# Application of Uniparted Hyperboloid in Architectural Planning and Design

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**Abstract.** The uniparted hyperboloid is a kind of ruled surface, which has a simple and beautiful appearance and is widely used in architectural planning and design. This paper analyzes the characteristics of uniparted hyperboloid from a mathematical point of view, and expounds how to flexibly apply uniparted hyperboloidin in architectural planning and design from an architectural point of view. There are two kinds of uniparted hyperboloid, which are ordinary uniparted hyperboloid and rotational uniparted hyperboloid. The latter is more applied in architecture. The uniparted hyperboloid can be used in steel structure buildings and reinforced concrete structure buildings. It can be used in both civil and industrial buildings. Architects should master the characteristics of the uniparted hyperboloid and be able to use it flexibly in architectural planning and design to create beautiful buildings.

**Keywords:** uniparted hyperboloid; ruled surface; architectural planning; architectural design; application

## 1 INTRODUCTION

Uniparted hyperboloid is a three-dimensional space surface, whose shape is symmetric about the *XOY* plane, *XOZ* plane, and *YOZ* plane. Meanwhile, uniparted hyperboloid is also a ruled surface, which can be obtained by moving a straight line along a specific trajectory. Uniparted hyperboloid is often used in architectural design, as it is a ruled surface, which brings great convenience to structural processing. Uniparted hyperbolic has a beautiful appearance and are commonly used in high-rise buildings in architectural design, which can create a strong artistic appeal. Many architects and structural engineers like uniparted hyperboloids and use them to create a variety of architectural forms.

In order to proficiently use uniparted hyperboloid in architectural design, designers must have a profound mathematical understanding of uniparted hyperboloid. They must master the spatial analytical equations of uniparted hyperboloid and the meanings they express. In the teaching of architecture and urban planning in China, higher mathematics is often ignored and does not involve deeper content, so the graduates of Architectural Colleges are not familiar with uniparted hyperboloid and can not cor-


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rectly use it in architectural design. If we master the mathematical principle of uniparted hyperboloid, we can design the exact shape of uniparted hyperboloid building according to the mathematical formula and image in architectural design. In practical applications, there are two uses for uniparted hyperboloid: one is a ordinary uniparted hyperboloid, and the other is a rotational uniparted hyperboloid. The rotational uniparted hyperboloid is a rotating surface, which has a more regular shape and more single components, so it is more conducive to structural treatment and construction is simpler.

## 2 ORDINARY UNIPARTED HYPERBOLOID

### 2.1 Equation and Image

The mathematical equation for an ordinary uniparted hyperboloid is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1 \quad (a, b, c > 0) \tag{1}$$

Its image is shown in Figure 1[1].

When the surface is cut with a plane parallel to *XOZ* and *YOZ*, a pair of hyperbolas is obtained. When the surface is cut with a plane parallel to *XOY*, an ellipse is obtained. The ellipse on the *XOY* plane is the smallest and is called the waist ellipse. There are two groups of straight generatrixes on the surface, all of which pass through the waist ellipse, and all lines in the same group of straight generatrixes do not intersect. As shown in Figure 2. The uniparted hyperboloid can be seen as a straight generatrix moving horizontally along the waist ellipse, or as an ellipse moving up and down along two pairs of hyperbolas in the *XOZ* plane and the *YOZ* plane (The vertices of the long and short axes of the ellipse are respectively located on four hyperbolas).

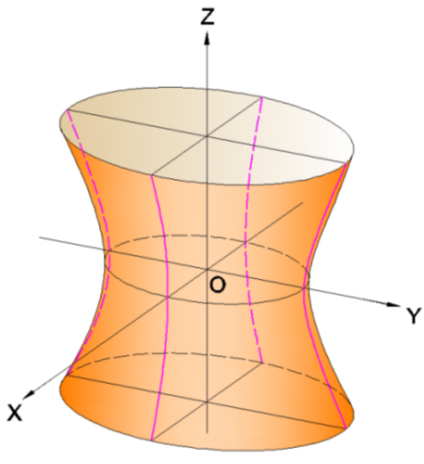


**Fig. 1.** The image of ordinary uniparted hyperboloid.

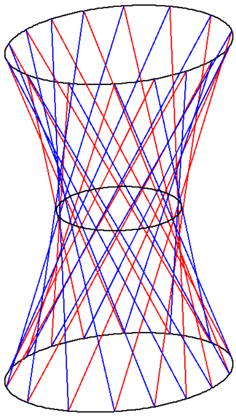

**Fig. 2.** Schematic diagram of straight generatrixes on the surface of ordinary uniparted hyperboloid.

Make equivalent transformation of equation (1) to obtain equation (2) or equation (3).

$$\begin{cases} u\left(\frac{x}{a}+\frac{z}{c}\right)=v\left(1-\frac{y}{b}\right) \\ v\left(\frac{x}{a}-\frac{z}{c}\right)=u\left(1+\frac{y}{b}\right) \end{cases} \quad (u^2+v^2\neq 0) \tag{2}$$

$$\begin{cases} p\left(\frac{x}{a}+\frac{z}{c}\right)=q\left(1+\frac{y}{b}\right) \\ q\left(\frac{x}{a}-\frac{z}{c}\right)=p\left(1-\frac{y}{b}\right) \end{cases} \quad (p^2+q^2\neq 0) \tag{3}$$

From equation (2) or (3), it can be seen that an uniparted hyperboloid is composed of a series of straight lines and is a ruled surface [2].

## 2.2 Application in Architectural Planning and Design

It is often seen in high-rise structures or special buildings. Nowadays, the main structural forms of high-rise buildings are steel structure and reinforced concrete structure. Here, we take these two structural forms as examples to elaborate respectively.

**Reinforced Concrete Structure.**
It is usually a reinforced concrete thin shell structure with beautiful shape, reasonable stress, light weight, material saving, firmness and durability. It is usually a reinforced concrete thin shell structure with beautiful shape, reasonable stress, light weight, material saving, firmness and durability. It is often used in industrial buildings, such as cooling towers, warehouses, etc. The architect first draws a sketch, then determines the size of the waist ellipse according to the sketch, and calculates the values of a and b. The architect sets the size and vertical position of the long axis of the bottom ellipse, substituting the values of a and b into equation (1) to calculate the

value of c, and then the size of the short axis of the bottom ellipse can be obtained. Now that a, b and c are known, we set the Z coordinate of the top ellipse and substitute it into equation (1) to find the size of the long and short axes of the top ellipse[3].

Because the surface is curved, it is usually cast-in-place. During construction, with the help of computer software simulation, the surface is divided into several blocks from bottom to top, and each block is approximately a plane, which can simplify the structure construction. The structure shall be poured from bottom to top, and the shape, size and angle of each block shall be noted. In case of door and window openings, they shall be reserved[4]. As shown in Figure 3.

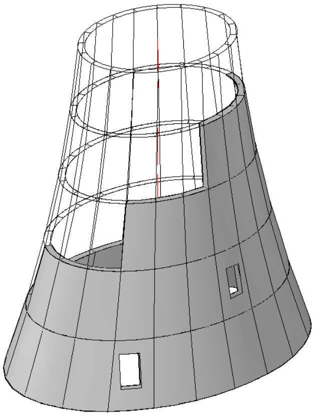

**Fig. 3.** Schematic diagram of reinforced concrete structure of ordinary uniparted hyperboloid.

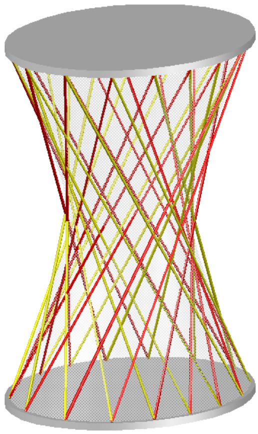

**Fig. 4.** Schematic diagram of steel structure of ordinary uniparted hyperboloid.

**Steel Structure.**

If used in public buildings, steel structure is usually used. First, the size, direction and position of the bottom ellipse and the top ellipse are determined, and the surface equation is obtained. Then, according to the surface equation, the straight generatrix of the surface is found. The bottom ellipse can be divided into several equal parts. According to the straight generatrix equation, the top coordinates of the straight generatrix passing through the equidistant points can be calculated, and the two points above and below are connected to obtain several straight generatrixes on the surface[5]. In practical engineering, these inclined straight generatrixes may be inclined steel columns, or steel tie rods or cables. Exterior walls or glass curtain walls are set between steel columns or steel tie rods. As shown in Figure 4. The interior is usually a steel frame with vertical steel columns and horizontal steel beams. The steel beam of each floor is connected with the external inclined steel column or steel tie rod [6-7].

The Canton Tower is a uniparted hyperboloid building with an elliptical horizontal cross section. The peripheral steel structure is composed of 24 inclined columns, 1104 inclined braces and 46 groups of annular beams[8]. The column is twisted up and down by 135 degrees. Different from the model, the top of the Canton Tower is an inclined plane, not a horizontal plane. The Canton Tower is graceful in shape and light in structure. As shown in Figure 5.

**Fig. 5.** The Canton Tower.

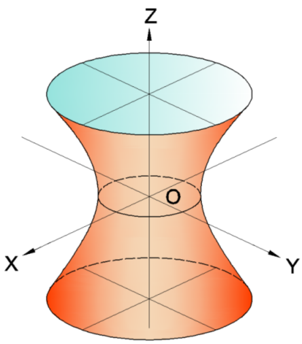


**Fig. 6.** The image of rotational uniparted hyperboloid.

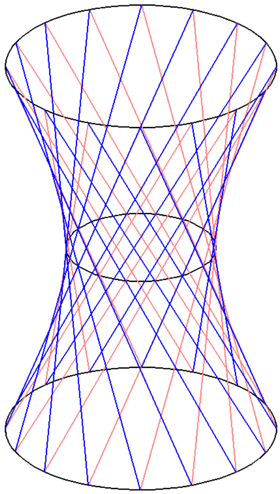

**Fig. 7.** Schematic diagram of straight generatrix on the surface of rotational uniparted hyperboloid.

# 3 ROTATIONAL UNIPARTED HYPERBOLOID

## 3.1 Equation and Image

The mathematical equation of the rotational uniparted hyperboloid is

$$\frac{x^2}{a^2}+\frac{y^2}{a^2}-\frac{z^2}{c^2}=1 \quad (a, c > 0) \tag{4}$$

The image is shown in Figure 6.

When the surface is cut with a plane parallel to the XOY plane, a horizontal circle is obtained, and the waist circle is located on the XOY plane. When the surface is cut with a plane parallel to the Z axis, a pair of hyperbolas is obtained. Obviously, the surface also has two sets of straight generatrixes, and all lines in the same set of straight generatrixes do not intersect. As shown in Figure 7. Rotational uniparted hyperboloid can be seen as an inclined straight line rotating around the Z axis, and the point closest to the Z axis on the straight line forms the waist circle[9]. It can also be seen a circle moving up and down along the Z axis, and the circumference is located on the hyperbola of the section passing through the Z axis.

Make an equivalent transformation of equation (4) to obtain equation (5) or equation (6).

$$\begin{cases} u\left(\frac{x}{a}+\frac{z}{c}\right)=v\left(1-\frac{y}{a}\right) \\ v\left(\frac{x}{a}-\frac{z}{c}\right)=u\left(1+\frac{y}{a}\right) \end{cases} \quad (u^2+v^2\neq 0) \tag{5}$$

$$\begin{cases} p\left(\frac{x}{a}+\frac{z}{c}\right)=q\left(1+\frac{y}{a}\right) \\ q\left(\frac{x}{a}-\frac{z}{c}\right)=p\left(1-\frac{y}{a}\right) \end{cases} \quad (p^2+q^2\neq 0) \tag{6}$$

From equation (5) or (6), it can be seen that rotational uniparted hyperboloid is composed of a series of straight lines and it is a ruled surface.

### 3.2 Application in Architectural Planning and Design

Similar to the usage of ordinary uniparted hyperboloid, the structure is also dominated by steel structure or reinforced concrete structure. Here, we still take these two structural forms as examples to elaborate respectively. In fact, the rotational uniparted hyperboloid has more regular shape and simpler structure treatment, so it is more applied in engineering than the ordinary uniparted hyperboloid.

**Reinforced Concrete Structure.**
It is usually a reinforced concrete thin shell structure, which is often used in industrial buildings, such as warehouses, cooling towers, etc. The architect first draws a sketch, determines the size of the waist circle according to the sketch, and calculates the value of a. The architect then sets the radius and vertical position of the bottom circle, substituting the value of a into equation (4) to calculate the value of c. Now a and c are both known, setting the Z coordinate of the top circle and substitute it into equation (4) the radius of the top circle can be calculated. Of course, the size of the surface circle at any elevation can also be calculated[10].

The construction method is usually cast-in-place, which can refer to the previous practice of reinforced concrete structure of ordinary uniparted hyperboloid, and the shape is also shown in Figure 3. The difference is that each wall at the same elevation has the same size and shape, and the structure is simpler[11]. There are many examples of building projects with such structures, as shown in Figure 8 for the cooling tower of a power plant.

**Fig. 8.** Cooling tower of a certain power plant.

**Steel Structure.**

It is mainly used in civil buildings and is often used in super high-rise buildings (h>100m). Architects design schemes and draw elevation sketches. The architect determines the value of a according to the size of the waist circle, which is the radius of the waist circle. Then we return to the bottom or top circle, make the Y coordinate be 0, substitute the X and Z coordinates of the bottom or top circle into equation (4), (a, c ＞ 0), and calculate the value of c. As shown in Figure 9. The top radius R1 and the bottom radius R2 in the figure are the X coordinate values of the upper and lower parts. a and c are known, and now the rotational uniparted hyperboloid has been determined. The radius of the surface circle at any elevation can also be obtained. We divide the bottom circle into several equal parts and draw the straight generatrixes of the surface from the equal points. According to equation (5) or (6), the X and Y coordinates of the top of the straight generatrixes can be obtained. It is OK to find one. All straight generatrixes are the same and can be obtained by rotation. In practical engineering, these straight generatrixes are inclined steel columns or steel tie rods in the structure, which are connected with the ring steel beams on each floor to form the peripheral structure of the building[12]. As shown in Figure 10.

Architects can also set a rotation axis first, and then define a straight generatrix, determine the tilt direction of the straight generatrix, and make the straight generatrix rotate 360 degrees around the rotation axis to obtain a building of rotational uniparted hyperboloid. As shown in Figure 11. At this time, it is mainly to determine the top position and bottom position of the straight generatrix first, and then determine how many generatrixes are rotated in the horizontal circle according to the needs of structure and shape[13].

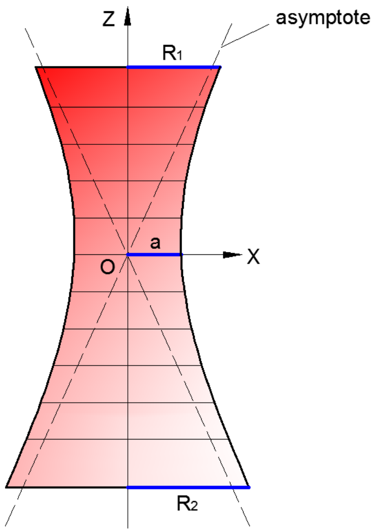


**Fig. 9.** Schematic diagram of rotational uniparted hyperboloid section.

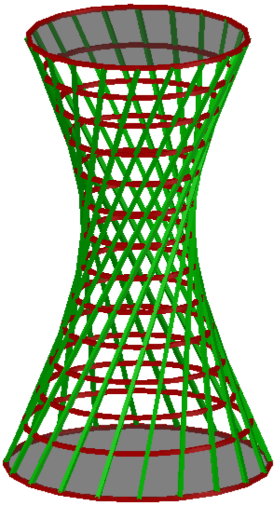

**Fig. 10.** Schematic diagram of steel structure of rotating uniparted hyperboloid.

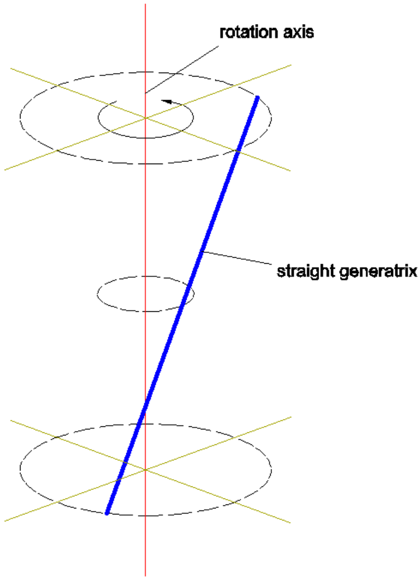


**Fig. 11.** Rotating straight generatrix to form rotational uniparted hyperboloid.

In reality, there are many civil buildings with the shape of rotational uniparted hyperboloid. For example, Kobe Port Tower in Japan, its exterior is composed of 32 red steel pipes, as shown in Figure 12. Another example is Forest Landscape Tower, which is supported by two-way straight generatrix steel pipes on the periphery and has a spiral wooden sightseeing path inside, as shown in Figure 13.

**Fig. 12.** Kobe Port Tower, Japan.

**Fig. 13.** Forest Landscape Tower, Denmark.

The shape of a rotational uniparted hyperboloid building can also be composed of hyperbolic members instead of linear members. According to the characteristics of the rotational hyperboloid, the outer surface of the building can be formed by a hyperbola rotating around a vertical straight line on the same plane[14]. Figure 14 is a schematic diagram of the building model. Several Hyperbolic bars are evenly distributed around the periphery of the model and intersect with the circular beams around each floor to form the overall structural frame. The hyperbolic shape and the radius of each layer of circular beam can be calculated by mathematical formula, and can also be simulated by computer software. Figure 15 shows Bayterek Tower, Kazakhstan. It is located in Astana, the capital of Kazakhstan, also known as the "tree of life". Its periphery is composed of several white hyperbolic steel frames[15].

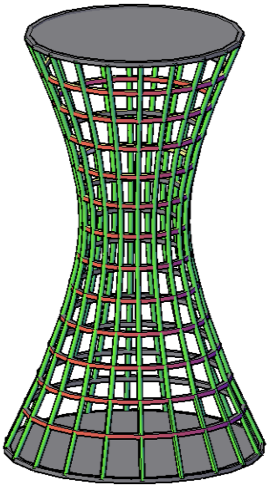

**Fig. 14.** Structural diagram of hyperbola member.

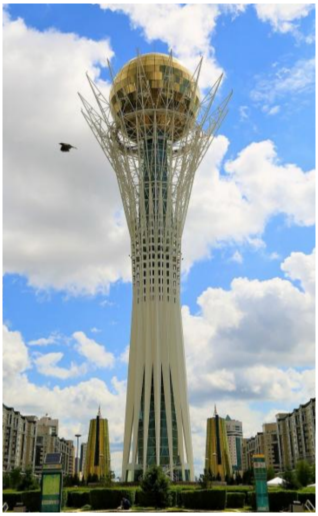

**Fig. 15.** Bayterek Tower, Kazakhstan.

Compared with the two kinds of uniparted hyperboloid, the latter one - rotational uniparted hyperboloid is more widely used in architecture. The structure of the rotational uniparted hyperboloid building is more simple, the components are more single, the construction is better handled, the cost is lower, and the appearance effect is similar, so it has been more widely used.

## 4 CONCLUSIONS

With its beautiful shape, unipartedt hyperboloid has been favored by many architects and is widely used in various building designs. Because it is a ruled surface, the curved surface can be formed by linear components in the structural treatment, which brings great convenience to the structural layout and increases its popularization and application in architectural planning and design. Uniparted hyperboloid can be used in steel structure building and reinforced concrete structure building. It can be used in civil buildings and industrial buildings. In architectural planning and design, if flexibly used, it can create a variety of lively, beautiful and unique buildings with great ornamental power. Among the two kinds of unipartedt hyperboloid, the rotational uniparted hyperboloid is more widely used in architecture.

Due to the lack of higher mathematics education in universities, young architects do not know much about uniparted hyperboloid and other spatial surfaces, which brings limitations to the future architectural design work. Young architects should strive to master the mathematical principle of uniparted hyperboloid and be familiar with its composition methods and skills, so as to flexibly apply it in architectural planning and design and broaden their own design ideas.

## FUND PROJECT

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