Differences in Analytical Solutions for Natural Frequencies of Offshore Wind Turbine Among Three Beam Models

Hengkai Niu[1,2], Ziquan Wang[1,2], Zhendong Shan[1,2*]

[1]Key Laboratory of Earthquake Engineering and Engineering Vibration, Institute of Engineering Mechanics, China Earthquake Administration, Harbin, China
[2]Key Laboratory of Earthquake Disaster Mitigation, Ministry of Emergency Management, Harbin, China

* Corresponding author: shanzhendong@gmail.com

Abstract. The transverse natural frequency of offshore wind turbines (OWTs) is one of the key factors to be considered in the design of OWTs, and the selection of a reasonable beam theory is crucial for the solution of the natural frequencies of OWTs. However, the additional shear stress caused by axial force is ignored in some literatures when using the Timoshenko beam theory for modeling OWTs. To this end, the transfer matrix method is used in this paper to compare the differences in the natural frequencies of OWTs based on the Euler-Bernoulli beam model, the Timoshenko beam model, and the Timoshenko beam model without considering the additional shear stresses. The results show that: the additional shear stress did not produce significant differences for the OWTs modeled with the Timoshenko beam model, but its physical significance is more reasonable. The natural frequencies of the OWTs are significantly affected by the axial force when the mass of the rotor nacelle assembly (RNA) is small. When the mass of the RNA is large, the OWT natural frequencies solved by the Euler-Bernoulli beam model are quite different from that by Timoshenko beam model.

Keywords: Offshore wind turbine; Natural frequency; Timoshenko beam; Transfer matrix method; Additional shear stress; Analytical solution

1 INTRODUCTION

The unfolding of crises such as energy security and extreme weather has amplified the significance and urgency of energy transition. As a superior, clean, and safe renewable energy source, wind energy plays a crucial role in fostering economic growth, strengthening energy security, and improving the environment. To prevent resonance hazards during turbine operation, the preferred range for the first natural frequency of the tower is situated between the first order of the rotor speed (1P) and the blade passing frequency (3P) [1]. DNVGL-ST-0126 [2] suggests a safety margin of 5% between 1P and 3P, necessitating an elevation in the accuracy of natural frequency calculations.

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The natural frequency of OWTs has been subject to increasing theoretical calculations year on year. Adhikari and Bhattacharya [3] simulated the lateral vibration of an OWT using the Euler-Bernoulli beam model. They simplified the OWT tower as a beam with a uniform cross-section and employed two springs to mimic pile-soil interaction. Arany et al. [4] utilized the Timoshenko beam model to simulate the lateral vibration of the OWT, considering the beam's rotational inertia and shear deformation, and employing three springs to represent the pile foundation. Absawy et al. [5] analyzed the lateral natural frequencies of a variable cross-section tower using the recursive differential method (RDM), reducing the error from 19% under the constant cross-section assumption to 4.2% under the variable cross-section assumption. Wang et al. [6] transformed a variable-section beam into a multi-segment uniform beam, discretized the tower unit into several segments, incorporated three coupled springs to model pile-soil interaction, and considered fluid-structure interaction with an additional mass, effectively solving the problem of calculating the natural frequency of a variable-section tower using Eulerian beam model. Pezeshki et al. [7] employed the nonlinear Stokes' wave theory and wave-structure and soil-foundation interactions to develop an analytical solution for the dynamic response of an OWT under wave loading, obtaining corresponding natural frequencies and modes.

From the above research literature, it can be found that researchers usually use the Euler-Bernoulli beam model or the Timoshenko beam model to simulate OWTs. However, when using the Timoshenko beam model to simulate OWTs, the additional shear stress caused by axial force is not considered. Therefore, to investigate the impact of the additional shear stress that was not considered in the aforementioned literature, this article incorporates the additional shear stress into the Timoshenko beam model, thereby improving the Timoshenko beam model. For variable cross-section tower, this article uses the transfer matrix method to calculate the natural frequency of the OWT system. Transfer matrix method can effectively solve variable cross-section problems, and can improve calculation accuracy. Based on the Euler-Bernoulli beam model, the Timoshenko beam model, and the Timoshenko beam model without considering the additional shear stress, this paper analyzes the differences in natural frequencies of OWTs when using these three beam models. The influence of additional shear stress on the natural frequencies of OWTs has been studied, and the effect of changes in axial force on the differences among the three models has been analyzed.

2 CALCULATION MODEL

The simplified model of OWT structure is shown in Figure 1(a), which consists of pile-soil spring, tower and rotor-nacelle assembly (RNA) three parts. RNA is simplified as a concentrated mass. The foundation is represented by three springs. The tower is hollow within, with its diameter progressively widening from top to bottom, the walls maintaining their thickness remains unaltered.

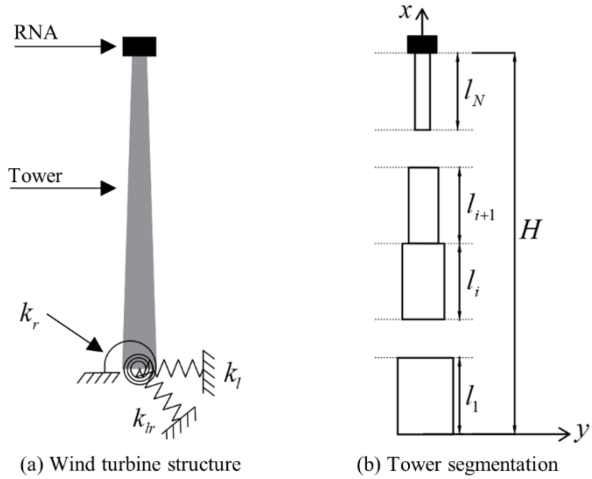

Fig. 1. Offshore Wind Turbine Calculation Model

2.1 Governing Equations

In practice, the cross section of the OWT is continuously increasing from the top to bottom. Simplifying the variable cross-section tower to a constant cross-section tower will reduce the calculation accuracy of the natural frequency. Therefore, this article uses the transfer matrix method to calculate the natural frequency of OWT. Using the segmented approach, the wind turbine tower is divided into interconnected segments [8]. As shown in Figure 1(b), the tower height is divided into N sections. The derivation process for both the Euler-Bernoulli beam model and the Timoshenko beam model is similar. In this paper, taking the Timoshenko beam model as an example, we establish the transverse vibration equation for the ith segment of the tower [9].

$$\kappa A_i G\left(w_{i,xx} - \psi_{i,x}\right) - P\alpha_{i,x} - \rho A_i w_{i,tt} = 0 \tag{1}$$

$$EI_i \psi_{i,xx} + \kappa A_i G\left(w_{i,x} - \psi_i\right) - P\alpha_i - \rho I_i \psi_{i,tt} + Pw_{i,x} = 0 \tag{2}$$

The bending moment and shear force of the ith tower are [10]:

$$M_i = EI_i \psi_{i,x} \tag{3}$$

$$V_i = \kappa A_i G\left(w_{i,x} - \psi_i\right) - P\alpha_i \tag{4}$$

where $P\alpha_i$ represents the additional shear stress caused by axial force. Abohadima et al. [10] pointed out that $\alpha_i = \psi_i$ is reasonable, so $\alpha_i = \psi_i$ will be followed in the following derivation; w_i and ψ_i are the lateral displacement and section rotation of the ith segment of the tower, respectively; A_i is the section area; I_i is the section moment of inertia; E and G are the elastic modulus and shear modulus of the material, respectively; κ is the section shear coefficient; ρ is the material density; P is the axial force.

2.2 Boundary Conditions

Bottom bending moment of wind turbine tower ($x=0$):

$$M_i - k_r \psi_i - k_{lr} w_i = 0 \tag{5}$$

Bottom shear of wind turbine tower ($x=0$):

$$V_i + k_l w_i + k_{lr} \psi_i = 0 \tag{6}$$

Top bending moment of wind turbine tower ($x=H$):

$$M_i = 0 \tag{7}$$

Top shear of wind turbine tower ($x=H$):

$$V_i + m_{RNA} w_{i,tt} = 0 \tag{8}$$

3 DERIVATION OF THE NATURAL FREQUENCIES

By substituting $w_i = v_i(x)e^{i\omega t}$ and $\psi_i = \theta_i(x)e^{i\omega t}$ into equations (1) and (2), and separating variables, we can obtain:

$$v_i^{(4)}(x) + \zeta_i v_i^{(2)}(x) + \eta_i v_i(x) = 0 \tag{9}$$

$$\theta_i^{(4)}(x) + \zeta_i \theta_i^{(2)}(x) + \eta_i \theta_i(x) = 0 \tag{10}$$

where:

$$\zeta_i = \frac{\rho\omega^2}{\kappa G} + \frac{\rho I_i \omega^2 - \kappa A_i G - P}{EI_i} + \frac{(\kappa A_i G + p)^2}{EI_i \kappa A_i G} \tag{11}$$

$$\eta_i = \frac{\rho\omega^2}{\kappa G}\left(\frac{\rho I_i \omega^2 - \kappa A_i G - P}{EI_i}\right) \tag{12}$$

According to equations (9) and (10), the vibration mode function of the ith tower section can be obtained as follows:

$$\begin{aligned} v_i(x) = {} & C_{1i}\sin(\lambda_i x_i) + C_{2i}\cos(\lambda_i x_i) \\ & + C_{3i}\sinh(\mu_i x_i) + C_{4i}\cosh(\mu_i x_i) \end{aligned} \tag{13}$$

$$\begin{aligned} \theta_i(x) = {} & D_{1i}\sin(\lambda_i x_i) + D_{2i}\cos(\lambda_i x_i) \\ & + D_{3i}\sinh(\mu_i x_i) + D_{4i}\cosh(\mu_i x_i) \end{aligned} \tag{14}$$

where: $\lambda_i = \sqrt{\left(\zeta_i + \sqrt{\zeta_i^2 - 4\eta_i}\right)/2}$, $\mu_i = \sqrt{\left(-\zeta_i + \sqrt{\zeta_i^2 - 4\eta_i}\right)/2}$. C_{nji} and D_{nji} ($j = 1,2,3,4$) are the undetermined coefficients of the lateral displacement and the section angle of the tower, respectively. The relationship between C_{nji} and D_{nji} can be obtained from equation (1).

The continuous relationship of the displacement, angle, bending moment and shear force between the section i beam and the section $i+1$ beam at the point of connection is shown as follows:

$$\begin{gathered} v_{i+1}(x_i) = v_i(x_i),\ \theta_{i+1}(x_i) = \theta_i(x_i),\ E_{i+1}I_{i+1}\theta_{i+1,x}(x_i) = E_iI_i\theta_{i,x}(x_i) \\ k_{i+1}A_{i+1}G_{i+1}\left[v_{i+1,x}(x_i) - \theta_{i+1}(x_i)\right] - P\theta_{i+1}(x_i) \\ = k_iA_iG_i\left[v_{i,x}(x_i) - \theta_i(x_i)\right] - P\theta_i(x_i) \end{gathered} \tag{15}$$

Substituting Equations (13) and (14) into Equation (15) yields

$$\mathbf{T_{i+1}C_{i+1}} = \mathbf{T_iC_i} \tag{16}$$

where $\mathbf{C_{i+1}}$ and $\mathbf{C_i}$ are the coefficients to be determined for the $i+1$ section tower and the i section tower, respectively, and $\mathbf{T_{i+1}}$, $\mathbf{T_i}$ and $\mathbf{Z_i}$ are the corresponding coefficient matrices. The relationship between $\mathbf{C_N}$ and $\mathbf{C_1}$ is shown as follows:

$$\mathbf{C_N} = \mathbf{ZC_1} \tag{17}$$

where $\mathbf{Z} = \mathbf{Z_{N-1}Z_{N-2}}\cdots\mathbf{Z_2Z_1}$, $\mathbf{Z_i} = \mathbf{T_{i+1}}^{\mathbf{-1}}\mathbf{T_i}$.

Using the same procedure, boundary conditions (5)-(8) can be written as follows:

$$\mathbf{T_BC_1} = 0,\ \mathbf{T_TC_N} = 0 \tag{18}$$

From Equations (17) and (18), we obtain

$$\mathbf{RC_1} = 0 \tag{19}$$

where $\mathbf{R} = \left[\mathbf{T_B}, \mathbf{T_TZ}\right]^T$. According to Equation (19), the coefficients in the vector $\mathbf{C_1}$ cannot be zero at the same time, so for it to have a nonzero solution, the determinant of the coefficient matrix of $\mathbf{R}$ must be zero:

$$|\mathbf{R}| = 0 \tag{20}$$

Equation (20) is a transcendental equation that contains the lateral natural frequency ω of the OWT system.

4 PARAMETRIC ANALYSIS

4.1 Method Validation

Table 1 shows the calculated results of the three models, the calculated results of the RDM method and the OWT data are all from reference [5]. As shown in Table 1, the results of transfer matrix method are closer to the measured values. The difference between the Euler-Bernoulli beam model and Timoshenko beam model is about 1%, the results of Euler-Bernoulli beam model are the closest to the measured values. The results show that considering the additional shear stress caused by the axial force and not considering it yield almost no difference, that is case $\alpha_i = 0$ and $\alpha_i = \psi_i$ have no influence on the OWT which is calculated by Timoshenko beam model.

Table 1. Comparison of calculated results and measured values by transfer matrix method

Type	LelyA2: NM412-bladed	Irene Vorrink 600 kW	Walney 1 S 3.6 MW
Measured	0.634	0.546	0.35
Formulation (Error %)			
Euler-Bernoulli	0.73191 (15.44%)	0.52640 (3.59%)	0.34522 (1.37%)
Timoshenko $\alpha_i = 0$	0.72849 (14.9%)	0.52352 (4.12%)	0.34039 (2.75%)
Timoshenko $\alpha_i = \psi_i$	0.72846 (14.9%)	0.52349 (4.12%)	0.34036 (2.75%)
RDM	0.74 (16.72%)	0.52 (4.76%)	0.335 (4.29%)

4.2 Differences Among Three Beam Models when RNA Quality Changes

Changes in RNA quality dominate changes in axial force, and as RNA quality changes, the natural frequency of the Walney 1 S 3.6 MW changes as shown in Figure 2. Before 10×10^5kg, the natural frequency of the wind turbine changes significantly. When the RNA mass of the wind turbine increases to 10×10^5kg, the change of natural frequency tends to be gentle, but the difference between the modeling based on Euler-Bernoulli beam model and the modeling based on Timoshenko beam model becomes more and more obvious. Therefore, when the RNA quality is low, attention should be paid to the changes in the wind turbine's natural frequency caused by the changes in RNA quality. When the RNA quality is high, the differences between the analytical solutions calculated by different beam models should not be ignored.

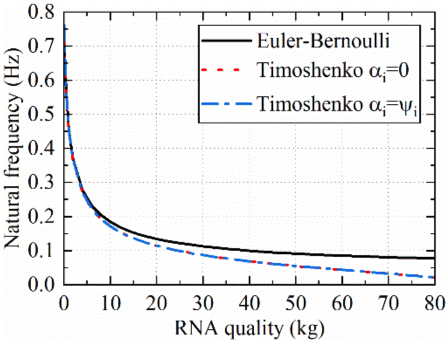

Fig. 2. Effect of RNA quality on natural frequency of Walney 1 S 3.6 MW

Figure 3 shows the variation of the displacement of the top of the tower with the change of RNA mass when subjected to external load (F =1000N, ω =2Hz). The cases with larger top displacement are still below 10×10^5kg, and the differences between different beam models are also significant. After the RNA mass of the wind turbine increases to 10×10^5kg, the top displacement at the tower becomes relatively stable, and there is almost no difference between the beam models.

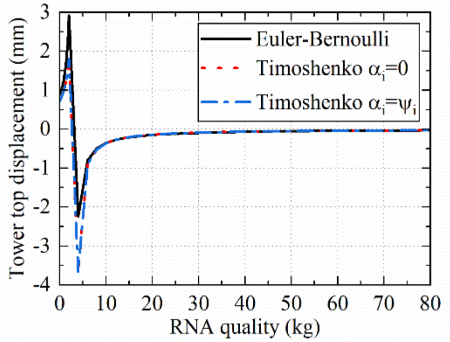

Fig. 3. Effect of RNA quality on top displacement of Walney 1 S 3.6 MW

Based on the above analysis, it can be concluded that the influence of axial force on the wind turbine system is mainly manifested in the stage when the RNA quality is small. The selection of different beam models will have a significant impact on the modal response of the wind turbine. The additional shear stress caused by axial force does not affect the wind turbine modeled by the Timoshenko beam model.

5 CONCLUSIONS

This article establishes an analytical calculation method for the modal response of OWT based on the transfer matrix method, considering the effect of additional shear stress caused by axial force in the Timoshenko beam model. The differences between modeling with the Euler-Bernoulli beam model and the Timoshenko beam model are compared. The changes in the natural frequency and displacement response of the OWT caused by changes in the RNA mass are analyzed. The main conclusions are as follows:

1.When using the Timoshenko beam model to simulate OWTs, the additional shear stress caused by axial force does not have a significant impact on the natural frequency of OWTs, but the additional shear stress is more physically meaningful.

2.The differences between the Euler-Bernoulli beam model and the Timoshenko beam model were small when calculating the natural frequencies of OWTs, around 1%. The Euler-Bernoulli beam model gave analytical results closer to the measured values.

3.When the RNA quality is low, the natural frequency of OWTs decreases significantly with the increase of RNA quality. When the RNA quality is high, the natural frequency and top displacement of OWTs tends to be stable without significant changes. There will be significant differences in the natural frequency calculated by the Euler-Bernoulli beam model and the Timoshenko beam model.

Finally, a simple excitation force is added in this article to observe the displacement response of the top of the tower under the action of external loads. However, in reality, wind load, wave load, and earthquake load are all relatively complex external excitations. In future studies, the research should consider how to incorporate the external excitations from the real environment into the modal response analysis of the OWT, and analyze the effect of external excitations on the vibration of the OWT.

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