Influence of the Elastic Supports on the Dynamic Buckling of Arch Structures under Explosive Impact

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Abstract. Elastic supported arch structures are widely used in engineering. Under dynamic loads, the bearing capacity of arch structures is closely related to their dynamic stability. In this paper, the effects of the elastic supports on the arch dynamic response and dynamic buckling are investigated by numerical method. The results show that the elastic supports change the stress distribution of the arch structure. The most dangerous position is no longer the arch springing but the spandrel. During the explosion phase, the elastic supports reduce the effective stress peak. But during the free vibration phase, the vibration is enhanced. It is difficult to reduce the vibration by elastic supports solely. The damping supports should be set up at the same time. The elastic supports also change the deformation laws. The plastic hinges appear first at the spandrel but not at the springing. When the dynamic buckling occurs, the plastic deformation zone is less than one of the rigid supported arches. The elastic supports still reduce the dynamic buckling critical load. When the stiffness of the elastic supports changes within a certain range, the stiffness is larger, the critical load smaller. The elastic supports increase the arches' ultimate load-bearing capacity. And the smaller the stiffness is, the bigger the ultimate load-bearing capacity is. The smaller the damping constant is, the smaller the dynamic buckling critical load is. But the damping supports have little effect on the ultimate load-bearing capacity.

Keywords: vibration theory; dynamic buckling; arch; elastic supports; explosive impact

1 INTRODUCTION

Arch structures are widely used in the bridges, dams, and other long-span projects. when the arch is supported directly on the foundation or other structures, under external loads, they will deform more or less. The supports are elastic really. In dynamic environments, the load-bearing capacity of the arch structures with elastic supports

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will be closely related to their dynamic stability. Therefore, the dynamic buckling study is very significant.

Currently, there are many studies on rigid supports structures. As the elastic supported arch structures are much simpler, they have been studied widely and deeply. Kiss [1] and Yang et al.[2] investigated the static stability of arch structure Based on the potential energy principle. Liu et al.[3] and Zang et al.[4] investigated the static stability of the circular shallow arch structure based on the virtual work principle. The analysis of the arch structure dynamic stability mainly is focused on the shallow arch structures[5-7]. YANG[8] et al. studied the dynamic snap buckling of concrete filled steel tubular arches. By the theoretical and experimental methods, Liu A.R. et al.[9-13]studied the dynamic stability of arch structure under an arbitrary step radial point load, parametric resonance, and a central concentrated load. The above research on the dynamic stability of arch structure is mostly suitable for shallow arch with small rise span ratio, and the load form is relatively simple. Tinge K. et al.[14] studied the dynamic buckling of arch subjected to explosive impact. The material and geometric nonlinearity are taken into account. The method of the arch dynamic buckling judgment is obtained according to B-R buckling criterion.

The additional inertial force will occur at the arch foot while the arch structure with elastic supports vibrating. Many scholars have studied the dynamic stability of elastic supported arch structure, but most of them focus on the relatively simple shallow arch structure. Chen et al.[15]studied the effects of elastic foundation on the snap-through buckling of a shallow arch under a moving point load. Xu et al.[16]studied the dynamic stability of shallow arch with elastic supports under pulse loads. Yi et al.[17-19]systematically studied the nonlinear dynamical behaviors of vertical elastic supported shallow arch. In above documents, the influence of the longitudinal inertia of the arch structure is ignored, and the external load is relatively simple. The conclusion is not applicable to the general arch structure with large rise to span ratio.

The research on the general elastic supported arch structure with large rise span ratio mainly focuses on its dynamic natural characteristics [20-21] and dynamic response characteristics [22-23]. However, the influence of elastic supports on the dynamic buckling of general arch structures under strong dynamic load has not been reported. In this paper, the arch dynamics with elastic supports are studied by applying the numerical method. The effects of the elastic supports on the arches stress distribution, the plastic deformation law, the dynamic buckling critical load, the dynamic buckling mode and the ultimate bearing capacity et al. are studied systematically. Also, the effects of the damping supports are investigated. Some useful conclusions are obtained.

2 ELASTIC SUPPORTED ARCH MODEL

Lightweight and high-strength steel is widely used in large roof truss structures. Fig.1 shows the elastic supported arch with span L=2m, arch rise f=0.4m, rise span ration $d=f/L$=0.2,cross section width b = 0.2m, cross section thickness h=0.05m, the spring height is 0.3m, the stiffness is K, and the height of the rigid cushion block under the spring is 0.2m. The arch material is alloy steel. The mass density is ρ=2700Kg/m^3, the elastic modulus is $E = 8.0 \times 10^{10}$ N/m^2, Poisson ration is $\gamma = 0.3$,and the yield strength is $\sigma_s = 175$Mpa . The natural period of the corresponding rigid supported arch is $T = 0.056s$

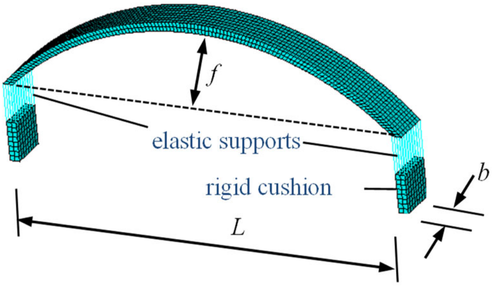


Fig. 1. Elastic supported arch structure model

When analysis is performed by PKPM, the arch structure is discretized 8-node hexahedron elements. The material is assumed to be perfect elastic-plastic. The elastic supports are linear elastic springs. The dampers assumed to be linear viscous dampers. The arch longitudinal (perpendicular to paper plane) vibration isn't taken into account. Also, the springing vibration is assumed to be vertical.

The effects of the elastic supports on the arch dynamics depend on the relative stiffness primarily. So $k = \frac{KL^3}{EI}$ is defined the relative stiffness coefficient. Where EI is the arch bending stiffness, K is the elastic supports stiffness.

3 EFFECTS OF THE ELASTIC SUPPORTS ON THE ARCH DYNAMIC RESPONSE

The load act on the arch is usually assumed to be distributed when explosion is far away. The explosion impact is simplified triangular pulse $p(t)$. $p(t)$ can be written as

$$\begin{cases} p(t) = P_0(1 - t/t_0) & 0 \le t \le t_0 \\ 0 & t > t_0 \end{cases} \tag{1}$$

Where $t_0 = 0.1T$ is the explosive duration time, $P_0 = 0.5\text{MPa}$ starting at t_0 is the pulse amplitude. According to the research, the arch deformation is elastic with $P_0 = 0.5\text{MPa}$.

The effects of the elastic supports on the arch dynamic response are analyzed when the relative stiffness coefficient $k = 1 \sim 1000$ (the elastic supports stiffness K = 2.083 × 104N / m ~ 2.083 × 107N / m). According to the study, when the elastic supports stiffness coefficient is small (k <5), the effective stress amplitude of the arch reduce obviously, but the vertical displacement of the springing is very big. These elastic supports with small stiffness can't be used in the engineering.

Fig. 2 shows the effective stress and time histories of the rigid supported arch. Fig.3 and fig.4 show the effective stress and time histories of the elastic supported arch with *k*=10 and *k*=300 respectively。

It can be seen from figure 2,when the rigid supported arch vibration is elastic under the small load ($P_0 = 0.5$MPa), the effective stress variation law of the different parts is the same on the whole. In an explosion instant, the effective stress reaches the maximum. After the explosion disappearing, as the time increases, the effective stress decreases. The arch foot is the most dangerous part where the effective stress is biggest. The effective stress in arch crown and arch shoulder is relatively smaller.

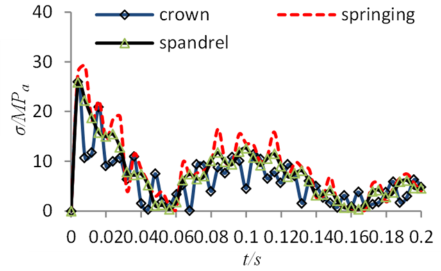


Fig. 2. effective-stress of the rigid supported arch vs. time

It can be seen from Fig.3 and Fig.4, the elastic supports change the stress distribution law. The spandrel is the most dangerous part where the effective stress is the

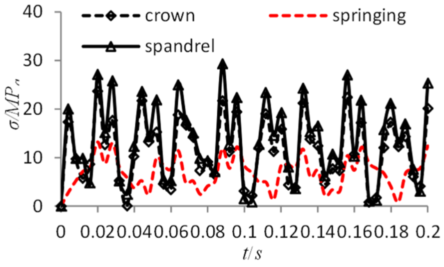


Fig. 3. effective-stress of the elastic supported arch vs. time（*k*=10）

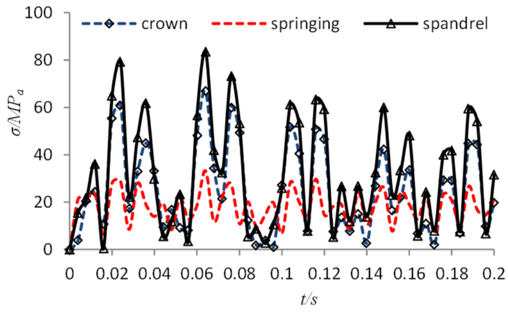


Fig. 4. effective-stress of the elastic supported arch vs. time（*k*=300）

biggest. The springing effective stress is the smallest. During the explosion phase(t=0～0.1t_0), the elastic supports can effectively reduce the peak stress. And the smaller the relative stiffness coefficient is, the more reduction the stress has. At the end of the explosion, when k = 10 and k = 300, the stress reduction is 89.8% and 23.4% at springing, 33.3% and 21.0% at arch crown, 22.5% and 15.4% at spandrel. But after the explosion disappears, the peak stress of the elastic supported arch is greater than one of the rigid supported arch. The elastic supports can't reduce vibration, but make it enhance. And the bigger the stiffness is, the more considerably the vibration enhances. The reason is maybe that more vibration models can be stimulated under the explosive impact, after the explosion disappearing, the lower model vibration interacts with the higher model vibration by internal resonance. So the arch vibration exhibits a strong coupling effect. The arch vibration is enhanced.

4 EFFECTS OF THE ELASTIC SUPPORTS ON THE ARCH DYNAMIC BUCKLING

In order to study the effects of the elastic suports on the arch dynamic buckling, the rigid suported arch dynamic buckling is studied first. In which the explosion duration time is t_0=0.1T and the dynamic buckling criteria is B-R criteria.

Under every blasting impact load amplitude P_0, the arch crown vertical displacement amplitude y_{Dmax} in free vibration is calculated. As the load amplitude P_0 increases,the y_{Dmax}～P_0 relation curve is obtained shown in Fig.5. Fig.6(a) shows the counter-intuitive response after dynamic buckling, and Figure 6(b) shows the failure vibration mode, Where the light-colored parts are plastic deformation zones. Fig.7 shows the time histories of the arch crown vertical displacement.

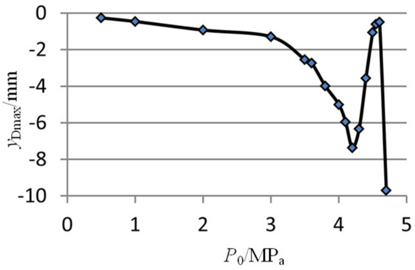
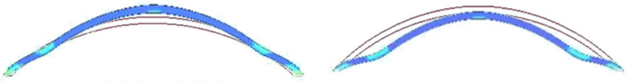


Fig. 5. The load amplitude vs. the displacement amplitude of the rigid supported arch top

It can be seen from Fig.5, there are two inflection points in the yDmax～P0 relation curve.According to studying, in the rigid supported arches, when P0 <1.5Mpa, the plastic deformation doesn't occurs. The arch vibrates elasticly around the static balance. As is shown in fig. 7 with P0=1.0Mpa.



(a) P_0=4.5Mpa (b) P_0=4.7Mpa

Fig. 6. The rigid supported arch vibration mode

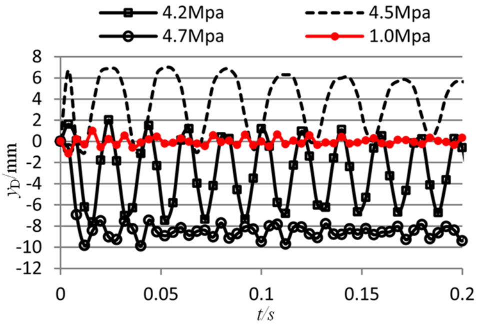


Fig. 7. The displacement of the rigid supported arch top vs. time

As the load amplitude P_0 increases, the plastic deformation begins at the springing first. The dynamic equilibrium position move down gradually. When the load amplitude P0=4.2Mpa, the crown vertical displacement amplitude reaches the maximum ,i.e. the first inflection point shown in figure 5. At this time , the plastic hinges appear at the spandrels about 1/6 span far away from the springing. If the load continues to increase, the plastic zone expands rapdly. The dynamic equilibrium position moves up because of the lateral deformation of arch both spandrels. When the load increases to 4.5Mpa, the dynamic equilibrium position reaches the highest , over the static equilibrium position, i.e. the second inflection point shown in figure 5. After the explosion impact disappears, the counter-intuitive dynamic response appears because of the elastic restoring force shown in figure 6(a) and figure 7. If the load continues to increase, the arch plastic deformation increases rapidly ,the arch stiffness decreases rapidly, the arch crown vertical displacement increases suddenly, the arch loses the carrying capacity completely, and the crushing damage appears.

When the buckling is judged by B-R criterion, there is no uniform standard about what is great response change. Based on the study above, when the load amplitude increases to the inflection point in Fig.5, the load has little increasment, Although the response hasn't large change, the dynamic balance position changes suddenly. According to the literature[5], the load corresponding to the first inflection point in Fig.5 can be looked as the dynamic buckling critical load, i.e. P_0=4.2Mpa. The load corresponding to the second inflection in point Fig.5 can be looked as the ultimate bearing capacity, i.e.P_0=4.5Mpa.

It can be obtained from study above, when the arch with elastic supports dynamic buckling occurs, the plastic hinges appear in the springing and spandrel, then the lateral deformation of both spandrels make the dynamic equilibrium of the crown moves up. The counter-intuitive dynamic response appears because of the elastic restoring force.

The following research is that the effects of the elastic supports on the arch dynamic buckling. According to the conclusion in Section 2, the elastic supports may enhance the arch vibration. It is difficult to improve the arch bearing capacity if the elastic supports are solely set up in the engineering. So the damping supports on both ends of the springing should be set up at the same time. In the following analysis, the damping constant c = 2.0E + 3N • s / m, the relative stiffness coefficient of the elastic supports *k*=50～1000. The displacement is the relativity, i.e. the difference between the crown vertical displacement and the springing displacement. Fig. 8 shows the crown vertical displacement y_{Dmax} and the load amplitude P_0 relationship with *k*=200. Fig. 9 shows the elastic supported arch vibration modes with P_0=4.1MP and P_0=5.5MP where the light color parts are the plastic deformation zones. Fig. 10 shows the vertical displacement time histories of the elastic supported arch crown.

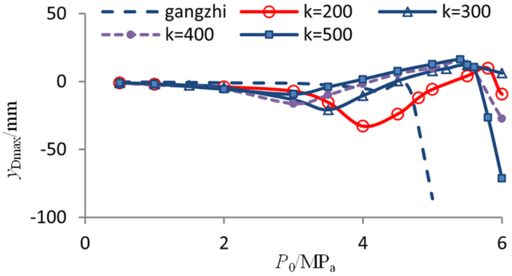


Fig. 8. The load amplitude vs. the displacement amplitude

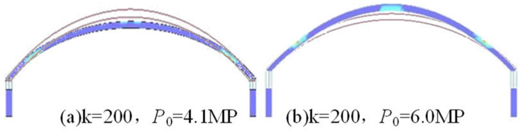

(a)k=200，P_0=4.1MP (b)k=200，P_0=6.0MP

Fig. 9. The elastic supported arch vibration mode（t=0.144s）

According to the analysis, the elastic supports can greatly reduce the springing stress. Under the explosion impact, the springing is no longer the most dangerous part.

As the explosion impact amplitude P_0 increases, the plastic hinges occur first in 1/6 arch span away from the springing (named spandrel following).

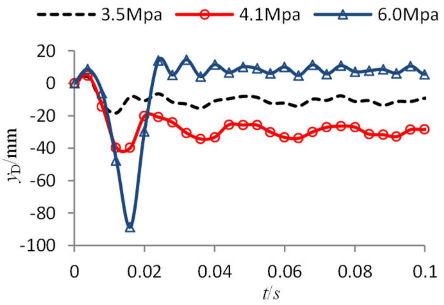


Fig. 10. The displacement of the elastic supported arch top vs. time

In fig. 8, it can be seen that there are also two inflection points in every y_{Dmax}～P_0 relation curves. When the load amplitude P_0<4.1Mpa with k=200, as the load mplitude P_0 increases, the plastic hinges appear first in both spandrels of the arch, the dynamic equilibrium postion of the crown is under the static equilibrium(fig.8 with k=200 and fig.10) . when the load amplitude P_0=4.1Mpa, the plastic deformation also appears in the crown(figure9(a)). The vertical displacement of the crown reaches the maximum (the first inflection point in figure8 with k=200). When the load amplitude P_0>4.1Mpa, the plastic deformation expands rapidly(figure9(b)). As the load continues,based on the analysis above, the two inflection points in y_{Dmax}～P_0 curve correspond respectively to the dynamic balancing development boundary and the ultimate bearer status. Which is same with the rigid supported arch. Therefore, the load amplitude corresponding to the first inflection point is considered the dynamic buckling critical load. the load mplitude corresponding to the second inflection point is considered the ultimate load.

According to the study, as the stiffness coefficient changes within a certain range, the elastic supports make the arch dynamic buckling critical load decrease. In Fig. 8, It can also be seen that the dynamic buckling critical load of the rigid supported arch P0 = 4.2Mpa, while the dynamic buckling critical load of the elastic supported arch with k=200 P0=4.1Mpa. The elastic supports make the dynamic buckling critical load reduce. And the bigger the stiffness coefficient is, the smaller the dynamic buckling critical load is. The reasonable reason may be that the elastic supports reduce the stress of the arch feet, when the plastic hings occur first in the spandrel, the arch dynamic buckling happens, while the deformation of the arch feet is still elastic(fig.9(a)). Under the same conditions, the plastic hinges of the rigid supported arch occur first in the arch feet. When the dynamic buckling occurs with the plastic hinges coming into being in the spandrel, the whole plastic deformation parts are greater than ones of the elastic supported arch. So the external load the rigid supported arch can withstand is bigger. I.e. the elastic supports make the buckling critical load decrease.The bigger the stiffness coefficient of the elastic supports, the greater the stress of the arch, the more easily the dynamic buckling happens with the plastic hinges in the spandrel coming into being. Therefore, the bigger the stiffness coefficient, the smaller the dynamic buckling critical load.

According to study, when the elastic stiffness coefficient is very large, and the plastic deformation law tends to be consistent with the rigid supported arch. The hinges occur first in the arch feet, then the spandrel and the crown. The dynamic buckling critical load tends to be same with the rigid supported arch.

At the same time, in Figure 8, it can also be seen that the elastic supports can increase the ultimate load, and the smaller the stiffness , the more it increases. For example, the ultimate load of the rigid supported arch P_0=4.5Mpa, while at the same condition, the ultimate load of the elastic supported arch with k=200 P_0=6.0Mpa, increased by about 33.3%. The results above are obtained without consideration of the limit bearing capacity of the elastic supports themselves. The ultimate status is based on the arch structure damage. Because the elastic supports and the damping supports play a role of the energy dissipation and inducing vibration, the elastic supported arch can withstand greater explosion load than the rigid supported arch.

5 EFFECTS OF THE DAMPING SUPPORTS ON THE ARCH DYNAMIC BUCKLING

The damping supports enable the arch vibration decay rapidly, and eliminate the

vibration enhances in free vibration. The following section contains the effects of the damping supports on the arch dynamic buckling and the ultimate bearing capacity, and the results are shown in figure 11. Where the relative stiffness of the elastic supports k = 200, and the unit of the damping constant c is N • s / m, and the crown vertical displacement of the elastic supported arch is the relative displacement of the crown and the springing.

As it can be seen from Figure 11, the smaller the damping constant, the smaller the dynamic buckling critical load ,the greater the crown displacement amplitude when the dynamic buckling occur. But the damping supports have little effect on the ultimate bearing capacity of the arch.

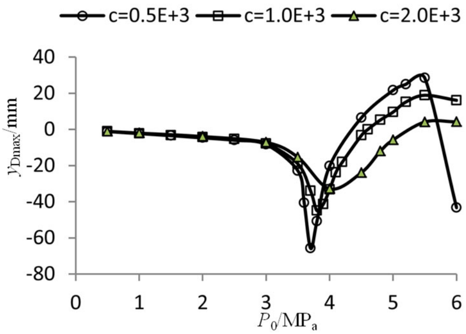


Fig. 11. The load amplitude vs. the displacement amplitude of the elastic supported arch-top(k=200)

6 CONCLUSION

In this paper, the effects of the elastic supports on the arch dynamic response and dynamic buckling are studied by applying the numerical method. The results show that:

(1) Under explosive impact, the elastic supports make the arch stress distribution change. The most dangerous position is no longer the springing but the spandrel. During the explosion phase, the general elastic supports can reduce the effective stress amplitude. But During the free vibration phase after the explosion disappearing, only the elastic supports with very small stiffness can reduce the effective stress amplitude, and the general elastic supports can enhance the vibration. The elastic supports can't reduce the arch vibration solely. The damping supports should be set up at the same time.

(2)The Elastic supports can reduce the effective stress peak of the springing. As the external load magnitude increase, the plastic deformation development law of the arch with elastic supports is different from the arch with rigid supports. The plastic hinges of the elastic supported arch appear first in the spandrels (while the plastic hinges of the rigid supported arch appear first in the springing.). When the dynamic buckling occurs, the deformation in the spinging is still elastic. The whole plastic deformation parts are less than the rigid supported arch. The elastic supports make the dynamic buckling critical load reduce. and the greater the stiffness, the more the critical load reduce. when the elastic stiffness coefficient is very large, the plastic deformation development law tends to be consistent with the rigid supported arch. The dynamic buckling critical load also tends to be same with the rigid supported arch.

(3)Only depends on the elastic supports, it is difficult to improve blast-resistant load-bearing capacity. The damping supports should be set up at the same time. The smaller the damping constant is, the smaller the dynamic buckling critical load is. But they have little effect on the ultimate bearing capacity of arch.

The conclusions above are obtained without considering the limit bearing capacity of the elastic supports themselves. But In the engineering, the limit displacement and the limit shock resistance of the elastic supports are great influence on the arch blast-resistant capacity. These problems are yet to be studied.

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