NMTCM Polynomial Analysis in the Study of Anticancer Drugs

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Abstract. Topological coindices serve as numerical descriptors that capture the structural intricacies of a molecular graph. These indices offer valuable insights into the spatial arrangement of atoms and bonds within a molecule, shedding light on its connectivity and overall structural relationships. In the context of this research, the focus is on calculating the M-polynomial and NM-polynomial for anticancer drugs. These polynomials are employed as tools to extract specific degree-based topological indices. The study further entails the computation and comparative analysis of NM-polynomials and molecular descriptors for selected anticancer drugs. The investigation delves into the practical and essential exploration of molecular graph structures, particularly those relevant to effective cancer treatment. Additionally, the research establishes a mathematical relationship between the NM-polynomial and $NMTCM$ polynomials.
Keywords: topological indices, topological coindices, M-polynomial, molecular graph, NM-polynomial.

INTRODUCTION

A topological index is a numeric measure linked to a graph, reflecting its topology or connectivity. In contrast, the topological coindex is a complementary graph invariant frequently employed in the analysis of chemical graphs and molecular structures [1, 2, 3, 4].

The genetic conditions category, encompassing cancer, a potentially fatal ailment, is distinguished by the uncontrolled multiplication of abnormal blood cells, disrupting crucial bodily functions and heightening vulnerability to infections. Carcinogens, such as specific components found in tobacco smoke, play a role in the onset of cancer. This condition has the potential to spread to different body parts, exhibiting symptoms like weight loss, irregular bleeding, prolonged coughing and the presence of lumps [5].

Key factors contributing to this cancerous state involve the use of chewing tobacco, obesity, unhealthy dietary habits, lack of physical activity and excessive alcohol consumption. Various treatment options, including surgery, radiation, chemotherapy, hormone therapy, targeted therapy, among others are available to address this severe ailment. The cancer illnesses category also encompasses alkylates and metabolites, which are targeted by anticancer drugs.

This article represents the forefront of progress in comprehending and researching anticancer drugs. It places special emphasis on introducing the *NMTCM* polynomial and exploring its applications within the realm of topological indices and coindices. The content encapsulates the most recent developments in this field, highlighting the innovative approaches taken to enhance our understanding of anticancer drug properties and their molecular structures.

This article maintains its focus on investigating anticancer drugs and introducing the concept of a *NMTCM* polynomial. This polynomial evaluates contributions from pairs of nonadjacent vertices, measuring the potential influence of distant vertex pairs on the molecule's attributes [6]. The *NMTCM* polynomial's strength lies in its comprehensiveness, providing valuable information on degree-based graph invariants. Additionally, it yields numerous degree-based topological co-indices, linking chemical features of the material under research, derived in closed forms using elementary calculus.

METHODOLOGY

Definition 1. *[7] For a connected graph χ of order n, we have*

$$\overline{m}_{ij} = |\overline{E}_{ij}| = \begin{cases} \frac{n_i(n_i-1)}{2} - m_{ii}, & for \quad i = j \\ n_i n_j - m_{ij}, & for \quad i < j. \end{cases}$$

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where $n_i = |V_i|$ *for* $V_i = \{v \in V(G)|\ d(v) = i\}$, $m_{ij} = |E_{ij}|$, $E_{ij} = \{pq \in E(G)|\ d(p) = i,\quad d(q) = j\}$ *and* $\overline{m_{ij}} = |\overline{E_{ij}}|$ *for* $\overline{E_{ij}} = \{pq \in E(\overline{G})|\ d(p) = i$ *and* $d(q) = j\}$.

Using the notation mentioned above, we expand the notion of an *NM*-polynomial to apply to non-adjacent pairs of vertices and introduce the concept of *NMTCM* polynomial (NM polynomials with neighbourhood coindex) in the following manner:

$$NMTCM(\chi;p,q) = \overline{NM}(\chi;p,q) = \sum_{i\leq j} \overline{nm}_{ij}(\chi)p^i q^j$$

where, $\overline{m}_{ij}(\chi), i, j \geq 1$, be the number of edges $pq \notin E(\chi)$ such that $\{(d(p),d(q)) = \{i,j\}\}$.

TABLE 1. Definition of topological indices and corresponding topological coindices.

TI	Degree based index	Degree based coindex
$M_1(\chi)$ [8]	$M_1(\chi) = \sum_{pq\in E(\chi)} [\Psi_p + \Psi_q]$	$\overline{NM_1}(\chi) = \sum_{pq\notin E(\chi)} [\Psi_p + \Psi_q]$.
$M_2(\chi)$ [8]	$M_2(\chi) = \sum_{pq\in E(\chi)} [\Psi_p.\Psi_q]$	$\overline{NM_2}(\chi) = \sum_{pq\notin E(\chi)} [\Psi_p.\Psi_q]$.
$mM_2(\chi)$ [9]	$mM_2(\chi) = \sum_{pq\in E(\chi)} \left[\frac{1}{\Psi_p.\Psi_q}\right]$	$\overline{NmM_2}(\chi) = \sum_{pq\notin E(\chi)} \left[\frac{1}{\Psi_p.\Psi_q}\right]$.
$I(\chi)$ [10]	$I(\chi) = \sum_{pq\in E(\chi)} \left[\frac{\Psi_p.\Psi_q}{\Psi_p+\Psi_q}\right]$	$\overline{NI}(\chi) = \sum_{pq\notin E(\chi)} \left[\frac{\Psi_p.\Psi_q}{\Psi_p+\Psi_q}\right]$
$ReZG_3(\chi)$ [11]	$ReZG_3(\chi) = \sum_{pq\in E(\chi)} \Psi_p\Psi_q[\Psi_p + \Psi_q]$	$\overline{NReZG_3}(\chi) = \sum_{pq\notin E(\chi)} \Psi_p\Psi_q[\Psi_p + \Psi_q]$
$SDD(\chi)$ [12]	$SDD(\chi) = \sum_{pq\in E(\chi)} \left[\frac{\Psi_p^2+\Psi_q^2}{\Psi_p.\Psi_q}\right]$	$\overline{NSDD}(\chi) = \sum_{pq\notin E(\chi)} \left[\frac{\Psi_p^2+\Psi_q^2}{\Psi_p.\Psi_q}\right]$
$A(\chi)$ [13]	$A(\chi) = \sum_{pq\in E(\chi)} \left[\frac{\Psi_p.\Psi_q}{\Psi_p+\Psi_q-2}\right]^3$	$\overline{NA}(\chi) = \sum_{pq\notin E(\chi)} \left[\frac{\Psi_p.\Psi_q}{\Psi_p+\Psi_q-2}\right]^3$
$F(\chi)$ [14]	$F(\chi) = \sum_{pq\in E(\chi)} [\Psi_p^2 + \Psi_q^2]$	$\overline{NF}(\chi) = \sum_{pq\notin E(\chi)} [\Psi_p^2 + \Psi_q^2]$
$H(\chi)$ [15]	$H(\chi) = \sum_{pq\in E(\chi)} \left[\frac{2}{\Psi_p+\Psi_q}\right]$	$\overline{NH}(\chi) = \sum_{pq\notin E(\chi)} \left[\frac{2}{\Psi_p+\Psi_q}\right]$

DERIVING TOPOLOGICAL COINDEX *NM*−POLYNOMIALS

The study establishes a noteworthy correlation between the physicochemical properties of potential anticancer drugs in question and the examined topological coindices. This indicates that topological coindices have the potential to serve as valuable tools for future QSPR (Quantitative Structure-Property Relationship) analysis in exploring anti-cancer medications [16, 17]. Figure 1 presents 3D structures of some of the considered anticancer drugs, while their corresponding molecular structures can be found in figure 2.

TABLE 2. Derivation of degree based topological coindices.

Topological coindex	**Derivation from** $f(p,q) = NMTCM(\chi : p,q)$
First Zagreb coindex: $\overline{M_1}(\chi)$	$(D_p + D_q)(f(p,q))_{p=q=1}$
Second Zagreb coindex: $\overline{M_2}(\chi)$	$(D_pD_q)(f(p,q))_{p=q=1}$
Second modified Zagreb coindex: $\overline{mM_2}(\chi)$	$(S_pS_q)(f(p,q))_{p=q=1}$
Inverse sum indeg coindex: $\overline{I}(\chi)$	$(S_pjD_pD_q)(f(p,q))_{p=q=1}$
Redefined third Zagreb coindex: $\overline{ReZG_3}(\chi)$	$D_pD_q(D_p + D_q)(f(p,q))_{p=q=1}$
Symmetric division coindex: $\overline{SDD}(\chi)$	$(D_pS_q + S_pD_q)(f(p,q))_{p=q=1}$
Augmented zagreb coindex: $\overline{A}(\chi)$	$(S_p^3Q_{-2}JD_p^3D_q^3)(f(p,q))_{p=q=1}$
Forgotten topological coindex: $\overline{F}(\chi)$	$(D_p^2 + D_q^2)(f(p,q))_{p=q=1}$
Harmonic coindex: $\overline{H}(\chi)$	$(2S_pJ)(f(p,q))_{p=q=1}$

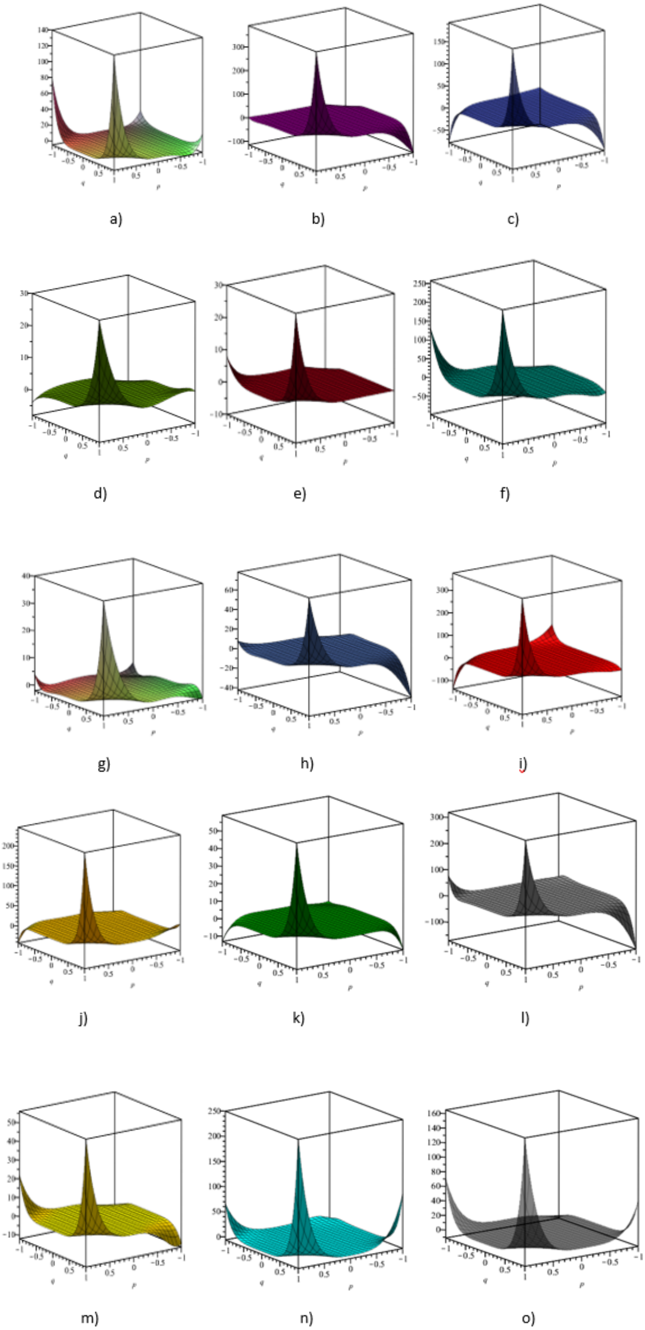
The presented table 2 is related to some of the most important degree-based topological coindices with *NM*-polynomial [18, 19] and the following fixed notations.

$$D_p = p\frac{\partial(f(p,q))}{\partial p}, D_q = q\frac{\partial(f(p,q))}{\partial q}, S_p = \int_0^p \frac{f(p,t)}{t}dt, S_q = \int_0^q \frac{f(p,t)}{t}dt$$

$$J(f(p,q)) = f(p,p), Q_\phi(f(p,q)) = p^\phi f(p,q)$$

VISUALIZING ANTICANCER DRUGS IN 3D GRAPHS

In this section, a combination of combinatorial computation, edge partition technique, vertex partition technique, and nonedge counting method is employed to determine the $NMTCM$ polynomial of several anticancer drugs, including Amathaspiramide-E $(A_m - E)$, Aminopterin $(A_m - P)$, Aspidostomide $(A_s - E)$, Carmustine $(C_s - N)$, Caulibugulone-E $(C_b - E)$, Convolutamide-A $(C_n - A)$, Convolutamine-F $(C_t - F)$, Convolutamydine-A $(C_m - A)$, Daunorubicin $(D_r - N)$, Deguelin $(D_g - N)$, Melatonin $(M_e - N)$, Minocycline $(M_i - E)$, Perfragilin-A $(P_f - A)$, Podophyllotoxin $(P_p - N)$, Pterocellin-B $(P_t - B)$, Raloxifene $(R_f - N)$ and Tambjamine-K $(T_j - K)$. These graphs can help illustrate various aspects of the drugs efficacy, toxicity, mechanisms of action, or other relevant factors. Additionally, various well-known topological coindices are derived for the provided anticancer drugs. This section also presents $NMTCM$ polynomial graphs in three dimensions for a diverse set of anticancer drugs. A $3D$ surface plot is an effective way to visualize the dose-response relationship of an anticancer drug. In this type of plot, the drug concentration is represented on the x-axis, the time of treatment on the y-axis and the drug's response (e.g., cell viability or tumor size) on the z-axis.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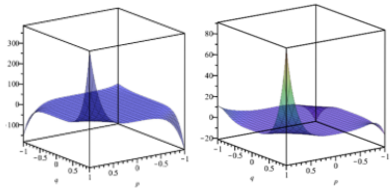

FIGURE 1. Molecular graphs of $TCOM$ of a)$(A_m - E)$, b)$(A_m - P)$, c)$(A_s - E)$, d)$(C_s - N)$, e)$(C_b - E)$, f)$(C_n - A)$, g)$(C_t - F)$, h)$(C_m - A)$, i)$(D_r - N)$, J)$(D_g - N)$, k)$(M_e - N)$, l)$(M_i - E)$, m)$(P_f - A)$, n)$(P_p - N)$, o)$(P_t - B)$, p)$(R_f - N)$, q)$(T_j - K)$

DISCUSSION AND MAIN RESULTS

This segment employs combinatorial calculations, the edge partition method, the vertex partition method and the non-edge counting technique to obtain $NMTCM$ polynomials of molecular graphs of anticancer drugs. Additionally, we provide insights into several prominent topological co-indices for the given molecular graphs.

Theorem 2. *The NMTCM polynomial for $A_m - E$ given by*

$$TCOM(A_m - E; p, q) = 2p^2q^4 + 28p^3q^6 + 13p^3q^7 + 9p^3q^8 + 2p^4q^4 + 16p^4q^6 + 8p^4q^7 + 13p^6q^6 + 16p^6q^7 + 10p^6q^8 + 10p^6q^{10} + 2p^7q^7 + 5p^7q^8 + 4p^7q^{10} + 2p^8q^{10}.$$

Proof. Let χ_1 be the molecular graph of amathaspiramide E. From the figure, it is straightforward to deduce that $|V(\chi_1)| = 22$ and $|E(\chi_1)| = 24$. χ_1 vertex set can be classified into seven classes based on their degrees.
$n_2^* = 1$, $n_3^8 = 5$, $n_4^* = 3$, $n_6^* = 6$, $n_7^* = 3$, $n_8^* = 2$, $n_{10}^* = 2$
Using definition, we have, $\overline{nm}_{12} = n_1n_2 - m_{12} = 41$ similarly,

$\overline{nm}_{ij}$	$\overline{nm}_{24}$	$\overline{nm}_{36}$	$\overline{nm}_{37}$	$\overline{nm}_{38}$	$\overline{nm}_{44}$	$\overline{nm}_{46}$	$\overline{nm}_{47}$	$\overline{nm}_{66}$	$\overline{nm}_{67}$	$\overline{nm}_{68}$
frequency	2	28	13	9	2	16	8	13	16	10

$\overline{nm}_{ij}$	$\overline{nm}_{6(10)}$	$\overline{nm}_{77}$	$\overline{nm}_{78}$	$\overline{nm}_{7(10)}$	$\overline{nm}_{8(10)}$	$\overline{nm}_{(10)(10)}$
frequency	10	2	5	4	2	0

By definition of *NMTCM* polynomial,

$$\begin{aligned}
Let, NM(\chi_1,p,q) &= \sum_{i\leq j} \overline{nm}_{ij}(\chi)p^iq^j \\
&= \sum_{2\leq 4} \overline{nm}_{24}(\chi_1))p^2q^4 + \sum_{3\leq 6} \overline{nm}_{36}(\chi_1)p^3q^6 + \sum_{3\leq 7} \overline{nm}_{37}(\chi_1))p^3q^7 \\
&+ \sum_{3\leq 8} \overline{nm}_{38}(\chi_1)p^3q^8 + \sum_{4\leq 4} \overline{nm}_{44}(\chi_1)p^4q^4 + \sum_{4\leq 6} \overline{nm}_{46}(\chi_1))p^4q^6 \\
&+ \sum_{4\leq 7} \overline{nm}_{47}(\chi_1)p^4q^7 + \sum_{6\leq 6} \overline{nm}_{66}(\chi_1)p^6q^6 + \sum_{6\leq 7} \overline{nm}_{67}(\chi_1)p^6q^7 \\
&+ \sum_{6\leq 8} \overline{nm}_{68}(\chi_1)p^6q^8 + \sum_{6\leq 10} \overline{nm}_{6(10)}(\chi_1)p^6q^{10} + \sum_{7\leq 7} \overline{nm}_{77}(\chi_1)p^7q^7 \\
&+ \sum_{7\leq 8} \overline{nm}_{78}(\chi_1)p^7q^8 + \sum_{7\leq 10} \overline{nm}_{7(10)}(\chi_1)p^7q^{10} + \sum_{8\leq 10} \overline{nm}_{8(10)}(\chi_1)p^8q^{10} \\
&+ \sum_{10\leq 10} \overline{nm}_{10(10)}(\chi_1)p^{10}q^{10}. \\
&= \mid E_{(2,4)} \mid p^2q^4 + \mid E_{(3,6)} \mid p^3q^6 + \mid E_{(3,7)} \mid p^3q^7 + \mid E_{(3,8)} \mid p^3q^8 \\
&+ \mid E_{(4,4)} \mid p^4q^4 + \mid E_{(4,6)} \mid p^4q^6 + \mid E_{(4,7)} \mid p^4q^7 + \mid E_{(6,6)} \mid p^6q^6 \\
&+ \mid E_{(6,7)} \mid p^6q^7 + \mid E_{(6,8)} \mid p^6q^8 + \mid E_{(6,10)} \mid p^6q^{10} + \mid E_{(7,7)} \mid p^7q^7 \\
&+ \mid E_{(7,8)} \mid p^7q^8 + \mid E_{(7,10)} \mid p^7q^{10} + \mid E_{(8,10)} \mid p^8q^{10} + \mid E_{(10,10)} \mid p^{10}q^{10} \\
&= 2p^2q^4 + 28p^3q^6 + 13p^3q^7 + 9p^3q^8 + 2p^4q^4 + 16p^4q^6 + 8p^4q^7 + 13p^6q^6 \\
&+ 16p^6q^7 + 10p^6q^8 + 10p^6q^{10} + 2p^7q^7 + 5p^7q^8 + 4p^7q^{10} + 2p^8q^{10}.
\end{aligned}$$

Hence, we get the required *NMTCM* polynomial equation of $A_m - E$. □

With the given *NMTCM* polynomial equation at hand, we can now proceed to analyze its implications.

Corollary 3. *Let χ_1 be a molecular graphs of $A_m - E$ then*

1. $\overline{NM}_1(\chi_1) = 1628$
2. $\overline{NM}_2(\chi_1) = 4687$
3. $\overline{NmM}_2(\chi_1) = 5.19$
4. $\overline{NI}(\chi_1) = 395.50$
5. $\overline{NRZ}_3(\chi_1) = 60190$
6. $\overline{NSDD}(\chi_1) = 324.036$
7. $\overline{NAZ}(\chi_1) = 7232.632$
8. $\overline{NF}(\chi_1) = 10468$
9. $\overline{NH}(\chi_1) = 25.225.$

Proof. Let,

$$f(p,q) = NMTCM(\chi;p,q) = \overline{NM}(\chi;p,q) = \sum_{i\leq j} \overline{nm}_{ij}(\chi)p^iq^j$$

Then, you can utilize the above formulas to perform the following calculations

$$NM(\chi_1,p,q)=2p^2q^4+28p^3q^6+13p^3q^7+9p^3q^8+2p^4q^4+16p^4q^6+8p^4q^7+13p^6q^6+16p^6q^7$$
$$+10p^6q^8+10p^6q^{10}+2p^7q^7+5p^7q^8+4p^7q^{10}+2p^8q^{10}.$$
$$D_p(\chi_1)=4p^2q^4+84p^3q^6+39p^3q^7+27p^3q^8+8p^4q^4+64p^4q^6+32p^4q^7+78p^6q^6+96p^6q^7$$
$$+60p^6q^8+60p^6q^{10}+14p^7q^7+35p^7q^8+28p^7q^{10}+16p^8q^{10}.$$
$$D_q(\chi_1)=8p^2q^4+168p^3q^6+91p^3q^7+72p^3q^8+8p^4q^4+96p^4q^6+56p^4q^7+78p^6q^6+112p^6q^7$$
$$+80p^6q^8+100p^6q^{10}+14p^7q^7+40p^7q^8+40p^7q^{10}+20p^8q^{10}.$$
$$(D_p+D_q)(\chi_1)=12p^2q^4+252p^3q^6+130p^3q^7+99p^3q^8+16p^4q^4+160p^4q^6+88p^4q^7+156p^6q^6$$
$$+208p^6q^7+140p^6q^8+160p^6q^{10}+28p^7q^7+75p^7q^8+68p^7q^{10}+36p^8q^{10}.$$
$$D_pD_q(\chi_1)=16p^2q^4+504p^3q^6+273p^3q^7+216p^3q^8+32p^4q^4+384p^4q^6+224p^4q^7+468p^6q^6$$
$$+672p^6q^7+480p^6q^8+600p^6q^{10}+98p^7q^7+280p^7q^8+280p^7q^{10}+160p^8q^{10}.$$
$$(D_p^2+D_q^2)(\chi_1)=40p^2q^4+1260p^3q^6+754p^3q^7+657p^3q^8+64p^4q^4+832p^4q^6+520p^4q^7+936p^6q^6$$
$$+1360p^6q^7+1000p^6q^8+1360p^6q^{10}+196p^7q^7+565p^7q^8+596p^7q^{10}+328p^8q^{10}.$$
$$D_pD_q(D_p+D_q)(\chi_1)=96p^2q^4+4536p^3q^6+2730p^3q^7+2376p^3q^8+256p^4q^4+3840p^4q^6+2464p^4q^7$$
$$+5616p^6q^6+8736p^6q^7+6720p^6q^8+9600p^6q^{10}+1372p^7q^7+4200p^7q^8+4760p^7q^{10}+2880p^8q^{10}.$$
$$S_q(\chi_1)=0.5p^2q^4+24.66p^3q^6+1.85p^3q^7+1.12p^3q^8+0.5p^4q^4+2.66p^4q^6+1.14p^4q^7+2.16p^6q^6$$
$$+2.28p^6q^7+1.25p^6q^8+p^6q^{10}+.28p^7q^7+.625p^7q^8+0.4p^7q^{10}+0.2p^8q^{10}.$$
$$S_pS_q(\chi_1)=0.25p^2q^4+8.22p^3q^6+0.61p^3q^7+0.37p^3q^8+0.12p^4q^4+0.66p^4q^6+0.28p^4q^7+0.36p^6q^6$$
$$+0.38p^6q^7+0.20p^6q^8+0.16p^6q^{10}+0.04p^7q^7+0.089p^7q^8+0.057p^7q^{10}+0.025p^8q^{10}.$$
$$(D_pS_q+S_pD_q)(\chi_1)=5p^2q^4+129.98p^3q^6+35.88p^3q^7+27.36p^3q^8+4p^4q^4+34.64p^4q^6+18.56p^4q^7$$
$$+25.96p^6q^6+32.34p^6q^7+20.83p^6q^8+22.66p^6q^{10}+3.96p^7q^7+10.08p^7q^8+8.51p^7q^{10}+4.1p^8q^{10}$$
$$2S_pJ(\chi_1)=0.66p^6+6.22p^9+2.6p^{10}+1.63p^{11}+0.5p^8+3.2p^{10}+1.45p^{11}+2.16p^{12}+2.46p^{13}$$
$$+1.42p^{14}+1.25p^{16}+0.28p^{14}+0.66p^{15}+0.47p^{17}+0.22p^{18}.$$
$$S_pJD_pD_q(\chi_1)=2.66p^6+56p^9+27.3p^{10}+19.63p^{11}+4p^8+38.4p^{10}+20.36p^{11}+39p^{12}+51.69p^{13}$$
$$+34.28p^{14}+37.5p^{16}+7p^{14}+18.66p^{15}+16.47p^{17}+8.88p^{18}.$$
$$S_p^3Q_{-2}JD_p^3D_q^3(\chi_1)=16p^4+476.08p^7+235.14p^8+170.66p^9+37.92p^6+432p^8+240.89p^9+606.52p^{10}$$
$$+890.61p^{11}+640p^{12}+787.17p^{14}+136.16p^{12}+399.67p^{13}+406.51p^{15}+250p^{16}.$$

Figure 1 displays the surface representation of NMTCM-polynomials for A_m-E drugs, depicting distinct behaviors resulting from the manipulation of parameters p and q. □

Theorem 4. *The NMTCM polynomial for A_m-P given by*

$$NMTCM(A_m-P;p,q)=12p^3q^4+74p^3q^5+54p^3q^6+20p^4q^5+15p^4q^6+51p^6q^5+25p^6q^6+82p^5q^6$$
$$+28p^5q^7+21p^6q^7+6p^6q^8+2p^7q^8.$$

Proof. Let χ_2 be the molecular graph of aminopterin. From the figure, it is straightforward to deduce that $|V(\chi_2)|=32$ and $|E(\chi_2)|=34$. χ_1 vertex set can be classified into six classes based on their degrees.
$n_3=7$, $n_4=2$, $n_5=11$, $n_6=8$, $n_7=3$, $n_8=1$,.
Using definition, we have,

$\overline{m}_{ij}$	$\overline{nm}_{(34)}$	$\overline{nm}_{35}$	$\overline{nm}_{36}$	$\overline{nm}_{45}$	$\overline{nm}_{(46)}$	$\overline{nm}_{55}$	$\overline{nm}_{66}$	$\overline{nm}_{56}$	$\overline{nm}_{57}$
frequency	12	74	54	20	15	51	25	82	28

$\overline{m}_{ij}$	$\overline{nm}_{(67)}$	$\overline{nm}_{68}$	$\overline{nm}_{78}$
frequency	21	6	2

By definition of *NMTCM* polynomial,

$$
\begin{aligned}
Let, NM(\chi_2,p,q) &= \sum_{i\leq j} \overline{nm_{ij}}(G)p^iq^j \\
&= \sum_{3\leq 4} \overline{nm}_{34}(\chi_2)p^3q^4 + \sum_{3\leq 5} \overline{nm}_{35}(\chi_2)p^3q^5 + \sum_{3\leq 6} \overline{nm}_{36}(\chi_2)p^3q^6 \\
&+ \sum_{4\leq 5} \overline{nm}_{45}(\chi_2)p^4q^5 + \sum_{4\leq 6} \overline{nm}_{46}(\chi_2)p^4q^6 + \sum_{5\leq 5} \overline{nm}_{55}(\chi_2)p^5q^5 \\
&+ \sum_{6\leq 6} \overline{nm}_{66}(\chi_2)p^6q^6 + \sum_{5\leq 6} \overline{nm}_{56}(\chi_2)p^5q^6 + \sum_{5\leq 7} \overline{nm}_{57}(\chi_2)p^5q^7 \\
&+ \sum_{6\leq 7} \overline{nm}_{67}(\chi_2)p^6q^7 + \sum_{6\leq 8} \overline{nm}_{68}(\chi_2)p^6q^8 + \sum_{7\leq 8} \overline{nm}_{78}(\chi_2)p^7q^8. \\
&= \mid E_{(3,4)} \mid p^3q^4 + \mid E_{(3,5)} \mid p^3q^5 + \mid E_{(3,6)} \mid p^3q^6 + \mid E_{(4,5)} \mid p^4q^5 \\
&+ \mid E_{(4,6)} \mid p^4q^6 + \mid E_{(5,5)} \mid p^5q^5 \mid E_{(6,6)} \mid p^6q^6 + \mid E_{(5,6)} \mid p^5q^6 \\
&+ \mid E_{(5,7)} \mid p^5q^7 + \mid E_{(6,7)} \mid p^6q^7 + \mid E_{(6,8)} \mid p^6q^8 + \mid E_{(7,8)} \mid p^7q^8 \\
&= 12p^3q^4 + 74p^3q^5 + 54p^3q^6 + 20p^4q^5 + 15p^4q^6 51p^6q^5 + 25p^6q^6 \\
&+ 82p^5q^6 + 28p^5q^7 + 21p^6q^7 + 6p^6q^8 + 2p^7q^8.
\end{aligned}
$$

Hence, we get the required *NMTCM*-polynomial equation of $A_m - P$. □

Corollary 5. *Let χ_2 be a molecular graphs of $A_m - P$ then*

1. $\overline{NM}_1(\chi_2) = 3927.$
2. $\overline{NM}_2(\chi_2) = 9883.$
3. $\overline{NmM}_2(\chi_2) = 17.486$
4. $\overline{NI}(\chi_2) = 951.433.$
5. $\overline{NRZ}_3(\chi_2) = 105392.$
6. $\overline{NSDD}(\chi_2) = 837.916.$
7. $\overline{NAZ}(\chi_2) = 11762.3$
8. $\overline{NF}(\chi_2) = 20881.$
9. $\overline{NH}(\chi_2) = 79.99.$

Theorem 6. *The NMTCM-polynomial for $A_s - E$ given by*

$$
\begin{aligned}
NMTCM(A_s - E; p,q) &= 17p^3q^5 + 28p^3q^6 + 27p^3q^7 + 4p^4q^7 + 1p^5q^5 + 14p^5q^6 + 14p^5q^7 + 8p^6q^6 \\
&+ 23p^6q^7 + 4p^6q^8 + 18p^6q^9 + 8p^7q^7 + 17p^7q^9 + 4p^7q^8 + 3p^8q^9 + 4p^9q^9.
\end{aligned}
$$

Proof. Let χ_3 be the molecular graph of $A_s - E$ From the figure, it is straightforward to deduce that $|V(\chi_3)| = 26$ and $|E(\chi_3)| = 28$. Vertex set can be classified into eight classes based on their degrees.
$n_2^* = 1$, $n_3^* = 6$, $n_4^* = 1$, $n_5^* = 3$, $n_6^* = 5$, $n_7^* = 5$, $n_8^* = 1$, $n_9^* = 4$.
Using definition, we have,

$\overline{m}_{ij}$	$\overline{m}_{24}$	$\overline{m}_{35}$	$\overline{m}_{36}$	$\overline{m}_{37}$	$\overline{m}_{47}$	$\overline{m}_{55}$	$\overline{m}_{56}$	$\overline{m}_{57}$	$\overline{m}_{66}$	$\overline{m}_{67}$
frequency	0	17	28	27	4	1	14	14	8	23

$\overline{m}_{ij}$	$\overline{m}_{68}$	$\overline{m}_{69}$	$\overline{m}_{77}$	$\overline{m}_{79}$	$\overline{m}_{78}$	$\overline{m}_{89}$	$\overline{m}_{99}$
frequency	4	18	8	17	4	3	4

By definition of *NMTCM* polynomial,

$$\begin{aligned} Let, NM(\chi_3,p,q) &= \sum_{i\le j} \overline{nm}_{ij}(G)p^iq^j \\ &= \sum_{2\le 4} \overline{nm}_{24}(\chi_3)p^2q^4 + \sum_{3\le 5} \overline{nm}_{35}(\chi_3)p^3q^5 + \sum_{3\le 6} \overline{nm}_{36}(\chi_3)p^3q^6 \\ &+ \sum_{3\le 7} \overline{nm}_{37}(\chi_3)p^3q^7 + \sum_{4\le 7} \overline{nm}_{47}(G)p^4q^7. + \sum_{5\le 5} \overline{nm}_{55}(\chi_3)p^5q^5 \\ &+ \sum_{5\le 6} \overline{nm}_{56}(\chi_3)p^5q^6 + \sum_{5\le 7} \overline{nm}_{57}(\chi_3)p^5q^7 + \sum_{6\le 6} \overline{nm}_{66}(\chi_3)p^6q^6 \\ &+ \sum_{6\le 7} \overline{nm}_{67}(G)p^6q^7 + \sum_{6\le 8} \overline{nm}_{68}(\chi_3)p^6q^8 + \sum_{6\le 9} \overline{nm}_{69}(\chi_3)p^6q^9 \\ &+ \sum_{7\le 7} \overline{nm}_{77}(\chi_3)p^7q^7 + \sum_{7\le 9} \overline{nm}_{79}(\chi_3)p^7q^9 + \sum_{7\le 8} \overline{nm}_{78}(G)p^7q^8. \\ &+ \sum_{8\le 9} \overline{nm}_{89}(\chi_3)p^8q^9 + \sum_{9\le 9} \overline{nm}_{99}(\chi_3)p^9q^9. \end{aligned}$$

$$\begin{aligned} NM(\chi_3,p,q) &= |E_{(2,4)}|p^2q^4 + |E_{(3,5)}|p^3q^5 + |E_{(3,6)}|p^3q^6 + |E_{(3,7)}|p^3q^7 \\ &+ |E_{(4,7)}|p^4q^7 + |E_{(5,5)}|p^5q^5 + |E_{(5,6)}|p^5q^6 + |E_{(5,7)}|p^5q^7 \\ &+ |E_{(6,6)}|p^6q^6 + |E_{(6,7)}|p^6q^7 + |E_{(6,8)}|p^6q^8 + |E_{(6,9)}|p^6q^9 \\ &+ |E_{(7,7)}|p^7q^7 + |E_{(7,9)}|p^7q^9 + |E_{(7,8)}|p^7q^8 + |E_{(8,9)}|p^8q^9 \\ &+ |E_{(9,9)}|p^9q^9 \\ &= 17p^3q^5 + 28p^3q^6 + 27p^3q^7 + 4p^4q^7 + 1p^5q^5 + 14p^5q^6 + 14p^5q^7 + 8p^6q^6 \\ &+ 23p^6q^7 + 4p^6q^8 + 18p^6q^9 + 8p^7q^7 + 17p^7q^9 + 4p^7q^8 + 3p^8q^9 + 4p^9q^9. \end{aligned}$$

Hence, we get the required *NMTCM*-polynomial equation of $A_s - E$. □

Corollary 7. *Let χ_3 be a molecular graphs of $A_s - E$ then topological coindices for χ_3*

1. $\overline{NM}_1(\chi_3) = 2322.$
2. $\overline{NM}_2(\chi_3) = 7018.$
3. $\overline{NmM}_2(\chi_3) = 6.797.$
4. $\overline{NI}(\chi_3) = 553.66$
5. $\overline{NRZ}_3(\chi_3) = 90568.$.
6. $\overline{NSDD}(\chi_3) = 435.52$
7. $\overline{NAZ}(\chi_3) = 8858.7.$
8. $\overline{NF}(\chi_3) = 15170$
9. $\overline{NH}(\chi_3) = 34.11.$

Theorem 8. *The NMTCM-polynomial for $C_s - N$ given by*

$$NMTCM(C_s - N; p,q) = 7p^2q^3 + 5p^2q^4 + 5p^3q^4 + 5p^3q^5 + 2p^3q^6 + 3p^4q^5 + 1p^4q^7 + p^5q^6 + p^5q^7.$$

Proof. Let χ_4 be the molecular graph of $C_s - N$ From the figure, it is straightforward to deduce that $|V(\chi_4)| = 11$ and $|E(\chi_4)| = 12$. Vertex set can be classified into three classes based on their degrees.
$n_2^* = 3$, $n_3^* = 3$, $n_4^* = 2$, $n_5^* = 2$, $n_6^* = 1$, $n_7^* = 1$.
Using definition, we have,

$\overline{m}_{ij}$	$\overline{nm}_{23}$	$\overline{nm}_{24}$	$\overline{nm}_{34}$	$\overline{nm}_{35}$	$\overline{nm}_{36}$	$\overline{nm}_{45}$	$\overline{nm}_{47}$	$\overline{nm}_{56}$	$\overline{nm}_{57}$	$\overline{nm}_{67}$
frequency	7	5	5	5	2	3	1	1	1	0

By definition of *NMTCM* polynomial,

$$
\begin{aligned}
Let, M(\chi_4,p,q) &= \sum_{i\leq j} \overline{m}_{ij}(G)p^i q^j \\
&= \sum_{2\leq 3} \overline{m}_{23}(\chi_4)p^2q^3 + \sum_{2\leq 4} \overline{m}_{24}(\chi_4))p^2q^4 + \sum_{3\leq 4} m_{34}(\chi_4))p^3q^4 \\
&+ \sum_{3\leq 5} \overline{m}_{35}(\chi_4)p^3q^5 + \sum_{3\leq 6} \overline{m}_{36}(\chi_4)p^3q^6 + \sum_{4\leq 5} \overline{m}_{45}(\chi_4)p^4q^5 \\
&+ \sum_{4\leq 7} \overline{m}_{47}(\chi_4)p^4q^7 + \sum_{5\leq 6} \overline{m}_{56}(\chi_4)p^5q^6 + \sum_{5\leq 7} \overline{m}_{57}(\chi_4)p^5q^7 \\
&+ \sum_{6\leq 7} \overline{m}_{67}(\chi_4)p^6q^7. \\
&= | E_{(2,3)} | p^2q^3 + | E_{(2,4)} | p^2q^4 + | E_{(3,4)} | p^3q^4 + | E_{(3,5)} | p^3q^5 \\
&+ | E_{(3,6)} | p^3q^6 + | E_{(4,5)} | p^4q^5 + | E_{(4,7)} | p^4q^7 + | E_{(5,6)} | p^5q^6 + | E_{(5,7)} | p^5q^7. \\
&= 7p^2q^3 + 5p^2q^4 + 5p^3q^4 + 5p^3q^5 + 2p^3q^6 + 3p^4q^5 + 1p^4q^7 + p^5q^6 + p^5q^7.
\end{aligned}
$$

Hence, we get the required *NMTCM*-polynomial equation of $Cs - E$. □

Now, using this *NMTCM*-polynomial equation of $Cs - E$, we have,

Corollary 9. *Let χ_4 be a molecular graphs of $Cs - E$ then topological co-indices for χ_4 are given by*

1. $\overline{M}_1(\chi_4) = 219$.
2. $\overline{M}_2(\chi_4) = 406$.
3. $\overline{NmM}_2(\chi_4) = 2.8917$.
4. $\overline{I}(\chi_4) = 51.854$.
5. $\overline{NRZ}_3(\chi_4) = 3392$.
6. $\overline{NSDD}(\chi_4) = 67.03$
7. $\overline{NAZ}(\chi_4) = 456.50$
8. $\overline{NF}(\chi_4) = 899$.
9. $\overline{NH}(\chi_4) = 8.776$.

Theorem 10. *The NMTCM-polynomial for $C_b - E$ given by*

$$NMTCM(C_b - E; p,q) = 2p^2q^4 + 3p^3q^6 + 3p^3q^7 + 5p^4q^7 + 4p^5q^4 + 2p^5q^8 + 3p^6q^7 + 3p^6q^8 + 2p^4q^4 + 3p^7q^8.$$

Proof. Let χ_5 be the molecular graph of $C_b - E$ From the figure, it is straightforward to deduce that $|V(\chi_5)| = 14$ and $|E(\chi_5)| = 15$. Vertex set can be classified into seven classes based on their degrees.
$n_2^* = |v_2| = 1$, $n_3^* = |v_3| = 2$, $n_4^* = |v_4| = 3$, $n_5^* = |v_5| = 2$, $n_6^* = |v_6| = 2$, $n_7^* = |v_7| = 2$, $n_8^* = |v_8| = 2$. Using definition, we have,

$\overline{nm}_{ij}$	$\overline{nm}_{24}$	$\overline{nm}_{36}$	$\overline{nm}_{37}$	$\overline{nm}_{47}$	$\overline{nm}_{54}$	$\overline{nm}_{58}$	$\overline{nm}_{66}$
frequency	2	3	3	5	4	2	0

$\overline{nm}_{ij}$	$\overline{nm}_{67}$	$\overline{nm}_{68}$	$\overline{nm}_{44}$	$\overline{nm}_{77}$	$\overline{nm}_{78}$	$\overline{nm}_{88}$
frequency	3	3	2	0	3	0

By definition of *NMTCM* polynomial,

$$\begin{aligned}
Let, NM(\chi_5, p, q) &= \sum_{i\leq j} \overline{nm}_{ij}(G)p^i q^j \\
&= \sum_{2\leq 4} \overline{nm}_{24}(\chi_5)p^2q^4 + \sum_{3\leq 6} \overline{nm}_{36}(\chi_5)p^3q^6 + \sum_{3\leq 7} \overline{nm}_{37}(\chi_5)p^3q^7 \\
&+ \sum_{4\leq 7} \overline{nm}_{47}(\chi_5)p^4q^7 + \sum_{5\leq 4} \overline{nm}_{54}(\chi_5)p^5q^4 + \sum_{5\leq 8} \overline{nm}_{58}(\chi_5)p^5q^8 \\
&+ \sum_{6\leq 6} \overline{nm}_{66}(\chi_5)p^6q^6 + \sum_{6\leq 7} \overline{nm}_{67}(\chi_5)p^6q^7 + \sum_{6\leq 8} \overline{nm}_{68}(\chi_5)p^6q^8 \\
&+ \sum_{4\leq 4} \overline{nm}_{44}(\chi_5)p^4q^4 + \sum_{7\leq 7} \overline{nm}_{77}(\chi_5)p^7q^7 + \sum_{7\leq 8} \overline{nm}_{78}(\chi_5)p^7q^8 \\
&+ \sum_{8\leq 8} \overline{nm}_{88}(\chi_5)p^8q^8
\end{aligned}$$

$$\begin{aligned}
NM(\chi_5, p, q) &= \mid E_{(2,4)} \mid p^2q^4 + \mid E_{(3,6)} \mid p^3q^6 + \mid E_{(3,7)} \mid p^3q^7 \\
&+ \mid E_{(4,7)} \mid p^4q^7 + \mid E_{(5,4)} \mid p^5q^4 + \mid E_{(5,8)} \mid p^5q^8. \\
&+ \mid E_{(6,6)} \mid p^6q^6 + \mid E_{(6,7)} \mid p^6q^7 + \mid E_{(6,8)} \mid p^6q^8 \\
&+ \mid E_{(4,4)} \mid p^4q^4 + \mid E_{(7,7)} \mid p^7q^7 + \mid E_{(7,8)} \mid p^7q^8 \\
&+ \mid E_{(8,8)} \mid p^8q^8. \\
&= 2p^2q^4 + 3p^3q^6 + 3p^3q^7 + 5p^4q^7 + 4p^5q^4 + 2p^5q^8 + 3p^6q^7 + 3p^6q^8 + 2p^4q^4 + 3p^7q^8.
\end{aligned}$$

Hence, we get the required *NMTCM*-polynomial equation of $C_b - E$. □

Now, using this *NMTCM*-polynomial equation of $C_b - E$, we have,

Corollary 11. *Let χ_5 be a molecular graphs of $C_b - E$ then topological co-indices for χ_5 are given by*

1. $\overline{M}_1(\chi_5) = 328.$
2. $\overline{M}_2(\chi_5) = 903$
3. $\overline{NmM}_2(\chi_5) = 1.297$
4. $\overline{I}(\chi_5) = 77.89$
5. $\overline{RZ_3}(\chi_4) = 10942.$
6. $\overline{NSDD}(\chi_5) = 67.42$
7. $\overline{NAZ}(\chi_5) = 1093.93.$
8. $\overline{NF}(\chi_5) = 1974.$
9. $\overline{NH}(\chi_5) = 5.825.$

Theorem 12. *The NMTCM-polynomial for $C_n - A$ given by*

$$NMTCM(C_n - A; p,q) = 5p^2q^3 + 33p^3q^6 + 10p^3q^7 + 46p^4q^4 + 21p^4q^5 + 10p^4q^9 + 10p^5q^6 + 15p^5q^8 + 13p^6q^6 + 10p^6q^7 + 10p^6q^8 + 5p^6q^9 + 1p^8q^9 + 3p^7q^8 + 1p^7q^9 + 65p^3q^4.$$

Proof. Let χ_6 be the molecular graph of $C_n - A$ From the figure, it is straightforward to deduce that $|V(\chi_6)| = 31$ and $|E(\chi_6)| = 32$. Vertex set can be classified into eight classes based on their degrees.
$n_2^* = |v_2| = 1$, $n_3^* = |v_3| = 6$, $n_4^* = |v_4| = 11$, $n_5^* = |v_5| = 2$, $n_6^* = |v_6| = 6$, $n_7^* = |v_7| = 2$, $n_8^* = |v_8| = 2$, $n_9^* = |v_9| = 1$.

Using definition, we have,

$\overline{m}_{ij}$	$\overline{nm}_{23}$	$\overline{nm}_{36}$	$\overline{nm}_{37}$	$\overline{nm}_{44}$	$\overline{nm}_{45}$	$\overline{nm}_{49}$	$\overline{nm}_{56}$
frequency	5	33	10	46	21	10	10

$\overline{m}_{ij}$	$\overline{nm}_{58}$	$\overline{nm}_{66}$	$\overline{nm}_{67}$	$\overline{nm}_{68}$	$\overline{nm}_{69}$	$\overline{nm}_{89}$	$\overline{nm}_{78}$	$\overline{nm}_{79}$	$\overline{nm}_{34}$
frequency	15	13	10	10	5	1	3	1	65

By definition of *NMTCM* polynomial,

$$\begin{aligned}
Let, NM(\chi_6,p,q) &= \sum_{i\le j} \overline{nm}_{ij}(G)p^iq^j \\
&= \sum_{2\le 3} \overline{nm}_{23}(\chi_6)p^2q^3 + \sum_{3\le 6} \overline{nm}_{36}(\chi_6)p^3q^6 + \sum_{3\le 7} \overline{nm}_{37}(\chi_6)p^3q^7 \\
&+ \sum_{4\le 4} \overline{nm}_{44}(\chi_6)p^4q^4 + \sum_{4\le 5} \overline{nm}_{45}(\chi_6)p^4q^5 + \sum_{4\le 9} \overline{nm}_{49}(\chi_6)p^4q^9 \\
&+ \sum_{5\le 6} \overline{nm}_{56}(\chi_6)p^5q^6 + \sum_{5\le 8} \overline{nm}_{58}(G)p^5q^8 + \sum_{6\le 6} \overline{nm}_{66}(\chi_6)p^6q^6 \\
&+ \sum_{6\le 7} \overline{nm}_{67}(\chi_6)p^6q^7 + \sum_{6\le 8} \overline{nm}_{68}(\chi_6)p^6q^8 + \sum_{6\le 9} \overline{nm}_{69}(\chi_6)p^6q^9 \\
&+ \sum_{8\le 9} \overline{nm}_{89}(\chi_6)p^8q^9 + \sum_{7\le 8} \overline{nm}_{78}(\chi_6)p^7q^8 + \sum_{7\le 9} \overline{nm}_{79}(\chi_6)p^7q^9 \\
&+ \sum_{3\le 4} \overline{nm}_{34}(\chi_6)p^3q^4. \\
&= |E_{(2,3)}|\,p^2q^3 + |E_{(3,6)}|\,p^3q^6 + |E_{(3,7)}|\,p^3q^7 + |E_{(6,6)}|\,p^6q^6 + |E_{(6,7)}|\,p^6q^7 \\
&+ |E_{(6,8)}|\,p^6q^8 + |E_{(6,9)}|\,p^6q^9 + |E_{(8,9)}|\,p^8q^9 + |E_{(7,8)}|\,p^7q^8 \\
&+ |E_{(7,9)}|\,p^7q^9 + |E_{(3,4)}|\,p^3q^4. \\
&= 5p^2q^3 + 36p^3q^3 + 10p^3q^7 + 46p^4q^4 + 21p^4q^5 + 10p^4q^9 + 10p^5q^6 \\
&+ 15p^5q^8 + 13p^6q^6 + 10p^6q^7 + 10p^6q^8 + 5p^6q^9 + 1p^8q^9 + 3p^7q^8 \\
&+ 1p^7q^9 + 65p^3q^4.
\end{aligned}$$

Hence, we get the required *NMTCM*-polynomial equation of $C_n - A$. □

Now, using this *NMTCM*-polynomial equation of $C_n - A$, we have,

Corollary 13. *Let χ_6 be a molecular graphs of Convolutamide A then topological co-indices for χ_6 are given by*

1. $\overline{M}_1(\chi_6) = 2448.$
2. $\overline{M}_2(\chi_6) = 5971.$
3. $\overline{NmM}_2(\chi_6) = 14.44$
4. $\overline{I}(\chi_6) = 587.15$
5. $\overline{RZ_3}(\chi_6) = 65102.$
6. $\overline{NSDD}(\chi_6) = 560.04$
7. $\overline{NAZ}(\chi_6) = 7065.19.$
8. $\overline{NF}(\chi_6) = 12988.$
9. $\overline{NH}(\chi_6) = 58.118.$

Theorem 14. *The NMTCM-polynomial for $C_t - F$ given by*

$$NMTCM(C_t - F; p,q) = 7p^2q^3 + 3p^2q^4 + 7p^3q^4 + 10p^3q^6 + 3p^3q^7 + 1p^4q^5 + 3p^4q^8 + p^5q^8 + p^6q^6 + 4p^6q^8.$$

Proof. Let χ_7 be the molecular graph of $C_t - F$ From the figure, it is straightforward to deduce that $|V(\chi_7)| = 15$ and $|E(\chi_7)| = 15$. Vertex set can be classified into seven classes based on their degrees.
$n_2^* = |v_2| = 2$, $n_3^* = |v_3| = 4$, $n_4^* = |v_4| = 2$, $n_5^* = |v_5| = 1$, $n_6^* = |v_6| = 3$, $n_7^* = |v_7| = 1$, $n_8^* = |v_8| = 2$.
Using definition, we have,

$\overline{nm}_{ij}$	$\overline{nm}_{23}$	$\overline{nm}_{24}$	$\overline{nm}_{34}$	$\overline{nm}_{36}$	$\overline{nm}_{37}$	$\overline{nm}_{45}$	$\overline{nm}_{48}$	$\overline{nm}_{58}$	$\overline{nm}_{66}$	$\overline{nm}_{68}$	$\overline{nm}_{78}$
frequency	7	3	7	10	3	1	3	1	1	4	0

By definition of *NMTCM* polynomial,

$$\begin{aligned}
Let, M(\chi_7, p,q) &= \sum_{i\leq j} \overline{nm}_{ij}(G)p^iq^j \\
&= \sum_{2\leq 3} \overline{nm}_{23}(\chi_7)p^2q^3 + \sum_{2\leq 4} \overline{nm}_{24}(\chi_7)p^2q^4 + \sum_{3\leq 4} \overline{nm}_{34}(\chi_7)p^3q^4 \\
&+ \sum_{3\leq 6} \overline{nm}_{36}(\chi_7)p^3q^6 + \sum_{3\leq 7} \overline{nm}_{37}(\chi_7)p^3q^7 + \sum_{4\leq 5} \overline{nm}_{45}(\chi_7)p^4q^5. \\
&+ \sum_{4\leq 8} \overline{nm}_{48}(\chi_7)x4q^8 + \sum_{5\leq 8} \overline{nm}_{58}(\chi_7)p^5q^8 + \sum_{6\leq 6} \overline{nm}_{66}(\chi_7)p^6q^6 \\
&+ \sum_{6\leq 8} \overline{nm}_{68}(\chi_7)p^6q^8 + \sum_{7\leq 8} \overline{nm}_{78}(\chi_7)p^7q^8 \\
&= \mid E_{(2,3)} \mid p^2q^3 + \mid E_{(2,4)} \mid p^2q^4 + \mid E_{(3,4)} \mid p^3q^4 + \mid E_{(3,6)} \mid p^3q^6 \\
&+ \mid E_{(3,7)} \mid p^3q^7 + \mid E_{(4,5)} \mid p^4q^5 + \mid E_{(4,8)} \mid p^4q^8 + \mid E_{(5,8)} \mid p^5q^8 \\
&+ \mid E_{(6,6)} \mid p^6q^6 + \mid E_{(6,8)} \mid p^6q^8 + \mid E_{(7,8)} \mid p^7q^8 \\
&= 7p^2q^3 + 3p^2q^4 + 7p^3q^4 + 10p^3q^6 + 3p^3q^7 + 1p^4q^5 + 3p^4q^8 + p^5q^8 + p^6q^6 + 4p^6q^8.
\end{aligned}$$

Hence, we get the required *NMTCM*-polynomial equation of $C_t - F$. □

Now, using this *NMTCM*-polynomial equation of $C_t - F$, we have,

Corollary 15. *Let χ_7 be a molecular graphs of Convolutamine F then topological co-indices for χ_7 are given by*

1. $\overline{NM}_1(\chi_7) = 348.$
2. $\overline{NM}_2(\chi_7) = 777.$
3. $\overline{NmM}_2(\chi_7) = 3.093.$
4. $\overline{NI}(\chi_7) = 80.706.$
5. $\overline{NRZ}_3(\chi_7) = 8164.$
6. $\overline{NSDD}(\chi_7) = 92.63.$
7. $\overline{NAZ}(\chi_7) = 872.71$
8. $\overline{NF}(\chi_7) = 1792.$
9. $\overline{NH}(\chi_7) = 10.23.$

Theorem 16. *The NMTCM-polynomial for $C_m - A$ given by*

$$NMTCM(C_m - A; p, q) = 10p^3q^4 + 14p^3q^3 + 5p^3q^5 + 22p^3q^6 + 5p^3q^9 + p^4q^9 + 2p^5q^6 + 5p^6q^6 + 6p^6q^7 + 3p^6q^{10} + p^7q^{10} + p^7q^9 + 3p^4q^7.$$

Proof. Let χ_8 be the molecular graph of $C_m - A$ From the figure, it is straightforward to deduce that $|V(\chi_8)| = 17$ and $|E(\chi_8)| = 18$. Vertex set can be classified into seven classes based on their degrees.
$n_3^* = 6$, $n_4^* = 2$, $n_5^* = 1$, $n_6^* = 4$,$n_7^* = 2$, $n_9^* = 1$, $n_{10}^* = 1$.
Using definition, we have,

$\overline{nm}_{ij}$	$\overline{nm}_{34}$	$\overline{nm}_{33}$	$\overline{nm}_{35}$	$\overline{nm}_{36}$	$\overline{nm}_{39}$	$\overline{nm}_{49}$	$\overline{nm}_{56}$
frequency	10	14	5	22	5	1	2

$\overline{nm}_{ij}$	$\overline{nm}_{66}$	$\overline{nm}_{67}$	$\overline{nm}_{6(10)}$	$\overline{nm}_{7(10)}$	$\overline{nm}_{9(10)}$	$\overline{nm}_{79}$	$\overline{nm}_{47}$
frequency	5	6	3	1	0	1	3

By definition of *NMTCM* polynomial,

$$\begin{aligned}
Let, NM(\chi_8, p, q) &= \sum_{i \le j} \overline{nm}_{ij}(G) p^i q^j \\
&= \sum_{3\le 4} \overline{nm}_{34}(\chi_8)p^3q^4 + \sum_{3\le 3} \overline{nm}_{33}(\chi_8)p^3q^3 + \sum_{3\le 5} \overline{nm}_{35}(\chi_8)p^3q^5 \\
&+ \sum_{3\le 6} \overline{nm}_{36}(\chi_8)p^3q^6 + \sum_{4\le 9} \overline{nm}_{49}(\chi_8)p^4q^9 + \sum_{4\le 5} \overline{nm}_{45}(\chi_7)p^4q^5. \\
&+ \sum_{5\le 6} \overline{nm}_{56}(\chi_8)x5q^6 + \sum_{6\le 6} \overline{nm}_{66}(\chi_8)p^6q^6 + \sum_{6\le 7} \overline{nm}_{67}(\chi_8)p^6q^7 \\
&+ \sum_{6\le 10} \overline{nm}_{6(10)}(\chi_8)p^6q^{10} + \sum_{7\le 10} \overline{nm}_{7(10)}(\chi_8)p^7q^{10} + \sum_{9\le 10} \overline{nm}_{9(10)}(\chi_8)p^9q^{10} \\
&+ \sum_{7\le 9} \overline{nm}_{79}(\chi_8)p^7q^9 + \sum_{4\le 7} \overline{nm}_{47}(\chi_8)p^4q^7 \\
&= |E_{(3,4)}| p^3q^4 + |E_{(3,3)}| p^3q^3 + |E_{(3,5)}| p^3q^5 + |E_{(3,6)}| p^3q^6 \\
&+ |E_{(3,9)}| p^3q^9 + |E_{(4,9)}| p^4q^9 + |E_{(5,6)}| p^5q^6 + |E_{(6,6)}| p^6q^6 \\
&+ |E_{(6,7)}| p^6q^7 + |E_{(6,10)}| p^6q^{10} + |E_{(7,10)}| p^7q^{10} + |E_{(9,10)}| p^9q^{10} \\
&+ |E_{(7,9)}| p^7q^9 + |E_{(4,7)}| p^4q^7. \\
&= 10p^3q^4 + 14p^3q^3 + 5p^3q^5 + 22p^3q^6 + 5p^3q^9 + p^4q^9 + 2p^5q^6 + 5p^6q^6 \\
&+ 6p^6q^7 + 3p^6q^{10} + p^7q^{10} + p^7q^9 + 3p^4q^7.
\end{aligned}$$

Hence, we get the required *NMTCM*-polynomial equation of $C_m - A$. □

Now, using this *NMTCM*-polynomial equation of $C_m - A$, we have,

Corollary 17. *Let χ_8 be a molecular graphs of convolutamydine A then topological co-indices for χ_8 are given by*

1. $\overline{NM}_1(\chi_8) = 739.$
2. $\overline{NM}_2(\chi_8) = 1777.$
3. $\overline{NmM}_2(\chi_8) = 4.678.$
4. $\overline{NI}(\chi_8) = 172.27.$
5. $\overline{NRZ}_3(\chi_8) = 19946.$
6. $\overline{NSDD}(\chi_8) = 176.9.$
7. $\overline{NAZ}(\chi_8) = 2039.35.$
8. $\overline{NF}(\chi_8) = 4083.$
9. $\overline{NH}(\chi_8) = 17.90.$

Theorem 18. *The NMTCM-polynomial for $D_r - N$ given by*

$$\begin{aligned} NMTCM(D_r - N; p, q) &= 2p^2q^4 + 20p^3q^6 + 76p^3q^7 + 5p^4q^5 + 26p^4q^7 + 18p^4q^8 \\ &+ 8p^4q^8 + 17p^5q^7 + 5p^5q^8 + 23p^6q^6 + 67p^6q^7 + 22p^6q^8 + 22p^7q^8 \\ &+ 21p^7q^9 + 35p^7q^7 + 8p^8q^9 + 2p^9q^9. \end{aligned}$$

Proof. Let χ_9 be the molecular graph of $D_r - N$ From the figure, it is straightforward to deduce that $|V(\chi_9)| = 38$ and $|E(\chi_9)| = 42$. Vertex set can be classified into eight classes based on their degrees.
$n_2^* = 1$, $n_3^* = 9$, $n_4^* = 3$, $n_5^* = 2$, $n_6^* = 8$, $n_7^* = 9$, $n_8^* = 3$, $n_9^* = 3$.
Using definition, we have,

$\overline{m}_{ij}$	$\overline{nm}_{24}$	$\overline{nm}_{36}$	$\overline{nm}_{37}$	$\overline{nm}_{45}$	$\overline{nm}_{47}$	$\overline{nm}_{48}$	$\overline{nm}_{57}$
frequency	2	20	76	5	26	8	17

$\overline{m}_{ij}$	$\overline{nm}_{58}$	$\overline{nm}_{66}$	$\overline{nm}_{67}$	$\overline{nm}_{68}$	$\overline{nm}_{78}$	$\overline{nm}_{79}$	$\overline{nm}_{77}$	$\overline{nm}_{89}$	$\overline{nm}_{99}$
frequency	5	23	67	22	22	21	35	8	2

By definition of *NMTCM* polynomial,

$$
\begin{aligned}
Let, NM(\chi_9,p,q) &= \sum_{i\leq j} \overline{nm}_{ij}(G)p^iq^j \\
&= \sum_{2\leq 4} \overline{nm}_{24}(\chi_9)p^2q^4 + \sum_{3\leq 6} \overline{nm}_{36}(\chi_9)p^3q^6 + \sum_{3\leq 7} \overline{nm}_{37}(\chi_9)p^3q^7 \\
&+ \sum_{4\leq 5} \overline{nm}_{45}(\chi_9)p^4q^5 + \sum_{4\leq 7} \overline{nm}_{47}(\chi_9)p^4q^7 + \sum_{4\leq 9} \overline{nm}_{48}(\chi_9)p^4q^8 \\
&+ \sum_{5\leq 7} \overline{nm}_{57}(\chi_9)p^5q^7 + \sum_{5\leq 8} \overline{nm}_{58}(G)p^5q^8 \\
&+ \sum_{6\leq 6} \overline{nm}_{66}(\chi_9)p^6q^6 + \sum_{6\leq 7} \overline{nm}_{67}(\chi_9)p^6q^7 + \sum_{6\leq 8} \overline{nm}_{68}(\chi_9)p^6q^8 \\
&+ \sum_{7\leq 8} \overline{nm}_{78}(\chi_9)p^7q^8 + \sum_{7\leq 9} \overline{nm}_{79}(\chi_9)p^7q^9 + \sum_{7\leq 7} \overline{nm}_{77}(\chi_9)p^7q^7 \\
&+ \sum_{8\leq 9} \overline{nm}_{89}(\chi_9)p^8q^9 + \sum_{9\leq 9} \overline{nm}_{99}(\chi_9)p^9q^9. \\
&= |\, E_{(2,4)} \,|\, p^2q^4 + |\, E_{(3,6)} \,|\, p^3q^6 + |\, E_{(3,7)} \,|\, p^3q^7 + |\, E_{(4,5)} \,|\, p^4q^5 \\
&+ |\, E_{(4,7)} \,|\, p^4q^7 + |\, E_{(4,8)} \,|\, p^4q^8 + |\, E_{(5,7)} \,|\, p^5q^7 + |\, E_{(5,8)} \,|\, p^5q^8 \\
&+ |\, E_{(6,6)} \,|\, p^6q^6 + |\, E_{(6,7)} \,|\, p^6q^7 + |\, E_{(6,8)} \,|\, p^6q^8 + |\, E_{(7,8)} \,|\, p^7q^8 \\
&+ |\, E_{(7,9)} \,|\, p^7q^9 + |\, E_{(7,8)} \,|\, p^7q^8 + |\, E_{(7,9)} \,|\, p^7q^9 + |\, E_{(7,7)} \,|\, p^7q^7 \\
&+ |\, E_{(8,9)} \,|\, p^8q^9 + |\, E_{(9,9)} \,|\, p^9q^9 \\
&= 2p^2q^4 + 20p^3q^6 + 76p^3q^7 + 5p^4q^5 + 26p^4q^7 + 18p^4q^8 \\
&+ 8p^4q^8 + 17p^5q^7 + 5p^5q^8 + 23p^6q^6 + 67p^6q^7 + 22p^6q^8 + 22p^7q^8 \\
&+ 21p^7q^9 + 35p^7q^7 + 8p^8q^9 + 2p^9q^9.
\end{aligned}
$$

Hence, we get the required *NMTCM*-polynomial equation of $D_r - N$. □

Now, using this *NMTCM*-polynomial equation of $D_r - N$, we have,

Corollary 19. *Let χ_9 be a molecular graphs of Daunorubicin then topological co-indices for χ_9 are given by*

1. $\overline{NM}_1(\chi_9) = 4431.$
2. $\overline{NM}_2(\chi_9) = 13557.$
3. $\overline{NmM}_2(\chi_9) = 11.233.$
4. $\overline{NI}(\chi_9) = 1056.91$
5. $\overline{NRZ}_3(\chi_9) = 178684.$.
6. $\overline{NSDD}(\chi_9) = 807.81.$
7. $\overline{NAZ}(\chi_9) = 17202.4.$
8. $\overline{NF}(\chi_9) = 29267.$
9. $\overline{NH}(\chi_9) = 59.19.$

Theorem 20. *The NMTCM-polynomial for $D_g - N$ given by*

$$
\begin{aligned}
NMTCM(D_g - N; p,q) &= 10p^2q^4 + 2p^3q^8 + 28p^4q^6 + 34p^4q^7 + 11p^4q^9 + 5p^4q^{11} \\
&+ 23p^5q^6 + 27p^5q^7 + 9p^5q^5 + 11p^5q^8 + 8p^6q^9 + 9p^6q^6 + 13p^7q^7 \\
&+ 26p^6q^7 + 17p^7q^8 + 10p^7q^9 + 2p^8q^8 + 2p^8q^{11}.
\end{aligned}
$$

Proof. Let χ_{10} be the molecular graph of $D_g - N$ From the figure, it is straightforward to deduce that $|V(\chi_{10})| = 31$ and $|E(\chi_{10})| = 35$. Vertex set can be classified into nine classes based on their degrees.
$n_2^* = 2$, $n_3^* = 12$, $n_4^* = 6$, $n_5^* = 5$, $n_6^* = 5$, $n_7^* = 6$, $n_8^* = 3$, $n_9^* = 2$, $n_{11}^* = 1$.
Using definition, we have,

$\overline{m}_{ij}$	$\overline{nm}_{24}$	$\overline{nm}_{38}$	$\overline{nm}_{46}$	$\overline{nm}_{47}$	$\overline{nm}_{49}$	$\overline{nm}_{4(11)}$	$\overline{nm}_{56}$	$\overline{nm}_{57}$	$\overline{nm}_{55}$	$\overline{nm}_{58}$	$\overline{nm}_{69}$
frequency	10	2	28	34	11	5	23	27	9	11	8

$\overline{m}_{ij}$	$\overline{nm}_{66}$	$\overline{nm}_{77}$	$\overline{nm}_{67}$	$\overline{nm}_{78}$	$\overline{nm}_{79}$	$\overline{nm}_{9(11)}$	$\overline{nm}_{88}$	$\overline{nm}_{8(11)}$
frequency	9	13	26	17	10	0	2	2

By definition of *NMTCM* polynomial,

$$\begin{aligned}
Let, NM(\chi_9, p, q) &= \sum_{i \le j} \overline{nm}_{ij}(G) p^i q^j \\
&= \sum_{2\le 4} \overline{nm}_{24}(\chi_{10}) p^2 q^4 + \sum_{3\le 8} \overline{nm}_{38}(\chi_{10}) p^3 q^8 + \sum_{4\le 6} \overline{nm}_{46}(\chi_{10}) p^4 q^6 \\
&+ \sum_{4\le 7} \overline{nm}_{47}(\chi_{10}) p^4 q^7 + \sum_{4\le 9} \overline{nm}_{49}(\chi_{10}) p^4 q^9 + \sum_{4\le 11} \overline{nm}_{4(11)}(\chi_{10}) p^4 q^{11} \\
&+ \sum_{5\le 6} \overline{nm}_{56}(\chi_{10}) p^5 q^6 + \sum_{5\le 7} \overline{nm}_{57}(G) p^5 q^7 + \sum_{5\le 5} \overline{nm}_{57}(\chi_{10}) p^5 q^7 \\
&+ \sum_{5\le 5} \overline{nm}_{55}(\chi_{10}) p^5 q^5 + \sum_{5\le 8} \overline{nm}_{58}(\chi_{10}) p^5 q^8 + \sum_{6\le 9} \overline{nm}_{69}(\chi_{10}) p^6 q^9 \\
&+ \sum_{6\le 6} \overline{nm}_{66}(\chi_{10}) p^6 q^6 + \sum_{7\le 7} \overline{nm}_{77}(\chi_{10}) p^7 q^7 + \sum_{6\le 7} \overline{nm}_{67}(\chi_{10}) p^6 q^7 \\
&+ \sum_{7\le 8} \overline{nm}_{78}(\chi_{10}) p^7 q^8 + \sum_{7\le 9} \overline{nm}_{79}(\chi_{10}) p^7 q^9 + \sum_{9\le 11} \overline{nm}_{9(11)}(\chi_{10}) p^9 q^{11} \\
&+ \sum_{8\le 8} \overline{nm}_{88}(\chi_{10}) p^8 q^8 + \sum_{8\le 11} \overline{nm}_{8(11)}(\chi_{10}) p^8 q^{11}.
\end{aligned}$$

$$\begin{aligned}
NM(\chi_9, p, q) &= |E_{(2,4)}| p^2 q^4 + |E_{(3,8)}| p^3 q^8 + |E_{(4,6)}| p^4 q^6 + |E_{(4,7)}| p^4 q^7 \\
&+ |E_{(4,9)}| p^4 q^9 + |E_{(4,11)}| p^4 q^{11} + |E_{(5,6)}| p^5 q^6 + |E_{(5,7)}| p^5 q^7 \\
&+ |E_{(5,5)}| p^5 q^5 + |E_{(5,8)}| p^5 q^8 + |E_{(6,9)}| p^6 q^9 + |E_{(6,6)}| p^6 q^6 \\
&+ |E_{(7,7)}| p^7 q^7 + |E_{(6,7)}| p^6 q^7 + |E_{(7,9)}| p^7 q^9 + |E_{(7,8)}| p^7 q^8 \\
&+ |E_{(8,11)}| p^9 q^{11} + |E_{(8,8)}| p^8 q^8 + |E_{(8,11)}| p^8 q^{11} \\
&= 10p^2q^4 + 2p^3q^8 + 28p^4q^6 + 34p^4q^7 + 11p^4q^9 + 5p^4q^{11} \\
&+ 23p^5q^6 + 27p^5q^7 + 9p^5q^5 + 11p^5q^8 + 8p^6q^9 + 9p^6q^6 + 13p^7q^7 \\
&+ 26p^6q^7 + 17p^7q^8 + 10p^7q^9 + 2p^8q^8 + 2p^8q^{11}.
\end{aligned}$$

Hence, we get the required *NMTCM*-polynomial equation of $(D_g - N)$. □

Now, using this *NMTCM*-polynomial equation of $(D_g - N)$, we have,

Corollary 21. *Let χ_{10} be a molecular graphs of $D_g - N$ then topological co-indices for χ_{10} are given by*

1. $\overline{NM}_1(\chi_{10}) = 2997.$
2. $\overline{NM}_2(\chi_{10}) = 9039.$
3. $\overline{NmM}_2(\chi_{10}) = 8.023.$
4. $\overline{NI}(\chi_{10}) = 719.75.$
5. $\overline{NRZ}_3(\chi_{10}) = 115990.$.
6. $\overline{NSDD}(\chi_{10}) = 539.23.$
7. $\overline{NAZ}(\chi_{10}) = 11324.4.$
8. $\overline{NF}(\chi_{10}) = 19509.$
9. $\overline{NH}(\chi_{10}) = 111.74$

Theorem 22. *The NMTCM-polynomial for $M_e - N$ given by*

$$NMTCM(M_e - N; p, q) = 2p^2q^4 + 4p^3q^4 + 15p^4q^5 + 5p^4q^6 + 13p^5q^5 + 11p^5q^6 + 8p^5q^7 + p^6q^8.$$

Proof. Let χ_{11} be the molecular graph of $M_e - N$ From the figure, it is straightforward to deduce that $|V(\chi_{11})| = 17$ and $|E(\chi_{11})| = 18$. Vertex set can be classified into three classes based on their degrees.
$n_2^* = 1$, $n_3^* = 2$, $n_4^* = 3$,$n_5^* = 6$, $n_6^* = 2$, $n_7^* = 2$,$n_8^* = 1$.
Using definition, we have,

$\overline{m}_{ij}$	$\overline{nm}_{24}$	$\overline{nm}_{34}$	$\overline{nm}_{45}$	$\overline{nm}_{46}$	$\overline{nm}_{55}$	$\overline{nm}_{56}$	$\overline{nm}_{57}$	$\overline{nm}_{66}$	$\overline{nm}_{68}$	$\overline{nm}_{78}$
frequency	2	4	15	5	13	11	8	0	1	0

By definition of *NMTCM* polynomial,

$$\begin{aligned} Let, NM(\chi_{11}, p, q) &= \sum_{i \le j} \overline{nm}_{ij}(G) p^i q^j \\ &= \sum_{2\le 4} \overline{nm}_{24}(\chi_{11}) p^2q^4 + \sum_{3\le 4} \overline{nm}_{34}(\chi_{11}) p^3q^4 + \sum_{4\le 5} \overline{nm}_{45}(\chi_{11}) p^4q^5 \\ &+ \sum_{4\le 6} \overline{nm}_{46}(\chi_{11}) p^4q^6 + \sum_{5\le 5} \overline{nm}_{55}(\chi_{11}) p^5q^5 + \sum_{5\le 6} \overline{nm}_{56}(\chi_{11}) p^5q^6 \\ &+ \sum_{5\le 7} \overline{nm}_{57}(\chi_{11}) p^5q^7 + \sum_{6\le 6} \overline{nm}_{66}(G) p^6q^6 + \sum_{6\le 8} \overline{nm}_{68}(\chi_{11}) p^6q^8 \\ &+ \sum_{7\le 8} \overline{nm}_{78}(\chi_{11}) p^7q^8 + \sum_{6\le 8} \overline{nm}_{68}(\chi_{11}) p^6q^8 + \sum_{7\le 8} \overline{nm}_{78}(\chi_{11}) p^7q^8. \end{aligned}$$

$$\begin{aligned} NM(\chi_{11}, p, q) &= \mid E_{(2,4)} \mid p^2q^4 + \mid E_{(3,4)} \mid p^3q^4 + \mid E_{(4,5)} \mid p^4q^5 + \mid E_{(4,6)} \mid p^4q^6 \\ &+ \mid E_{(5,5)} \mid p^5q^5 + \mid E_{(5,6)} \mid p^5q^6 + \mid E_{(5,7)} \mid p^5q^7 + \mid E_{(6,6)} \mid p^6q^6 \\ &+ \mid E_{(6,8)} \mid p^6q^8 + \mid E_{(7,8)} \mid p^7q^8 \\ &= 2p^2q^4 + 4p^3q^4 + 15p^4q^5 + 5p^4q^6 + 13p^5q^5 + 11p^5q^6 + 8p^5q^7 + p^6q^8 \end{aligned}$$

Hence, we get the required *NMTCM*-polynomial equation of $M_e - N$. □

Now, using this *NMTCM*-polynomial equation of $M_e - N$, we have,

Corollary 23. *Let χ_{11} be a molecular graphs of $M_e - N$ then topological co-indices for χ_{11} are given by*

1. $\overline{NM}_1(\chi_{11}) = 486.$
2. $\overline{NM}_2(\chi_{11}) = 507.$
3. $\overline{NmM}_2(\chi_{11}) = 32.21.$
4. $\overline{NI}(\chi_{11}) = 114.28.$
5. $\overline{NRZ}_3(\chi_{11}) = 2336.$.
6. $\overline{NSDD}(\chi_{11}) = 26632.$
7. $\overline{NAZ}(\chi_{11}) = 886.9.$
8. $\overline{NF}(\chi_{11}) = 1128.$
9. $\overline{NH}(\chi_{11}) = 186.49.$

Theorem 24. *The NMTCM-polynomial for $M_i - E$ given by*

$$\begin{aligned} NMTCM(M_i - E; p,q) = & \, 49p^3q^5 + 10p^3q^6 + 31p^3q^7 + 64p^3q^8 + 17p^4q^8 + 8p^4q^{10} \\ & + 2p^4q^{11} + 9p^5q^5 + 4p^5q^6 + 28p^5q^8 + 9p^5q^9 + 14p^5q^{10} + p^6q^9 \\ & + 16p^7q^8 + 4p^7q^9 + 7p^7q^{10} + 13p^8q^8 + 10p^8q^9 + 15p^8q^{10} \\ & + 4p^8q^{11} + 2p^{10}q^{10} + 2p^{10}q^{11}. \end{aligned}$$

Proof. Let χ_{12} be the molecular graph of $M_i - E$ From the figure, it is straightforward to deduce that $|V(\chi_{12})| = 35$ and $|E(\chi_{12})| = 38$. Vertex set can be classified into three classes based on their degrees.
$n_3^* = 11$, $n_4^* = 3$, $n_5^* = 5$, $n_6^* = 1$, $n_7^* = 3$, $n_8^* = 6$, $n_9^* = 2$, $n_{10}^* = 3$, $n_{11}^* = 1$.
Using definition, we have,

$\overline{m}_{ij}$	$\overline{m}_{(35)}$	$\overline{m}_{36}$	$\overline{m}_{37}$	$\overline{m}_{38}$	$\overline{m}_{48}$	$\overline{m}_{4(10)}$	$\overline{m}_{4(11)}$	$\overline{m}_{55}$	$\overline{m}_{56}$	$\overline{m}_{58}$	$\overline{m}_{59}$
frequency	49	10	31	64	17	8	2	9	4	28	9

$\overline{m}_{ij}$	$\overline{m}_{5(10))}$	$\overline{m}_{69}$	$\overline{m}_{78}$	$\overline{m}_{79}$	$\overline{m}_{7(10)}$	$\overline{m}_{88}$	$\overline{m}_{89}$	$\overline{m}_{8(10)}$	$\overline{m}_{8(11)}$	$\overline{m}_{10(10)}$	$\overline{m}_{10(11)}$
frequency	14	1	16	4	7	13	10	15	4	2	2

By definition of *NMTCM* polynomial,

$$\begin{aligned} Let, NM(\chi_{12}, p, q) &= \sum_{i \leq j} \overline{nm}_{ij}(G) p^i q^j \\ &= \sum_{3 \leq 5} \overline{nm}_{35}(\chi_{12}) p^3 q^5 + \sum_{3 \leq 6} \overline{nm}_{36}(\chi_{12}) p^3 q^6 + \sum_{3 \leq 7} \overline{nm}_{36}(\chi_{12}) p^3 q^7 \end{aligned}$$

$$
\begin{aligned}
&+\sum_{3\leq 8}\overline{nm}_{38}(\chi_{12})p^3q^8+\sum_{4\leq 8}\overline{nm}_{48}(\chi_{12})p^4q^8+\sum_{4\leq 10}\overline{nm}_{4(10)}(\chi_{12})p^4q^{10}\\
&+\sum_{4\leq 11}\overline{nm}_{4(11)}(\chi_{12})p^4q^{11}+\sum_{5\leq 5}\overline{nm}_{55}(G)p^5q^5+\sum_{5\leq 6}\overline{nm}_{56}(\chi_{12})p^5q^6\\
&+\sum_{5\leq 8}\overline{nm}_{58}(\chi_{12})p^5q^8+\sum_{5\leq 9}\overline{nm}_{59}(\chi_{12})p^5q^9+\sum_{5\leq 10}\overline{nm}_{5(10)}(\chi_{12})p^5q^{10}\\
&+\sum_{6\leq 9}\overline{nm}_{69}(\chi_{12})p^6q^9+\sum_{7\leq 8}\overline{nm}_{78}(\chi_{12})p^7q^8+\sum_{7\leq 9}\overline{nm}_{79}(\chi_{12})p^7q^9\\
&+\sum_{7\leq 10}\overline{nm}_{7(10)}(\chi_{12})p^7q^{10}+\sum_{8\leq 8}\overline{nm}_{88}(\chi_{12})p^8q^8+\sum_{8\leq 9}\overline{nm}_{89}(\chi_{12})p^8q^9\\
&+\sum_{8\leq 10}\overline{nm}_{8(10)}(\chi_{12})p^8q^{10}+\sum_{8\leq 11}\overline{nm}_{8(11)}(\chi_{12})p^8q^{11}\\
&+\sum_{10\leq 10}\overline{nm}_{10(10)}(\chi_{12})p^{10}q^{10}+\sum_{10\leq 11}\overline{nm}_{10(11)}(\chi_{12})p^{10}q^{11}.\\
&=|E_{(3,5)}|p^3q^5+|E_{(3,6)}|p^3q^6+|E_{(3,7)}|p^3q^7+|E_{(3,8)}|p^4q^8\\
&+|E_{(4,10)}|p^4q^{11}+|E_{(5,5)}|p^5q^6+|E_{(5,8)}|p^5q^8+|E_{(5,9)}|p^5q^9\\
&+|E_{(5,10)}|p^5q^{10}+|E_{(6,9)}|p^6q^9+|E_{(7,8)}|p^7q^8\\
&+|E_{(7,9)}|p^7q^9+|E_{(7,10)}|p^7q^{10}+|E_{(8,8)}|p^8q^9\\
&+|E_{(8,10)}|p^8q^{10}+|E_{(8,11)}|p^8q^{11}+|E_{(10,10)}|p^{10}q^{10}+|E_{(10,11)}|p^{10}q^{11}\\
&=49p^3q^5+10p^3q^6+31p^3q^7+64p^3q^8+17p^4q^8+8p^4q^{10}\\
&+2p^4q^{11}+9p^5q^5+4p^5q^6+28p^5q^8+9p^5q^9+14p^5q^{10}+p^6q^9\\
&+16p^7q^8+4p^7q^9+7p^7q^{10}+13p^8q^8+10p^8q^9+15p^8q^{10}\\
&+4p^8q^{11}+2p^{10}q^{10}+2p^{10}q^{11}.
\end{aligned}
$$

Hence, we get the required $NMTCM$-polynomial equation of $M_i - E$. □

Now, using this $NMTCM$-polynomial equation of $M_i - E$, we have,

Corollary 25. *Let χ_{12} be a molecular graphs of $M_i - E$ then topological co-indices for χ_{12} are given by*

1. $\overline{NM}_1(\chi_{12}) = 3920$.
2. $\overline{NM}_2(\chi_{12}) = 11840$.
3. $\overline{NmM}_2(\chi_{12}) = 11.439$.
4. $\overline{NI}(\chi_{12}) = 893.67$.
5. $\overline{NRZ}_3(\chi_{12}) = 166606.$.
6. $\overline{NSDD}(\chi_{12}) = 783.74$.
7. $\overline{NAZ}(\chi_{12}) = 14570$.
8. $\overline{NF}(\chi_{12}) = 28872$.
9. $\overline{NH}(\chi_{12}) = 55.34$.

Theorem 26. *The NMTCM-polynomial for $P_f - A$ given by*

$$NMTCM(P_f - A; p, q) = 22p^3q^6 + 8p^3q^7 + 9p^6q^7 + 7p^6q^8 + 7p^6q^6 + 3p^7q^8.$$

Proof. Let χ_{13} be the molecular graph of $P_f - A$ From the figure, it is straightforward to deduce that $|V(\chi_{13})| = 17$ and $|E(\chi_{13})| = 18$. Vertex set can be classified into three classes based on their degrees.
$n_2^* = 1$, $n_3^* = 5$, $n_4^* = 1$, $n_5^* = 1$, $n_6^* = 5$, $n_7^* = 2r$.
Using definition, we have,

$\overline{nm}_{ij}$	$\overline{nm}_{24}$	$\overline{nm}_{36}$	$\overline{nm}_{37}$	$\overline{nm}_{45}$	$\overline{nm}_{57}$	$\overline{nm}_{67}$	$\overline{nm}_{68}$	$\overline{nm}_{66}$	$\overline{nm}_{78}$	$\overline{nm}_{88}$
frequency	0	22	8	0	0	9	7	7	3	0

By definition of *NMTCM* polynomial,

$$\begin{aligned}
Let, NM(\chi_{13}, p, q) &= \sum_{i\leq j} \overline{nm}_{ij}(G)p^i q^j \\
&= \sum_{2\leq 4} \overline{nm}_{24}(\chi_{13})p^2q^4 + \sum_{3\leq 6} \overline{nm}_{36}\chi_{13}p^3q^6 + \sum_{3\leq 7} m_{37}(\chi_{13})p^3q^7 \\
&+ \sum_{4\leq 5} \overline{nm}_{45}\chi_{13}p^4q^5 + \sum_{5\leq 7} \overline{nm}_{57}\chi_{13}p^5q^7 + \sum_{6\leq 7} m_{67}(\chi_{13})p^6q^7 \\
&+ \sum_{6\leq 8} \overline{nm}_{68}\chi_{13}p^6q^8 + \sum_{6\leq 6} \overline{nm}_{66}\chi_{13}p^6q^6 + \sum_{7\leq 8} m_{78}(\chi_{13})p^7q^8 \\
&+ \sum_{8\leq 8} \overline{nm}_{88}\chi_{13}p^8q^8. \\
&= | E_{(2,4)} | p^2q^4 + | E_{(3,6)} | p^3q^6 + | E_{(3,7)} | p^3q^7 + | E_{(4,5)} | p^4q^5 \\
&+ | E_{(5,7)} | p^5q^7 + | E_{(3,6)} | p^3q^6 + | E_{(6,7)} | p^6q^7 + | E_{(6,8)} | p^6q^8 \\
&+ | E_{(6,6)} | p^6q^6 + | E_{(7,8)} | p^7q^8 + | E_{(8,8)} | p^8q^8 \\
&= 22p^3q^6 + 8p^3q^7 + 9p^6q^7 + 7p^6q^8 + 7p^6q^6 + 3p^7q^8.
\end{aligned}$$

Hence, we get the required *NMTCM*-polynomial equation of $P_f - A$. □

Now, using this *NMTCM*-polynomial equation of $P_f - A$, we have,

Corollary 27. *Let χ_{13} be a molecular graphs of $P_f - A$ then topological co-indices for χ_{13} are given by*

1. $\overline{NM}_1(\chi_{13}) = 622$.
2. $\overline{NM}_2(\chi_{13}) = 1698$.
3. $\overline{NmM}_2(\chi_{13}) = 2.193$.
4. $\overline{NI}(\chi_{13}) = 146.07$
5. $\overline{NRZ}_3(\chi_{13}) = 20406$.
6. $\overline{NSDD}(\chi_{13}) = 129.93$.
7. $\overline{NAZ}(\chi_{13}) = 2029.11$.
8. $\overline{NF}(\chi_{13}) = 3762$.
9. $\overline{NH}(\chi_{13}) = 10.42$.

Theorem 28. *The NMTCM-polynomial for $P_p - N$ given by*

$$\begin{aligned}
NMTCM(P_p - N; p, q) &= 15p^5q^8 + 16p^6q^7 + 18p^6q^8 + 9p^6q^9 + 12p^7q^8 + 7p^7q^9 \\
&+ 5p^8q^8 + 6p^8q^9 + 15p^2q^4 + 9p^3q^6 + 7p^3q^7 + 14p^4q^4 \\
&+ 20p^4q^5 + 28p^4q^6 + 22p^4q^7 + 23p^4q^8 + 5p^5q^5 + 19p^5q^6.
\end{aligned}$$

Proof. Let χ_{14} be the molecular graph of $P_p - N$ From the figure, it is straightforward to deduce that $|V(\chi_{14})| = 30$ and $|E(\chi_{14})| = 34$. Vertex set can be classified into eight classes based on their degrees.
$n_2^* = 3$, $n_3^* = 2$, $n_4^* = 6$,$n_5^* = 4$, $n_6^* = 5$, $n_7^* = 4$,$n_8^* = 4$, $n_9^* = 2$.
Using definition, we have,

$\overline{nm}_{ij}$	$\overline{nm}_{(58)}$	$\overline{nm}_{67}$	$\overline{nm}_{68}$	$\overline{nm}_{69}$	$\overline{nm}_{78}$	$\overline{nm}_{(79)}$	$\overline{nm}_{88}$	$\overline{nm}_{89}$	$\overline{nm}_{99}$	$\overline{nm}_{24}$
frequency	15	16	18	9	12	7	5	6	0	15

$\overline{nm}_{ij}$	$\overline{nm}_{36}$	$\overline{nm}_{37}$	$\overline{nm}_{44}$	$\overline{nm}_{45}$	$\overline{nm}_{(46)}$	$\overline{nm}_{47}$	$\overline{nm}_{48}$	$\overline{nm}_{55}$	$\overline{nm}_{56}$
frequency	9	7	14	20	28	22	23	5	19

By definition of *NMTCM* polynomial,

$$
\begin{aligned}
Let, NM(\chi_{14}, p, q) &= \sum_{i\leq j} \overline{nm}_{ij}(G) p^i q^j \\
&= \sum_{5\leq 8} \overline{nm}_{58}(\chi_{14}) p^5 q^8 + \sum_{6\leq 7} \overline{nm}_{67} \chi_{14} p^6 q^7 + \sum_{6\leq 8} m_{68}(\chi_{14}) p^6 q^8 \\
&+ \sum_{6\leq 9} \overline{nm}_{69}(\chi_{14}) p^6 q^9 + \sum_{7\leq 8} \overline{nm}_{78} \chi_{12} p^7 q^8 + \sum_{7\leq 9} \overline{nm}_{79}(\chi_{14}) p^7 q^9. \\
&+ \sum_{8\leq 8} \overline{nm}_{88}(\chi_{14}) p^8 q^8 + \sum_{8\leq 9} \overline{nm}_{89} \chi_{12} p^8 q^9 + \sum_{9\leq 9} \overline{nm}_{99}(\chi_{14}) p^9 q^9. \\
&+ \sum_{2\leq 4} \overline{nm}_{24}(\chi_{14}) p^2 q^4 + \sum_{3\leq 6} \overline{nm}_{36} \chi_{12} p^3 q^6 + \sum_{3\leq 7} \overline{nm}_{37}(\chi_{14}) p^3 q^7. \\
&+ \sum_{4\leq 4} \overline{nm}_{44}(\chi_{14}) p^4 q^4 + \sum_{4\leq 5} \overline{nm}_{45} \chi_{12} p^4 q^5 + \sum_{4\leq 6} \overline{nm}_{46}(\chi_{14}) p^4 q^6. \\
&+ \sum_{4\leq 7} \overline{nm}_{47}(\chi_{14}) p^4 q^7 + \sum_{4\leq 8} \overline{nm}_{48} \chi_{12} p^4 q^8 + \sum_{5\leq 5} \overline{nm}_{55}(\chi_{14}) p^5 q^5. \\
&+ \sum_{5\leq 6} \overline{nm}_{56}(\chi_{14}) p^5 q^6 \\
&= |E_{(5,8)}| p^5 q^8 + |E_{(6,7)}| p^6 q^7 + |E_{(6,8)}| p^6 q^8 + |E_{(6,9)}| p^6 q^9 \\
&+ |E_{(7,8)}| p^7 q^8 + |E_{(7,9)}| p^7 q^9 + |E_{(8,8)}| p^8 q^8 + |E_{(8,9)}| p^8 q^9 + |E_{(9,9)}| p^9 q^9 \\
&+ |E_{(2,4)}| p^2 q^4 + |E_{(3,6)}| p^3 q^6 + |E_{(3,7)}| p^3 q^7 + |E_{(4,4)}| p^4 q^4 \\
&+ |E_{(4,5)}| p^4 q^5 + |E_{(4,6)}| p^4 q^6 + |E_{(4,7)}| p^4 q^7 + |E_{(4,8)}| p^4 q^8 \\
&+ |E_{(5,5)}| p^5 q^5 + |E_{(5,6)}| p^5 q^6 \\
&= 15p^5q^8 + 16p^6q^7 + 18p^6q^8 + 9p^6q^9 + 12p^7q^8 + 7p^7q^9 \\
&+ 5p^8q^8 + 6p^8q^9 + 15p^2q^4 + 9p^3q^6 + 7p^3q^7 + 14p^4q^4 \\
&+ 20p^4q^5 + 28p^4q^6 + 22p^4q^7 + 23p^4q^8 + 5p^5q^5 + 19p^5q^6.
\end{aligned}
$$

Hence, we get the required *NMTCM*-polynomial equation of $P_p - N$. □

Now, using this *NMTCM*-polynomial equation of $P_p - N$, we have,

Corollary 29. *Let χ_{14} be a molecular graphs of $P_p - N$ then topological co-indices for χ_{14} are given by*

1. $\overline{NM}_1(\chi_{14}) = 2846.$
2. $\overline{NM}_2(\chi_{14}) = 8259.$
3. $\overline{NmM}_2(\chi_{14}) = 9.839.$
4. $\overline{NI}(\chi_{14}) = 683.7$
5. $\overline{NRZ}_3(\chi_{14}) = 104410.$
6. $\overline{NSDD}(\chi_{14}) = 549.65.$
7. $\overline{NAZ}(\chi_{14}) = 10281.8.$
8. $\overline{NF}(\chi_{14}) = 17838.$
9. $\overline{NH}(\chi_{14}) = 46.55.$

Theorem 30. *The NMTCM-polynomial for $P_t - B$ given by*

$$NMTCM(P_t-B;p,q) = 5p^2q^4+11p^3q^4+13p^4q^4+20p^4q^5+29p^4q^6+10p^4q^7 + 9p^3q^6+18p^5q^6+7p^5q^7+9p^6q^6+8p^6q^7+11p^6q^8 + 11p^5q^8+2p^8q^8+p^8q^9+p^7q^9.$$

Proof. Let χ_{15} be the molecular graph of $P_t - B$ From the figure, it is straightforward to deduce that $|V(\chi_{15})| = 24$ and $|E(\chi_{15})| = 27$. Vertex set can be classified into eight classes based on their degrees.
$n_2^* = 1$, $n_3^* = 2$, $n_4^* = 6$,$n_5^* = 4$, $n_6^* = 5$, $n_7^* = 2$, $n_8^* = 3$, $n_9^* = 1$.
Using definition, we have,

$\overline{nm}_{ij}$	$\overline{nm}_{(24)}$	$\overline{nm}_{34}$	$\overline{nm}_{44}$	$\overline{nm}_{45}$	$\overline{nm}_{(46)}$	$\overline{nm}_{47}$	$\overline{nm}_{36}$
frequency	5	11	13	20	29	10	9

$\overline{nm}_{ij}$	$\overline{nm}_{56}$	$\overline{nm}_{57}$	$\overline{nm}_{66}$	$\overline{nm}_{67}$	$\overline{nm}_{(68)}$	$\overline{nm}_{58}$	$\overline{nm}_{88}$	$\overline{nm}_{89}$	$\overline{nm}_{79}$
frequency	18	7	9	8	11	11	2	1	1

By definition of *NMTCM* polynomial,

$$\begin{aligned}
Let, NM(\chi_{15},p,q) &= \sum_{i\le j} \overline{nm}_{ij}(G)p^iq^j \\
&= \sum_{2\le 4} \overline{nm}_{24}(\chi_{15})p^2q^4 + \sum_{3\le 4} \overline{nm}_{34}\chi_{15}p^3q^4 + \sum_{4\le 4} \overline{nm}_{44}(\chi_{15})p^4q^4 \\
&+ \sum_{4\le 5} \overline{nm}_{45}(\chi_{15})p^4q^5 + \sum_{4\le 6} \overline{nm}_{46}\chi_{15}p^4q^6 + \sum_{4\le 7} \overline{nm}_{47}\chi_{15}p^4q^7 \\
&+ \sum_{3\le 6} \overline{nm}_{36}(\chi_{15})p^3q^6 + \sum_{5\le 6} \overline{nm}_{56}\chi_{15}p^5q^6 + \sum_{5\le 7} \overline{nm}_{57}\chi_{15}p^5q^7 \\
&+ \sum_{6\le 6} \overline{nm}_{66}(\chi_{15})p^6q^6 + \sum_{6\le 7} \overline{nm}_{67}\chi_{15}p^6q^7 + \sum_{6\le 8} \overline{nm}_{68}\chi_{15}p^6q^8 \\
&+ \sum_{5\le 8} \overline{nm}_{58}(\chi_{15})p^5q^8 + \sum_{8\le 8} \overline{nm}_{88}\chi_{15}p^8q^8 + \sum_{8\le 9} \overline{nm}_{89}\chi_{15}p^8q^9. \\
&+ \sum_{7\le 9} m_{79}(\chi_{15})p^7q^9 \\
&= |E_{(2,4)}|p^2q^4 + |E_{(3,4)}|p^3q^4 + |E_{(4,4)}|p^4q^4 + |E_{(4,5)}|p^4q^5 \\
&+ |E_{(4,6)}|p^4q^6 + |E_{(4,7)}|p^4q^7 + |E_{(3,6)}|p^3q^6 + |E_{(5,6)}|p^5q^6 \\
&+ |E_{(5,7)}|p^5q^7 + |E_{(6,6)}|p^6q^6 + |E_{(6,7)}|p^6q^7 + |E_{(6,8)}|p^6q^8 \\
&+ |E_{(5,8)}|p^5q^8 + |E_{(8,8)}|p^8q^8 + |E_{(8,9)}|p^8q^9 + |E_{(7,9)}|p^7q^9 \\
&= 5p^2q^4+11p^3q^4+13p^4q^4+20p^4q^5+29p^4q^6+10p^4q^7+9p^3q^6 \\
&+ 18p^5q^6+7p^5q^7+9p^6q^6+8p^6q^7+11p^6q^8+11p^5q^8+2p^8q^8+1p^8q^9+p^7q^9.
\end{aligned}$$

Hence, we get the required *NMTCM*-polynomial equation of $P_t - B$. □

Now, using this *NMTCM*-polynomial equation of $P_t - B$, we have,

Corollary 31. *Let χ_{15} be a molecular graphs of $P_t - B$ then topological co-indices for χ_{15} are given by*

1. $\overline{NM}_1(\chi_{15}) = 1728.$
2. $\overline{NM}_2(\chi_{15}) = 4594.$
3. $\overline{NmM}_2(\chi_{15}) = 7.191.$
4. $\overline{NI}(\chi_{15}) = 429.6.$
5. $\overline{NRZ}_3(\chi_{15}) = 52454.$
6. $\overline{NSDD}(\chi_{15}) = 351.97.$
7. $\overline{NAZ}(\chi_{15}) = 5586.15.$
8. $\overline{NF}(\chi_{15}) = 9728.$
9. $\overline{NH}(\chi_{15}) = 33.005.$

Theorem 32. *The TCONM-polynomial for $R_f - N$ given by*

$$NMTCM(R_f - N; p,q) = 14p^3q^4 + 44p^3q^5 + 14p^3q^6 + 8p^4q^4 + 69p^4q^5 + 99p^5q^5 + 68p^5q^6 + 41p^5q^7 + 12p^6q^7 + 9p^6q^8 + 4p^6q^9 + 4p^7q^8.$$

Proof. Let χ_{16} be the molecular graph of $R_f - N$ From the figure, it is straightforward to deduce that $|V(\chi_{16})| = 34$ and $|E(\chi_{16})| = 38$. Vertex set can be classified into seven classes based on their degrees.
$n_3^* = 3$, $n_4^* = 5$, $n_5^* = 15$,$n_6^* = 5$, $n_7^* = 3$, $n_8^* = 2$,$n_9^* = 1$.
Using definition, we have,

$\overline{nm}_{ij}$	$\overline{nm}_{(34)}$	$\overline{nm}_{35}$	$\overline{nm}_{36}$	$\overline{nm}_{44}$	$\overline{nm}_{(45)}$	$\overline{nm}_{55}$	$\overline{nm}_{56}$
frequency	14	44	14	8	69	99	68

$\overline{nm}_{ij}$	$\overline{nm}_{57}$	$\overline{nm}_{58}$	$\overline{nm}_{67}$	$\overline{nm}_{68}$	$\overline{nm}_{(69)}$	$\overline{nm}_{78}$	$\overline{nm}_{89}$
frequency	41	29	12	9	4	4	4

By definition of *TCONM* polynomial,

$$\begin{aligned} Let, NM(\chi_{16},p,q) &= \sum_{i\le j} \overline{nm}_{ij}(G)p^iq^j \\ &= \sum_{3\le 4} \overline{nm}_{34}\chi_{16}p^3q^4 + \sum_{3\le 5} \overline{nm}_{35}(\chi_{16})p^3q^5 + \sum_{3\le 6} m_{36}(\chi_{16})p^3q^6 \\ &+ \sum_{4\le 4} \overline{nm}_{44}\chi_{16}p^4q^4 + \sum_{4\le 5} \overline{nm}_{45}(\chi_{16})p^4q^5 + \sum_{5\le 5} m_{55}(\chi_{16})p^5q^5 \\ &+ \sum_{5\le 6} \overline{nm}_{56}\chi_{16}p^5q^6 + \sum_{5\le 7} \overline{nm}_{57}(\chi_{16})p^5q^7 + \sum_{5\le 8} \overline{nm}_{58}(\chi_{16})p^5q^8 \\ &+ \sum_{6\le 7} \overline{nm}_{67}\chi_{16}p^6q^7 + \sum_{6\le 8} \overline{nm}_{68}(\chi_{16})p^6q^8 + \sum_{6\le 9} \overline{nm}_{69}(\chi_{16})p^6q^9 \\ &+ \sum_{7\le 8} \overline{nm}_{78}(\chi_{16})p^7q^8 + \sum_{8\le 9} \overline{nm}_{89}(\chi_{16})p^8q^9 \end{aligned}$$

$$\begin{aligned} NM(\chi_{16},p,q) &= |E_{(3,4)}|\ p^3q^4 + |E_{(3,5)}|\ p^3q^5 + |E_{(3,6)}|\ p^3q^6 + |E_{(4,4)}|\ p^4q^4 \\ &+ |E_{(4,5)}|\ p^4q^5 + |E_{(5,5)}|\ p^5q^5 + |E_{(5,6)}|\ p^5q^6 + |E_{(5,7)}|\ p^5q^7 \\ &+ |E_{(5,8)}|\ p^5q^8 + |E_{(6,7)}|\ p^6q^7 + |E_{(6,8)}|\ p^6q^8 + |E_{(6,9)}|\ p^6q^9 \\ &+ |E_{(7,8)}|\ p^7q^8 + |E_{(8,9)}|\ p^8q^9 \\ &= 14p^3q^4 + 44p^3q^5 + 14p^3q^6 + 8p^4q^4 + 69p^4q^5 + 99p^5q^5 + 68p^5q^6 + 41p^5q^7 \\ &+ 12p^6q^7 + 9p^6q^8 + 4p^6q^9 + 4p^7q^8. \end{aligned}$$

Hence, we get the required $NMTCM$-polynomial equation of $R_f - N$. □

Now, using this $NMTCM$-polynomial equation of $R_f - N$, we have,

Corollary 33. *Let χ_{16} be a molecular graphs of $R_f - N$ then topological co-indices for χ_{16} are given by*

1. $\overline{NM}_1(\chi_{16}) = 4720.$
2. $\overline{NM}_2(\chi_{16}) = 11074.$
3. $\overline{NmM}_2(\chi_{16}) = 17.542.$
4. $\overline{NI}(\chi_{16}) = 1044.53.$
5. $\overline{NRZ}_3(\chi_{16}) = 120858.$
6. $\overline{NSDD}(\chi_{16}) = 868.56.$
7. $\overline{NAZ}(\chi_{16}) = 13387.3.$
8. $\overline{NF}(\chi_{16}) = 23114.$
9. $\overline{NH}(\chi_{16}) = 83.076.$

Theorem 34. *The NMTCM-polynomial for $T_j - K$ given by*

$$NMTCM(T_j - K; p, q) = 5p^2q^4 + 10p^3q^4 + 13p^4q^4 + 19p^4q^5 + 4p^6q^7 + 4p^7q^7 + 13p^7q^5 + 23p^7q^4.$$

Proof. Let χ_{17} be the molecular graph of $T_j - K$ From the figure, it is straightforward to deduce that $|V(\chi_{17})| = 19$ and $|E(\chi_{17})| = 20$. Vertex set can be classified into six classes based on their degrees.
$n_2^* = 1$, $n_3^* = 2$, $n_4^* = 6$, $n_5^* = 4$, $n_6^* = 2$, $n_7^* = 4$.
Using definition, we have,

$\overline{nm}_{ij}$	$\overline{nm}_{24}$	$\overline{nm}_{34}$	$\overline{nm}_{44}$	$\overline{nm}_{45}$	$\overline{nm}_{67}$	$\overline{nm}_{77}$	$\overline{nm}_{75}$	$\overline{nm}_{74}$
frequency	5	10	13	19	4	4	13	23

By definition of $NMTCM$ polynomial,

$$\begin{aligned} Let, NM(\chi_{17}, p, q) &= \sum_{i \leq j} \overline{nm}_{ij}(G) p^i q^j \\ &= \sum_{2 \leq 4} \overline{nm}_{24} \chi_{17} p^2 q^4 + \sum_{3 \leq 4} \overline{nm}_{34} \chi_{17} p^3 q^4 + \sum_{4 \leq 4} \overline{nm} 44(\chi_{17}) p^4 q^4 \\ &+ \sum_{4 \leq 5} \overline{nm}_{45}(\chi_{17}) p^4 q^5 + \sum_{6 \leq 7} \overline{nm}_{67} \chi_{17} p^6 q^7 + \sum_{7 \leq 7} \overline{nm}_{77} \chi_{17} p^7 q^7 \\ &+ \sum_{7 \leq 5} \overline{nm}_{75} \chi_{17} p^7 q^5 \sum_{7 \leq 4} \overline{nm}_{74} \chi_{17} p^7 q^4 \\ &= | E_{(2,4)} | p^2 q^4 + | E_{(3,4)} | p^3 q^4 + | E_{(4,4)} | p^4 q^4 + | E_{(4,5)} | p^4 q^5 \\ &+ | E_{(6,7)} | p^6 q^7 + | E_{(7,7)} | p^7 q^7 + | E_{(7,5)} | p^7 q^5 + | E_{(7,4)} | p^7 q^4 \\ &= 5p^2q^4 + 10p^3q^4 + 13p^4q^4 + 19p^4q^5 + 4p^6q^7 + 4p^7q^7 + 13p^7q^5 + 23p^7q^4 \end{aligned}$$

Hence, we get the required $NMTCM$-polynomial equation of $T_j - K$. □

Now, using this $NMTCM$-polynomial equation of $T_j - K$, we have,

Corollary 35. *Let χ_{17} be a molecular graphs of $T_j - K$ then topological co-indices for χ_{17} are given by*

1. $\overline{NM}_1(\chi_{17}) = 912.$
2. $\overline{NM}_2(\chi_{17}) = 2211.$
3. $\overline{NmM}_2(\chi_{17}) = 4.584.$
4. $\overline{NI}(\chi_{17}) = 215.39.$
5. $\overline{NRZ}_3(\chi_{17}) = 23636.$
6. $\overline{NSDD}(\chi_{17}) = 195.248.$
7. $\overline{NAZ}(\chi_{17}) = 2604.83.$
8. $\overline{NF}(\chi_{17}) = 4734.$
9. $\overline{NH}(\chi_{17}) = 19.513.$

CONCLUSION

Introducing the *NMTCM* polynomial, this article highlights its capability to generate closed forms for diverse degree-based topological coindices, effectively predicting physiochemical properties. The investigation seeks to advance our understanding of developing innovative medications, specifically for anticancer treatment. The computed topological coindices demonstrate considerable predictive capabilities regarding the characteristics of potential anticancer drugs. This investigation strives to pinpoint the most impactful predictive topological coindices associated with the *NMTCM* polynomial. The findings aim to offer valuable insights into the development of cutting-edge medications, specifically those tailored for anticancer therapeutic purposes.

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