, there is no significant difference in students' perceptions of the factors influencing their mathematics learning based on gender.

The second study revealed significant gender differences in students' mathematical thinking abilities in observing the real world (A3), mathematical perspective (C3), and opinions on teaching methods (D1) through independent samples t-tests ($P < 0.05$). No significant differences were observed in the other thirteen dimensions ($P > 0.05$). This indicates that gender differences primarily exist in observing reality, mathematical perspective, and teaching methods.

The third research result indicates a significant association between students' understanding of mathematics curriculum standards and their existing cognition, evaluations of current learning, existing mathematical literacy, and opinions on teacher teaching methods.

The fourth study, based on theoretical research and questionnaire data, established a model to assess students' levels of mathematical thinking ability, represented as $Y = 0.056A + 0.27B + 0.42C + 0.25D$. Regression analysis results indicated a normal distribution of overall thinking abilities, with factors showing both positive and negative impacts. Students were categorized into four levels based on total scores. The findings highlighted generally high mathematical thinking abilities among students, with the majority at moderate to high levels and a small percentage at lower levels.

3.2 Strategies for Cultivating Students' Mathematical Thinking Skills Under the New Curriculum Standards

Firstly, there's an emphasis on thoroughly cultivating curriculum standards and constructing knowledge frameworks. The new curriculum standards, guided by principles such as integrating traditional Chinese culture and interdisciplinary collaboration, offer innovative pathways for teaching. Educators are encouraged to align with these standards, prioritizing the cultivation of students' core mathematical literacy and integrating metacognitive knowledge into teaching to develop sound cognitive structures[9].

Secondly, self-efficacy is identified as a crucial factor influencing students' psychological and behavioral aspects of learning. High self-efficacy fosters interest, motivation, and confidence in tackling tasks efficiently. Teachers are advised to boost students' learning confidence through tailored teaching and recognition from peers and parents, contributing to the development of mathematical thinking.

Thirdly, the integration of mathematical culture into teaching is highlighted. Mathematicians' thinking processes, when presented by teachers, can guide students to think

with a mathematician's mindset, experience logical thinking, and generate meaningful mathematical knowledge. This approach aims to overcome difficulties and instill a rigorous scientific spirit in students.

Fourthly, building an evaluation system that prioritizes individuals and students over results is recommended. The emphasis is on a comprehensive assessment of students' learning processes and outcomes, encouraging positive contributions to their growth. A scientifically reasonable evaluation system that combines process-oriented and result-oriented assessments is essential for motivating students and fostering healthy development.

Finally, the utilization of scenario-based instruction is introduced to ignite student emotions. This approach, suitable for middle school students, involves creating engaging situations through storytelling, experiments, outdoor teaching, and multimedia presentations. The goal is to spark students' curiosity, encourage proactive and positive thinking in problem-solving, and cultivate self-directed learning abilities[10].

In summary, these five strategies collectively aim to enhance students' mathematical thinking abilities by integrating innovative teaching methods, fostering self-efficacy, exploring mathematical history, addressing cognitive challenges, building a student-centered evaluation system, and utilizing scenario-based instruction.

Acknowledgment

This work is supported by the Jiangnan University Graduate Research Innovation Fund Project (KYCXJJ202350) and Ministry of Education Industry University Collaborative Education Project (220506627242057).

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