# Difficulties of Prospective Teachers in Proving Algebra Based on Understanding Concepts

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**Abstract.** The purpose of this research is to identify and analyse the difficulties of proving prospective teachers in working on algebraic structure proofs. The research method used is a qualitative descriptive proof method. The subjects used were prospective teachers who had taken the Group material and were at one of the universities in the city of Semarang, Indonesia. The research results show that there are several explanations for the difficulties in the mathematical proof carried out. The first difficulty in mathematical proof is based on Toulmin's argumentative point of view, the second difficulty is based on common mistakes that are often made in mathematical proof, the third error is based on the point of view of deductive proof. Each point of view has a different explanation of the difficulties experienced by prospective teachers in mathematical proof. The conclusion in this research is that quantifiers in mathematics have an important role in mathematical proof and further studies can examine effective proof methods.

Keywords: Difficulty of Mathematical Proof, Prospective Teacher, Mathematics Quantifier.

## 1. Introduction

Mathematical proof has an important role in the mathematics learning process[1]. According to Polya, there are two aspects in mathematics, namely the problems solving process and the mathematical proof process. Problem solving and mathematical proof skills need to be possessed in learning mathematics. Problem solving and mathematics learning receive serious attention in the learning process and in several research studies ranging from elementary level to higher education. There are several variables studied using the themes of problem solving and mathematical proof[2][3].

In this research, what is studied is related to mathematical proof using algebra. Based on a preliminary study conducted by researchers, there were several difficulties found by prospective teachers in mathematical proof. The main difficulty is not understanding the concept of the material well. In addition, knowledge about quantifiers has not been mastered thoroughly and mathematical proof methods are not applied proportionally[4]. The ability


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to use definitions, theorems and lemmas is still low so that the conclusions of the proof carried out are not fully supported by the correct theory.

Weaknesses in understanding the concept are quite important and influence the proof steps taken. Based on observations, prospective teachers who had difficulty carrying out algebra proofs found indications that when starting the proof they were still confused. The process of identifying questions that lead to proof is not well connected. Some prospective teachers do not understand well how to determine the initial steps of a proof. This has an impact on the work carried out. Not being used to working on evidence-based questions is one of the reasons prospective teachers do not have good evidence skills[5][6].

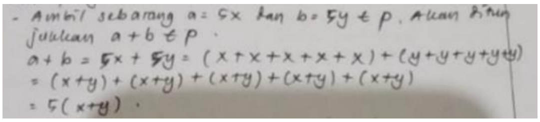
Several previous studies have examined students' difficulties in mathematical proofs[7],examined mathematical proofs with self-confidence, [8] studied difficulties in mathematical proofs based on the Adversity Quotient and [9] found a relationship between how prospective teachers overcome obstacles in mathematical proof and the enthusiasm they have for solving mathematical proof problems. In this research, we will explore mathematical proof errors using a combination of research indicators carried out by previous research. The aim of using combined indicators in this research is to describe in more depth the difficulties experienced by prospective teachers in mathematical proof and no other researcher has done this.

## 2. Methods

The method used in this research is qualitative descriptive research involving prospective teachers who have taken group material at one of the universities in the city of Semarang, Indonesia. The proof question given is "Prove, do all sets of integer multiples of five act as semigroups?".

## 3. Results and Discussions

Based on the answers that have been collected, three research subjects were taken who met the criteria to describe the evidential errors made. The first subject is seen with Toulmin's argumentation structure. The results of the first subject's work can be seen in Figure 1.



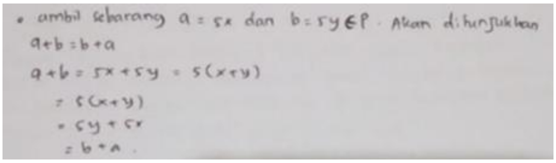
Take anything $a = 5x\ and\ b = 5y \in P$. Will be shown $a + b \in P$

$$a + b = 5x + 5 = (x + x + x + x + x) + (y + y + y + y + y)$$
$$= (x + y) + (x + y) + (x + y) + (x + y)$$
$$+ (x + y) = 5(x + y)$$

**Fig 1.** Results of Subject 1's work

Based on Figure 1, the part of Toulmin's argumentation structure in the form of data shows that prospective teachers can organize a fact and manipulate all sets of integer multiples of 5. For the warrant component, prospective teachers can write answers that lead to a conclusion reached. On the other hand, there are deficiencies in supporting the answer process carried out to clarify the provision of a conclusion. The first subject's weakness was that he did not completely write down a theorem or definition that underlies the work steps being carried out. Then in the answer given by the first Subject it is written 5y∈P. The P referred to by the first subject has no meaning that leads to an answer given. The symbol P in mathematics has another meaning and is not related to the proof in the problem. Each mathematical symbol only has one meaning and cannot be represented by other notations or meanings. After warrant, there is a Toulmin argumentation component, namely Claim, where prospective teachers are used as subjects to state claims about the final conclusion of the proof step which is still incorrect. The reason is that the warrant process contained several errors in the proof process, so that the final statement given was not meaningful.

The second subject for the proof process is observed from the point of view of things that usually occur in the process of mathematical proof. The results of the second subject's work can be observed in Figure 2.




Take anything $a = 5x \; and \; b = 5y \in P$. Will be shown $a + b = b + a$

$$a + b = 5x + 5y = 5(x + y)$$

$$5(x + y) = 5y + 5x = b + a$$

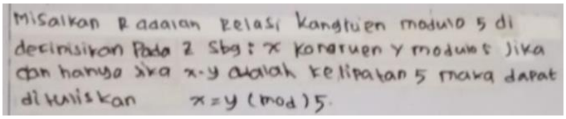
**Fig 2.** Subject 2 work results

Based on the results of the work of subject two, it shows that the application of strategies or methods of proof is not appropriate. The second subject wrote "Take anything a=5x and b=5y∈P". This shows that S2 does not yet fully know the methods and strategies used in mathematical proof. In fact, S2 does not know the proof steps that are written including what type of method they use. in the process of mathematical proof. This second subject has low knowledge about mathematical proof.

In line with difficulties using strategies or methods of mathematical proof, S2 includes difficulties in applying definitions and concepts. The ability to use concepts and definitions is still weak in mathematical proof based on the results of Master's work. This fragment of the answer represents another proof step that indicates S2 did not correctly use the concepts and definitions correctly. This also shows that S2 does not know the differences between definitions and concepts well so that the proof steps taken are not appropriate.

S2 did not construct a proof correctly because it did not use the definition correctly. S2 states 5y=b, even though at the beginning of the process the work was carried out to show a certain property 5y=a. Consistency in the proof process is very important to clarify the proof steps taken. Inconsistencies in the proof process can create different meanings so that the conclusion shown as the final result of the proof cannot be justified according to mathematical rules. The inconsistency carried out by S2 shows a lack of accuracy in the mathematical proof process. Small errors in mathematical proofs can change the meaning and conclusions provided.

The third subject that is observed from the proof steps is based on the type or type of error. The results of S3's work can be observed in Figure 3.


Misalkan R adalah relasi kongruen modulo 5 di definisikan pada Z sbg : x kongruen y modulo 5 jika dan hanya jika x-y adalah kelipatan 5 maka dapat dituliskan x=y (mod) 5.

Let R be a congruent relation modulo 5 defined on Z as x is congruent to y modulo 5 if and only if x=y is a multiple of 5 then it can be written x=y(mod)5

**Fig 3.** Subject 3's work results

Based on the results of subject 3's work by interpreting all sets of integer multiples of 5 to be x=y(mod)5. There are several indications of errors that appear, namely reading errors, comprehension errors, transformation errors, process skill errors, encoding errors. The reading error made by Subject 3 is related to the teacher candidate not being able to understand and identify the questions well. The Comprehension Error made by the third subject can be seen from the use of inaccurate definitions. Transformation Error is related to errors in interpreting all sets of integer multiples of 5. The encoding error made by subject 3 is related to writing x=y(mod)5 which means all sets of open numbers are multiples of 5. The process skill error made by subject 3 is related to proof skills which are still weak in the deductive proof process.

Errors Based on Toulmin's Argumentation Point of View

Proof errors are based on Toulmin's argumentation structure, namely in the warrant section of the proof process there are several steps that have not been written down thoroughly and are not supported by correct proof theory. In addition, there are several errors in writing the symbols which cause changes in meaning. Prospective teachers who have weaknesses in this aspect do not have sufficient knowledge about the application of theorems, definitions or lemmas in the mathematical proof process. This causes the evidentiary steps taken to not support the conclusions given[10].

Another weakness relates to understanding the function of symbols or signs in mathematics. Prospective teachers who experience difficulty in interpreting signs do not fully understand the meaning of each mathematical symbol or sign. Apart from that, subjects who have difficulty interpreting signs have weaknesses in applying signs in the mathematical proof process[11]. Knowledge of signs in mathematics is quite important, considering that mathematics is mostly interpreted in terms of signs[3].

Based on inappropriate evidentiary steps in the warrant process, the conclusions given are weak. Strong conclusions are supported by a correct theoretical basis and appropriate evidentiary steps. Starting from a correct understanding of the question, then supported by the use of relevant theory and proof steps according to the rules can support the final statement in a proof. Argumentation in mathematical proof has an important aspect in the process of showing the truth of a statement[12].

Errors Based on Mistakes Often Made in Mathematical Proof

In general, mistakes that are often made in mathematical proofs are not being able to apply proof strategies and methods properly, errors in applying concepts, inconsistency and lack of accuracy in mathematical proofs. The first common mistake is not implementing strategies and methods of proof properly. Applying the right proof method can make the steps of mathematical proof run effectively. Mathematical proofs are attempted to be carried out effectively and there are no statements outside the context of mathematical proofs[13].

The second mistake that is often made is applying the concept. Difficulty in understanding concepts and definitions as well as inaccuracies in their application can cause mathematical proof steps to produce incorrect conclusions. Knowledge of definitions and concepts is quite important in mathematical proof. If the mathematical proof does not include precise definitions and concepts, the proof steps will not lead to the desired conclusion[14].

The third error that often occurs is inconsistent application of the signs or symbols used. One of the causes is a lack of thoroughness in the proof process. Mathematical proof prioritizes the principle of consistency, without consistency the proof steps taken cannot be accepted as correct[15]. Consistency is not only in the use of signs or symbols, consistency in the application of definitions and concepts needs to be done to clarify the evidentiary conclusions provided[16].

Errors Based on the Point of View of Deductive Proof

In general, the mathematical proof process uses deductive proof. Therefore, types of errors ranging from Reading errors, Comprehension Errors, Transformation errors, Process skill errors and Encoding errors need to be avoided. The characteristics of deductive proof which starts from a general statement to become a specific statement needs to be supported by an appropriate proof flow. Identifying the right problem can help determine the appropriate strategy and proof method so that the application of the definition or concept in each step of the mathematical proof runs correctly[17]. Deductive proof is also influenced by deductive reasoning. The reasoning process in mathematical proof is an important aspect that needs to be given more attention in mathematical proof[15]

## 4. Conclusions

The mathematical proof process needs to use the right quantifier. The correct use of quantifiers can be trained by increasing the quality and quantity of mathematical proof practice. Educators who teach mathematical proof need to strengthen aspects of definitions, theorems or lemmas in the mathematical proof process. Educators also teach comprehensively the correct use of mathematical signs and symbols. The correct use of mathematical symbols and signs supports the proof steps carried out.

The characteristic of mathematical proof is deductive proof. Deductive proof needs to be supported by the use of appropriate proof strategies and methods. There are several methods of proof that can be used to make deductive proof effectively. Educators need to familiarize students with applying several appropriate strategies and methods of proof to increase knowledge about proof methods. Further research can inform effective proof methods in working on mathematical proofs.

## Authors' Contributions

Abdul Aziz Played A Role In Determining Subjects And Data Analysis. Iswahyudi Joko Suprayitno Played A Role In Creating Questions And Correcting Prospective Teachers' Answers. All Authors Contributed To The Preparation Of The Manuscript.

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